

Capital Income Taxation and Resource Allocation

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Chapter 3: Taxes and the Decision Problem of the Firm

Chapter 3

TAXES AND THE DECISION PROBLEM OF THE FIRM

After examining the intertemporal allocation mechanism in the context of household and firm decisions, we can now move on to an analysis of the role of taxation. This chapter gives an overview of alternative tax systems and studies the way in which they enter the decision problem of the firm. Except for decisions about optimal labor supply, the behavioral implications of taxation are not yet treated. Their analysis is postponed until the following Chapters 4 and 5.

3.1. An Overview of Tax Systems

The task of this Section is to define the theoretical tax systems to be analyzed, drawing on systems that exist in the OECD countries or have come under political consideration. Attention will be focussed primarily on alternative systems of capital income taxation that are characterized by different degrees of integration of personal income taxation and corporate taxation and by different depreciation rules. However, in a rudimentary form, other taxes are also included in the analysis.

Without substantial idealizations, the formulation of an appropriate model framework cannot be achieved. Thus the idea of the representative household and the representative firm will always be employed, and difficulties like, for example, non-proportional taxes¹ are neglected. A tax is

¹The assumption of proportional taxes is not as restrictive as it might appear at first glance. At any rate it is not more restrictive than the assumption of linear local approximations of tax functions with the aid of which it is even possible to represent certain elements of progressive taxes. Linear taxes differ from proportional taxes only by the additional consideration of absolute terms in the tax functions. These absolute terms can be interpreted as additional lump sum taxes that, by themselves, do not induce substitution effects. If they have any effect at all, it may be to induce income effects which, however, are excluded from this analysis (compare Introduction). This implies that the tax rates used here should normally be compared to *marginal* rather than *average* tax rates in real tax systems. Only in the context of share

always described by a tax rate and a tax base. The tax rate will be indicated by the Greek letter " τ " amended by a suitable subscript. It will turn out that the algebraic formulations in the later analysis often contain expressions of the kind $1 - \tau$. These expressions are called *tax factors* and are denoted by a " θ " with the same subscript as the corresponding τ .

3.1.1. The Basic Structure of a Simple Tax System

The tax systems of the various countries are complex and differ in many details but they are, nevertheless, closely related. Everywhere there are personal income taxes, corporate income taxes, consumption taxes, and taxes on the stock of capital. In a number of countries there are, moreover, capital gains taxes. Except for corporate income taxes, these taxes are similar in the different countries, and corporate income taxes can be reduced to a small number of different categories.

As a typical representative of the consumption taxes we assume a tax like the European *value-added tax* that traditionally has been a major source of government finance in the Latin countries but during the last two decades has come to dominate throughout the whole of Europe. With regard to the problem of intertemporal allocation, it seems that the value-added tax can be used as a first approximation for a number of specific consumption taxes in the different countries. At any rate, for the sake of this analysis it can be roughly identified with the *retail sales tax* that, judged by the revenue it generates, is the most important among all North American sales taxes.² The tax base is the sales, C , that firms make to private households. Set the net price of the consumption good (and hence the net and gross prices of the investment good) equal to unity. Then the flow of revenue generated at a particular point in time by the value-added tax is

$$T_v = \tau_v C, \quad (3.1)$$

where $\tau_v \geq 0$ indicates the tax rate. In economic terms, the value-added tax can also be identified in the present context with the expenditure tax that

valuation (see Chapters 6.1 and 10) will the assumption of proportionality really be needed. These remarks should not, however, discourage further research on the problem of progressive *marginal* tax rates. Any model that produces the so-called "Miller equilibrium" would have to incorporate this property explicitly. See Chapter 4.3.4 for details.

²See Musgrave and Musgrave (1973, pp. 323–326) for further details. It goes without saying that the similarity of the two taxes is limited and that there are problems for which a careful distinction would be necessary.

was praised by Mill (1865, pp. 488–492), Elster (1913, 1916), Mombert (1916), I. and H. W. Fisher (1942), Kaldor (1955), and many others. The difference is, however, that the expenditure tax is levied directly on the household and is progressive.

The stocks of capital and wealth in the economy are usually burdened with different taxes that are levied on households and firms. Because of the typically small percentage of these taxes in the total tax revenue we forego a very detailed description and content ourselves with a proportional tax on the stock of capital that is levied on firms only.³ With τ_k , $0 \leq \tau_k < f_K - \delta$, as the tax rate, the revenue is therefore

$$T_k = \tau_k K. \quad (3.2)$$

Corporate tax and personal income tax are parts of total income tax and should therefore be considered together. In the OECD countries, both taxes are in principle designed according to the definition of income as an increment of wealth as given by Schanz (1896), Haig (1921), and Simons (1938).⁴ This means, in particular, that true economic depreciation is allowed in calculating income and that debt interest is tax deductible.⁵ The significant deviations from these rules that are common in practice will be taken into account in Sections 3.1.3 and 3.1.5 where a generalized tax system is formulated. This generalized system will also be capable of representing other theoretically interesting tax systems or systems that have been proposed as a reform. For the time being, however, the presentation is solely concerned with highly idealized versions of the income tax systems that now exist in the OECD countries.⁶

The formal presentation of income taxation can use the aggregates of national income accounting. The net national product at market prices in the economy considered here is $f(K, L) - \delta K + T_v$. Accordingly, national income or national product at factor cost is $f(K, L) - \delta K - T_k$. Here, T_k is subtracted because (as for example in West Germany) the tax on the stock of capital is treated as an indirect tax; none of the results to be derived would change if T_k were treated as a direct tax. National income consists of wages (wL), interest income (rD_f) that the firms pay on their stock of debt (D_f), and accounting profits (Π), where the latter can be separated into

³For example, until recently, such a tax existed in West Germany.

⁴For an overview of the history of economic thought in the field of income definitions see Goodé (1977).

⁵A precise definition of the concept of true economic depreciation is given in Chapter 5.3.2.

⁶For obvious reasons the discussion exempts Yugoslavia.

distributed (Π^d) and retained profits ($\Pi - \Pi^d$):⁷

$$f(K, L) - \delta K - T_k \equiv wL + rD_r + \Pi^d + (\Pi - \Pi^d). \quad (3.3)$$

Various tax rates τ_i ; $0 \leq \tau_i < 1$; $i = w, p, d, r, c$, are applied to these income categories, where dividends and retentions are subject to double taxation.

The revenue from direct *taxation of retained profits* is

$$T_r = \tau_r(\Pi - \Pi^d), \quad (3.4)$$

and the revenue from the *corporate tax on dividends* is

$$T_d = \tau_d \Pi^d. \quad (3.5)$$

Again, the terminology refers to corporations rather than non-corporate firms without, however, excluding the latter from the analysis. As will be explained in the next section, the differences between the two types of firm are reflected in the size of the tax rates.

Households pay a *wage tax* amounting to

$$T_w = \tau_w wL \quad (3.6)$$

and a *tax on interest income*:

$$T_i = \tau_p r D_r. \quad (3.7)$$

In reality, there is an additional interest income tax on the returns from government bonds that the households possess. It is convenient, however, to take this tax into account through the assumption that government serves its debt at the net-of-tax market rate of interest $\theta_p r$ and allows in exchange an exemption of its interest payments to the household.

In addition to the corporate tax (3.5), all countries have a *personal tax on dividends* at the same rate τ_p as the tax on interest income [cf. (3.7)]:

$$T_{pd} = \tau_p(\Pi^d - T_d) = \tau_p \theta_d \Pi^d. \quad (3.8)$$

The total revenue resulting from the double taxation of dividends by personal and corporate tax is given by

$$T_d^* \equiv T_d + T_{pd} = \tau_d^* \Pi^d, \quad (3.9)$$

where τ_d^* is the *combined (or overall) tax rate on dividends* that is implicitly defined by the corresponding tax factor

⁷Throughout this book the word "profit" will, unless otherwise qualified, be used in the accounting sense. "Profit" is not "economic profit"; it includes the normal return on equity capital.

$$\theta_d^* \equiv \theta_d \theta_p \quad (3.10)$$

or explicitly by $\tau_d^* \equiv 1 - \theta_d^* = \tau_d + \tau_p - \tau_d \tau_p$.

The next tax considered is a personal *capital gains tax* that applies to the appreciation of company shares at the rate τ_c . To the extent that this appreciation results from retained profits, the capital gains tax is an additional tax on retentions. Admittedly, the taxation of retentions is quite indirect, and the precise relationship between the capital gains tax and the direct tax on retentions described in (3.4) has yet to be worked out. Nevertheless, it will turn out to be useful to define, analogously to (3.10), a *combined (or overall) tax factor for retentions*

$$\theta_r^* \equiv \theta_r \theta_c \quad (3.11)$$

and the corresponding tax rate $\tau_r^* \equiv 1 - \theta_r^* = \tau_r + \tau_c - \tau_r \tau_c$ that reflect the joint effect of the two taxes.

While the capital gains tax is unimportant in most OECD countries – typically there are speculation periods of a year or less beyond which capital gains are fully tax-exempt – it plays quite a significant role in Anglo-Saxon countries. For this reason it deserves consideration here. In the Anglo-Saxon countries, the tax operates on a realization, rather than on an accrual basis, and the tax base is often reduced, but not eliminated, after a speculation period of half a year or more. The analytical difficulties that would arise from an attempt to model these aspects explicitly are bypassed here through the optimistic assumption that the tax rate on realized capital gains can be represented by an equivalent tax rate (τ_c) on accrued capital gains. Because of the speculation periods and the interest advantage of paying only at realization, this equivalent tax rate is lower than the one written down in the tax law.

Let

$$M \equiv mz \quad (3.12)$$

be the market value of company shares with m as the price per share and z as the number of outstanding shares. Moreover, let Q denote the flow of funds resulting from issuing new shares. Then the revenue from the capital gains tax is:

$$T_c = \tau_c z \dot{m} + \tau_c (\dot{z}m - Q) = \tau_c (\dot{M} - Q). \quad (3.13)$$

Here $\tau_c z \dot{m}$ is the revenue from taxing the appreciation of existing shares and $\tau_c (\dot{z}m - Q)$ is the revenue from taxing the capital gains which shareholders get

in the case where the new shares are issued at a price below their market value.⁸

3.1.2. Systems of Capital Income Taxation

With the tax rates τ_p , τ_d , τ_r , and τ_c , from Equations (3.4)–(3.9) and (3.13), a general system of capital income taxation is described. If it is recalled that, by assumption, the tax rates refer to a representative firm and to a representative shareholder of this firm⁹ then some basic types of existing systems of capital income taxation or systems that have been discussed can be represented by this general system.¹⁰ Figure 3.1 illustrates such basic types in that it depicts various ordinal patterns of the tax factors defined above. According to the three elementary types of capital income – retentions, dividends, and interest income – the presentation concentrates on the tax factors θ_r^* , θ_d^* , and θ_p that measure the overall tax burdens on these income categories.

The simplest system is one where there are no capital gains taxes and the marginal tax rates on all three kinds of capital income are equal. It is realized in the sector of non-corporate firms since, with a non-corporate firm, the taxation of profits is determined solely through the personal income tax of the owner of the firm; there is no corporate income tax and wealth increases from an investment of taxed profits are tax exempt. Whether the owner of a non-corporate firm decides to invest an additional dollar of profit in his own firm, to consume it, or to invest it in the capital market, the returns in each case are subject to the same tax rate. Thus it holds that $\theta_r^* = \theta_d^* = \theta_p$ where $\theta_c = \theta_d = 1$.

In the industrial countries of the Western world, however, a significant section of the economy – in the United States, for example, about 85% – is

⁸Cf. the discussion of Equation (3.21) below.

⁹As far as corporations are concerned, the representative shareholder is the shareholder who has the median position with regard to the distribution of personal marginal income tax rates among the set of company shares (not among the set of shareholders). Thus, the representative shareholder is the one who decides on the policy of the firm when there is majority voting. If there are other voting rules, for example with non-corporate firms, the representative shareholder is at any rate the person who ultimately determines the firm's policy. This definition does not exclude the set of shareholders belonging to a company depending in turn on this company's financial policy (clienteles effect).

¹⁰Other categorizations can be found with Mennel (1971), Flämig (1974, pp. 56–63), King (1977, pp. 50–58), and in the information bulletin of the West German Ministry of Finance, for example. (See IFA 1986). Cf. also Musgrave and Musgrave (1973, Chapter 12), Wöhe (1978, Chapters II and III), Boadway (1979, Chapter 13–4), or Fullerton/King (1984).

organized in the form of corporate firms and is thus subject to corporate income taxation. In principle it would not be difficult to design the corporate income tax so that the tax factors are equal for all three types of capital income. It would be necessary for this purpose to attribute all parts of the accounting profit, even the retained profit, to the single shareholders and then to apply their personal income tax rates. This idea was proposed by the Carter Commission (1966). It is also central to the so-called *Teilhabersteuer* suggested by Engels and Stützel (1968), and goes back to Goode (1947) and to Dietzel (1859).¹¹ Up to now, however, no country has managed to struggle through to the *Teilhabersteuer*. The retained profits of the corporate firm, at least, are taxed everywhere without considering the personal aspects of the shareholder. The equality of all three tax factors is nowhere realized.

The systems of capital income taxation that are realized in practice can basically be reduced to three different types.

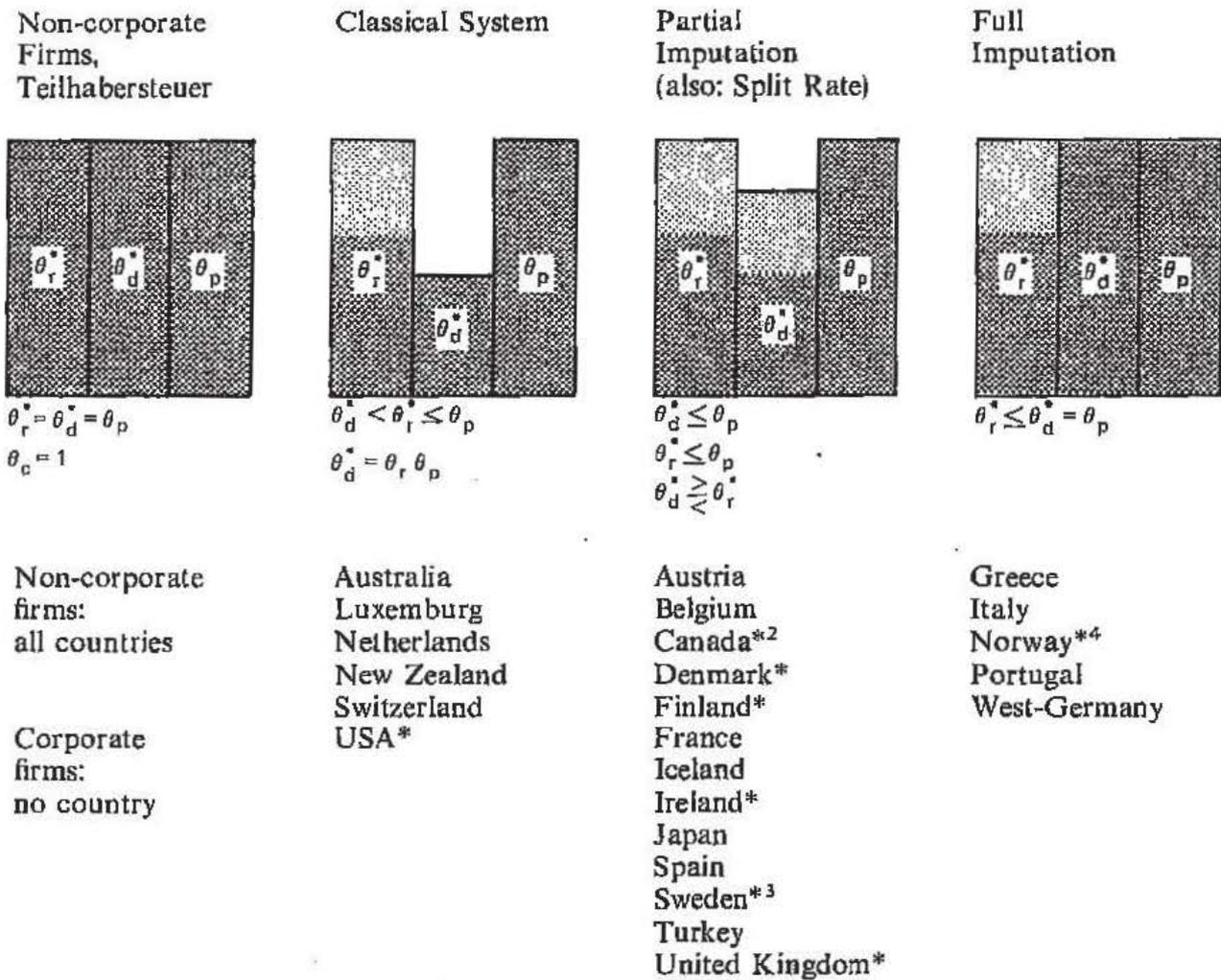
The first type denotes the so-called *classical system* of capital income taxation. All profits, whether retained or distributed, are burdened with a uniform corporate income tax rate. In addition, distributed profits are subject to personal income tax and so too is interest income earned in the capital market. In this system, $\theta_d = \theta_r$ and so the effective tax factor for a unit of distributed profits is $\theta_d^* = \theta_r \theta_p$. Because of the double taxation it falls short of both the tax factor for retained earnings and the tax factor for interest income of households.

The classical system was practised in the United Kingdom, Denmark, Germany, and many other countries, but today it seems a little less fashionable. It only exists in the United States and in a number of smaller countries some of which are listed in Figure 3.1.

The second type includes systems with *partial imputation* and others with a *split corporate tax rate* without imputation of the corporate tax to the personal income tax. The system with a split rate was employed in West Germany from 1953 until 1976. After 1958 the tax rate for retained profits was 51%. Distributed profits were first taxed at an effective corporate tax rate of 23.4% and then, a second time, at the personal income tax rate. Currently, among the countries listed in Figure 3.1, only Austria employs the system with a split rate.

Partial imputation systems are in use in Japan, France, the United

¹¹The verbal translation of "Teilhabersteuer" is "partnership tax" but "integrated tax" might be a translation that better describes its essence. [Goode is cited according to Carter Commission (1966, Chapter 19, p. 94) as I was unable to get the article; the author.]

Figure 3.1. Systems of capital income taxation.¹

*Countries subjecting capital gains realized more than one year after asset purchase to personal taxation

¹The illustrations are to be interpreted in the sense of ordinal relationships between the tax factors. The classification of the countries refers to the year 1986 and is based on information provided by the West German Ministry of Finance. See IFA (1986).

²Beginning in 1986, Canada is phasing in a \$500 000 lifetime exemption for capital gains realized by an individual.

³In Sweden there is a partial imputation system with an upper limit for the imputable distributions. For shareholders whose dividends exceed this upper limit the Swedish system coincides with the classical system with regard to marginal tax rates.

⁴Cf. Footnote 14 in the text.

Kingdom, Canada, Belgium, and in all the other countries listed in Figure 3.1. At first glance, the partial imputation systems seem different from the systems with a split rate but the difference is not substantial. It is true that the firm pays the full corporate tax on its total profit whether retained or distributed. However, a part of the corporate tax paid on dividends – hence

the name "partial imputation system" – can be deducted from personal income tax by the shareholder. Thus, in effect, distributed profits are burdened with a reduced corporate income tax rate as in the system with a split rate. It is easy to show that the difference between the French partial imputation system and the former German system, for example, is simply that the effective corporate tax rate on distributed profits (τ_d) is 22.5% instead of 23.4%.

Like the classical system, the partial imputation systems and the systems with a split corporate tax rate have the property that the overall marginal tax rate on distributed profits exceeds that of the interest income of the representative shareholder, that is, that $\theta_d^* < \theta_p$. A special characteristic of the two latter types of system, however, is that the marginal tax rate on distributed profits does not necessarily exceed the marginal tax rate on retained profits. With a given corporate tax rate, the relationship between these two marginal tax rates depends crucially on the personal marginal income tax rate of the representative shareholder. When the personal tax rate is sufficiently low, the overall tax rate on retained profits will exceed that on distributed profits.

Another type is the so-called *full imputation system* that was introduced in West Germany in 1977 following recommendations by two different councils of economic advisors.¹² In this system, dividends are first taxed within the firm, but the full amount of the tax is rebated to the shareholder household so that ultimately distributions are only subject to personal taxation. The marginal tax burden on dividends is the same in this system as that on interest income the shareholder household earns in the capital market:¹³ $\theta_d^* = \theta_p$, $\theta_d = 1$. The system is also practised in Italy. Norway, Greece, and Portugal have systems where the dividends can be deducted from the corporate tax base. This is more or less the same as in the full imputation system.¹⁴

All three corporate income tax systems can be combined with capital gains taxes on the appreciation of company shares. However, as indicated in Figure 3.1, only countries that have the classical system or the partial

¹²See Wissenschaftlicher Beirat (1967) and Steuerreformkommission (1971).

¹³The equality $\theta_d^* = \theta_p$ presupposes that the shareholders declare their interest income. If interest income is formally taxed but in fact only partially declared – a possibility that unfortunately cannot always be ruled out – then, as in the classical system or with a partial imputation system, it holds that $\theta_p > \theta_d^*$.

¹⁴There are, however, special provisions in Norwegian municipal taxation that can, in particular cases, even result in an overall dividend tax rate below the personal tax rate. See Lund (1986) for details. Here we neglect this exception.

imputation system tax capital gains realized more than one year after asset purchase. This seems consistent in so far as double taxation of distributed profits is complemented by double taxation of retained profits.

Even in countries where all capital gains are taxed, the tax rate is typically less than the tax rate on the interest income of a shareholder household: $\theta_c > \theta_p$. There are two reasons for this. On the one hand, it was agreed to utilize an equivalent capital gains tax rate on accrued capital gains that, as explained, falls short of the rate on realized capital gains which is stated in the tax law. On the other hand, even the legal tax rate is typically lower than the marginal personal income tax rate as realized capital gains are only partially included in the personal tax base when the speculation period has elapsed.

A somewhat peculiar situation prevails in the United States. Here, only 40% of realized capital gains used to be included in the tax base if the realization occurred more than half a year after the shares were purchased. With reference to a study by Bailey (1969), Fullerton et al. (1981, p. 684) have estimated that the equivalent capital gains tax rate of a shareholder was only one quarter of this marginal personal tax rate ($\tau_c \approx 0.25 \tau_p$). However, the 1986 tax reform changed the situation by expanding the speculation period to infinity: from 1988 onwards, realized capital gains will be fully subjected to personal income tax, regardless of the time span between the purchase and sale of the asset. Only the first reason for $\theta_c > \theta_p$ will therefore be operative in the United States. It is not clear precisely how this will affect the equivalent tax rate on accrued capital gains. There are counteracting effects in that, on the one hand, the average period of asset holding will probably fall and, on the other hand, not all capital gains are affected by the doubling of the tax base. Nevertheless, a good guess might be that, relative to the personal tax rate, the equivalent capital gains tax rate will be about two and a half times its pre-reform value; that is, about sixty percent of the personal tax rate: $\tau_d \approx 0.6 \tau_p$.

The classical system is characterized by $\theta_d^* = \theta_r \theta_p$. Thus the preferential treatment of capital gains relative to interest income implies that, in this system, the combined tax factor for retained profits exceeds the combined tax factor for distributed profits: $\theta_r^* \equiv \theta_c \theta_r > \theta_d^*$. With the partial imputation systems, on the other hand, the ambiguity with regard to the relative magnitudes of θ_r and θ_d^* carries over to θ_r^* and θ_d^* . With a high degree of imputation of dividends and a high capital gains tax rate, even the case $\theta_r^* < \theta_d^*$ would be possible here.

Despite all the differences between the various systems of capital income taxation, they have two characteristics in common. These should be stressed

here since they are, as will be shown, quite important for the working of the capital market. The first characteristic is that the marginal personal tax rate of the representative shareholder household lies between the marginal combined tax rate on distributed profits and the marginal tax rate on capital gains:

$$\theta_d^* \leq \theta_p \leq \theta_c. \quad (3.14)$$

The first part of this inequality follows from the fact that, in all systems, dividends are fully subject to personal income tax and, in most systems, are also subject to corporate income tax, either fully or partially.¹⁵ The second part follows from the fact that realized, and not accrued, capital gains are subject to personal income taxation, typically even at less than 100%.

The second characteristic refers to the relative magnitude of the direct and indirect tax burden on retained profits on the one hand and the tax burden on the interest income of the representative shareholder household on the other. While the previous discussion has shown that non-corporate firms are characterized by $\theta_r^* = \theta_p$ the relationship between these tax factors is more ambiguous for corporate firms. In countries where the maximum marginal personal tax rate exceeds the corporate tax rate and where there is no capital gains tax, θ_p can either exceed or fall short of θ_r^* , depending on the tax bracket of the representative shareholder household. There are, however, certain indications that

$$\theta_r^* \leq \theta_p \quad (3.15)$$

(or even $\theta_r^* \approx \theta_p$) characterizes the normal case.

First, in countries like Canada, Denmark, or West Germany, the progression of the personal income tax scale stops at the level of the corporate tax rate or even below. In these countries, the case $\theta_r^* > \theta_p$ can be excluded right away.

Secondly, in about half of the countries – among them the United States, Australia, and a number of European countries – the corporate tax rate on retained profits is more than three quarters of the maximum personal income tax rate.¹⁶ In these countries, too, the case $\theta_r^* > \theta_p$ does not seem to be very plausible for the representative shareholder, that is, for that

¹⁵Cf. the previous footnote.

¹⁶In 1986, 13 out of 24 OECD countries (excluding Yugoslavia) satisfied this criterion. The national product of these 13 countries amounts to more than 60% of aggregate OECD national product. See IFA (1986, Übersicht 1 and 4), ITS (1985), and Weltbank (1985, pp. 202 n.).

shareholder who occupies the median position with regard to the distribution of marginal personal tax rates.

Thirdly, in contrast to the direct check on retained profits of corporations, governments seem to have substantial difficulties in monitoring the interest income earned by households. It is well known that in a number of countries only a small percentage of interest incomes is actually taxed. Thus, even if $\theta_r^* > \theta_p$ were the case when tax payers are honest, $\theta_r^* \leq \theta_p$ could be the case when they are not.¹⁷

Fourthly, the capital gains tax that is levied in the eight countries denoted by an asterisk in Figure 3.1 supports Condition (3.15). Consider for example the United States. With the post-1986 corporate tax rate of $\tau_r = 0.34$ that applies to most firms and the cited value of the equivalent tax rate of $\tau_c \approx 0.6 \tau_p$ on accrued capital gains, it follows from the condition $(1 - \tau_r)(1 - \tau_c) = 1 - \tau_p$ that the critical value for the marginal personal tax rate of the representative shareholder is $\tau_p = \tau_r / [1 - (\theta_r \cdot 0.6)] \approx 0.56$. If the true value of τ_p is on or below this level then Condition (3.15) is satisfied, and this is indeed the case. The U.S. income tax reform of 1986 reduced the maximum marginal rate of federal income tax from 50% to 33%, and state income taxes are normally not more than 12% of income net of the federal tax.¹⁸ These pieces of information together imply that the upper limit of the overall marginal personal tax rate is 41% – a value far below the critical value.¹⁹ The federal rate of 33% only applies to a limited income range though. Most shareholders, including very rich ones, will pay a marginal federal tax rate of 28%. This reduces the upper limit of the sum of the marginal state and federal rates to about 37% which is even further below the critical value.

Before the 1986 tax reform, the situation in the United States was more ambiguous. During this time, the corporate tax rate was 46% for most corporations and so the value $\tau_c = 0.25\tau_p$ calculated by Fullerton et al. implies a critical value for the marginal personal tax rate of $\tau_p = \tau_r / [1 - (\theta_r \cdot 0.25)] \approx 0.54$. With a top federal income tax rate of 50% (70% before 1981), the sum of the marginal state and federal income tax rates in principle could exceed this critical level. On the other hand, in many

¹⁷There is no monitoring problem in countries like the United Kingdom or Sweden since these have source taxes on interest income. However, the tax rates are far below the respective corporate tax rates.

¹⁸See ITS (1985, Section U-28).

¹⁹To be on the safe side this calculation neglects the state corporation franchise taxes and the additional 5% corporate tax charged to phase-out the benefit of graduated rates for small corporations. To include these taxes would mean raising the critical value, and (3.15) would hold for an even stronger reason.

cases the representative shareholder's marginal personal tax rate was certainly below the top rate. As Bradford (1980, pp. 51, 55) reports, the U.S. treasury estimated that the average marginal personal income tax rate of shareholders was 40% in the year 1976. Although the average shareholder cannot necessarily be identified with the representative shareholder, this suggests that there may not have been many firms for which Condition (3.15) was violated.

Despite these arguments, empirical grounds cannot provide sufficient support for Condition (3.15) in the case of each and every firm and country. In Japan, the United Kingdom, France, Italy, Spain, and a number of smaller countries the maximum personal income tax rate exceeds the corporate tax rate sufficiently far to make the case $\theta_r^* > \theta_p$ appear possible or even plausible for many firms. There are however, in addition, two theoretical arguments in favor of Condition (3.15) that will be discussed in Chapters 4.2.2 and 4.3.4. One is Miller's (1977) argument that firms will adjust their debt-equity ratios to the point where, because of the progressivity of marginal personal tax rates, (3.15) holds with equality ($\theta_r^* = \theta_p$). The other is based on the non-existence of an intertemporal general equilibrium with $\theta_r^* > \theta_p$. These theoretical arguments perhaps support Condition (3.15) even more strongly than the empirical evidence.

3.1.3. *The Possibility of Accelerated Tax Depreciation*

With Equations (3.3) and (3.4) it was assumed that, for the calculation of taxable retentions, true economic depreciation of the capital stock is allowed, that is, a depreciation that reflects the true loss in market value (δK) of the capital stock employed. These equations therefore correspond to the Schanz-Haig-Simons definition of income which might have been the ideal that lay behind the construction of the income tax systems of the Western industrial nations.

It should be legitimate, to a first approximation, to use this ideal for an analysis of tax effects in some countries. Perhaps West Germany is such a country for, according to a study by Kopits (1975, p. 33), except for Luxembourg, it seems to be the most restrictive of all OECD countries in its depreciation allowances. However, it should not be overlooked that the attempt to formulate tax depreciation rules in line with true economic depreciation is, in the international context, more the exception than the rule. With the passage of time, many countries have significantly loosened their depreciation rules, and for a while some of them seemed to be no

closer to true economic depreciation than to an immediate write-off.

The United Kingdom and the United States in particular had, and continue to have, quite generous depreciation systems.²⁰ Until 1984 the U.K. allowed a 75% depreciation of industrial buildings in the first year and straight line depreciation thereafter. Plants could even be written off at a rate of 100% in the first year. From 1986 onwards there are no longer any initial depreciation allowances. However, a declining-balance depreciation for plants at the high rate of 25% p.a. remains. Only a little less generous than the previous British depreciation rules were those that the United States introduced in 1981 under the Reagan administration with the so-called Accelerated Cost Recovery System (ACRS). According to this system, the majority of plants could be fully depreciated over as few as five years.²¹ It was estimated that by the year 1986 when the new rules should have been fully operative, there would have been a loss in tax revenue of between \$54 and \$61 billion – figures that have the same order of magnitude as the total U.S. corporate income tax revenue in 1981.²² The expected revenue losses were so dramatic that the Reagan administration took fright and abolished, as a first reaction, the generous “safe-harbor leasing” arrangements for inter-firm loss transfers that had come with the reform and that were being intensively exploited by industry. Shortly after the President was re-elected in 1984, the Treasury even proposed returning to more conservative depreciation rules,²³ and this proposal ultimately resulted in the U.S. tax reform of 1986. According to the new law most plants can be depreciated in about 7 years. This is two years more than under ACRS, but still about three years less than under the Asset Depreciation Range System that preceded ACRS. It is certainly also less than the economic life of most plants.²⁴

²⁰ Other examples with generous depreciation systems include Canada and Australia, where plant is usually eligible for a depreciation period of not more than five years. By contrast, depreciation periods for plant in West Germany try to approximate “economic useful life” which is significantly more than 10 years. Cf. Footnote 24.

²¹ Exceptions were buildings that could be depreciated over 15 years and short-lived assets like automobiles that could be depreciated over three years.

²² See U.S. Joint Committee on Taxation (1981, Table 2, p. 58) and the table “Changes in Fiscal Year Receipts Resulting from the Conference Agreement on H.R. 4242, the Economic Recovery Tax Act of 1981, Office of the Secretary of the Treasury, Office of Tax Analysis, August 3, 1981” that was issued by the the U.S. Department of the Treasury. For the American corporate income tax revenue compare, for example, *Survey of Current Business* 64, 1984, Table S-14, where a corporate tax revenue of \$61.137 b. in 1981 and a revenue of only \$37.022 b. in 1983 is revealed. Cf. Auerbach (1982), Sunley (1982), and Sinn (1984b) for discussions of various economic effects of ACRS.

²³ See U.S. Department of the Treasury (1984).

²⁴ For West Germany, Jatzek and Leibfritz (1982, Table 5, p. 66) estimated an average life of

In the light of this information, the problem of accelerated tax depreciation can hardly be avoided in a realistic analysis of tax distortions, and the question arises of how the phenomenon can be formally represented. Despite certain differences in the depreciation allowances in the various countries, there are three aspects that are typical. First, because of the interest advantage of accelerated depreciation, the profitability of investment projects rises. Second, hidden reserves are created. With accelerated depreciation, the tax-written-down value of the capital stock falls short of the market value of capital, and the difference between these values is a reserve that is hidden from the tax balance sheet.²⁵ Third, the sum of tax depreciation allowances calculated for the total life span of an asset equals its initial purchasing value. All these aspects should be taken into account in the formulation of an appropriate model.

A way of modelling accelerated depreciation that seems attractive at first sight consists of the assumption that, at each point in time, tax depreciation exceeds true economic depreciation by a certain percentage. This assumption takes the first of the three aspects into account, but not the other two. The total volume of tax depreciation is more than 100% of the initial purchasing value of the equipment, and there are no hidden reserves. Thus, another, equally simple and quite familiar assumption will be made here.

It is assumed that a certain proportion α_1 , $0 \leq \alpha_1 \leq 1$, of an investment expenditure is depreciated immediately and the remainder $1 - \alpha_1$ gradually over time by keeping the tax depreciation at a level of $1 - \alpha_1$ times true economic depreciation. Since, at each point in time, gross investment is $I + \delta K$, and since true economic depreciation is δK , the flow of immediate depreciation on new investment is $\alpha_1(I + \delta K)$ and the flow of depreciation on existing assets is $(1 - \alpha_1)\delta K$. Hence, the aggregate current flow of tax depreciation is²⁶

$$B \equiv \alpha_1(I + \delta K) + (1 - \alpha_1)\delta K = \delta K + \alpha_1 I. \quad (3.16)$$

It consists of true economic depreciation plus the proportion α_1 of net investment. The formula for the revenue from taxing retained profits

13 years for plants expiring at the end of the seventies. In the presence of inflation and historical cost accounting a somewhat shorter depreciation period than this would be necessary to approximate true economic depreciation. According to the U.S. Department of the Treasury (1984, pp. 106 n.), the Asset Depreciation Range System can be seen as an attempt to be such an approximation.

²⁵Cf. Section 3.2.2. for further details.

²⁶The analysis abstracts from the possibility of diverging depreciation rules for existing and new assets. None of the allocative results derived in this book depends on this specification; only the tax treatment of new assets matters.

changes accordingly from (3.4) to:

$$T_r = \tau_r[f(K, L) - T_k - wL - \delta K - rD_f - \alpha_1 I - \Pi^d]. \quad (3.17)$$

The tax-written-down of the capital stock is $(1 - \alpha_1)K$, and the value of the hidden reserves is $K - (1 - \alpha_1)K = \alpha_1 K$.²⁷

Admittedly, the specification of the depreciation problem chosen cannot claim full generality. The assumptions that economic depreciation can be modelled as declining balance depreciation and that accelerated tax depreciation can be represented as a linear combination of economic depreciation and an immediate write-off clearly cannot do full justice to all aspects of depreciation that are relevant in practice. However, these assumptions are nevertheless flexible enough to capture the incentive effects relating to depreciation and, in fact, it is not difficult to translate the rules accountants use into the parameters of the model so as to produce an equivalence in present value terms. Suppose, using the discount rate i , the present values y_1 and y_2 of economic and tax depreciation have been calculated from empirical data. Then, equating the present value of declining balance depreciation, $\delta/(i + \delta)$, with the present value of economic depreciation, y_1 , gives $\delta = iy_1/(1 - y_1)$, and it follows from $y_2 = \alpha_1 \cdot 1 + (1 - \alpha_1)y_1$ that $\alpha_1 = (y_2 - y_1)/(1 - y_1)$. Assume, for example, the American Asset Depreciation Range System, which was in operation before 1981 and allowed plants to be depreciated in about 10 years on average, can be associated with true economic depreciation. For discount rates between 3% and 10% and assuming straight-line depreciation, it then follows from these equations that $0.21 \leq \delta \leq 0.22$ and that a reduction of the depreciation period from ten to five or seven years increases the depreciation parameter α_1 from zero to a value in the range $0.49 \leq \alpha_1 \leq 0.53$ or $0.25 \leq \alpha_1 \leq 0.32$, respectively. As stylized facts it can therefore be assumed that the 1981 introduction of the Accelerated Cost Recovery System increased the depreciation parameter α_1 from zero to about 0.5 and that the 1986 reform pushed this parameter back to a value of about 0.3.

²⁷ Since Equation (3.17) is algebraic and hence includes arbitrary negative values of taxable retained profits, it implicitly allows for a perfect loss-offset. This possibility is obviously unrealistic. It is true that many countries offer generous possibilities for carrying loss forward and backward and, as mentioned, for a while the United States even allowed for inter-company transfers of depreciation allowances. However, all of this by no means amounts to a perfect loss-offset. For this reason, Chapter 5 will study a special constraint on the financial planning of firms that allows for a temporary loss-offset but requires that the revenue from taxing retained and distributed profits be non-negative in a steady state. For the time being though we exclude the problem.

3.1.4. Alternative Assumptions on the Deductibility of Interest Costs

True economic depreciation and deductibility of debt interest are the basic ingredients of the Schanz–Haig–Simons concept of capital income taxation. A variation of this concept using different depreciation rules has been discussed. In addition, alternative assumptions on the deductibility of interest costs will now be considered.

On the one hand, a variable proportion α_3 ($\alpha_3 = 0$, $\alpha_3 = 1$) of non-deductible debt interest is to be allowed that can obtain values of either zero or one. On the other hand, it is assumed that a proportion α_2 , again either with a value of zero or one ($\alpha_2 = 0$, $\alpha_2 = 1$), of the total, that is actual and imputed, interest cost rK can be deducted from the tax base of the firm. Utilizing the extension introduced with (3.17), the general version of the tax function (3.4) for retained profits can therefore be written as

$$T_r = \tau_r [f(K, L) - T_k - wL - \delta K - \alpha_1 I - \alpha_2 rK - (1 - \alpha_3)rD_f - \Pi^d]. \quad (3.18)$$

This formulation is flexible enough to capture many different kinds of tax model.

Only four classes of models will be treated in this book though.²⁸ Not all variants of these classes will be discussed – if only because existence problems require a limitation of possible parameter constellations in the case of non-deductible debt interest. However, variants that do not belong to one or other of these classes can already at this stage be definitely excluded from the analysis.

- (1) $\theta_p < \theta_c$, $\alpha_2 = \alpha_3 = 0$. This class includes all empirically relevant tax systems. Because of (3.14) and (3.15) the direct marginal tax burden on interest income, retained profits, and dividends is strictly greater than the effective marginal tax burden on accrued capital gains. The depreciation rules are arbitrary and only actual interest cost is deductible.
- (2) $\theta_p = \theta_c$, $\alpha_1 = \alpha_2 = \alpha_3 = 0$. This constellation resembles Class (1); but it says that uniform taxation of interest income and accrued capital gains with full deductibility of actual interest cost is only examined in the context of true economic depreciation.
- (3) $\alpha_1 = 0$, $\alpha_2 = \alpha_3 = 1$. While the tax rates can have arbitrary values

²⁸The basic assumptions (3.14), (3.15); $0 \leq \theta_j \leq 1$, $j = d, r, p, c$; $0 \leq \alpha_1 \leq 1$; $\alpha_2 = 0$ or $\alpha_2 = 1$; $\alpha_3 = 0$ or $\alpha_3 = 1$ hold for all classes and are not repeated here.

and true economic depreciation prevails, it is assumed that imputed and actual interest cost is deductible.

- (4) $\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 1$. This case characterizes an immediate write-off and a non-deductibility of interest costs of any kind. Again the tax rates are arbitrary.

The reason for considering alternative possibilities for deductibility of interest cost is not that, in practice, the taxation of firms deviates significantly from the Schanz–Haig–Simons concept. Indeed, actual interest cost, and only actual interest cost, can be deducted from the tax base by firms in all OECD countries. Rather, this general formulation serves the purpose of understanding the importance of the Schanz–Haig–Simons concept and, in particular, of examining different proposals for a reform of capital income taxation.

Unlike the treatment of debt interest at the firm level the treatment of personal debt interest at the household level deviates from the Schanz–Haig–Simons concept in some countries. While interest earned by a household is always taxable, debt interest paid by a household can sometimes only to a limited extent be deducted from the personal tax base and sometimes not at all. We forego an explicit consideration of this aspect since it will turn that it is quite meaningless, in the framework of the present model, as long as the household possesses a strictly positive stock of tradable private assets and government bonds. Compare, in this context, the discussion of the borrowing constraint of private households in Chapter 8, Equations (8.12) and (8.53) through (8.55).

3.2. The Optimization Problem of the Firm under the Influence of Taxation

After considering various tax systems, the discussion can now turn towards the role of taxation in the decision problem of the firm. For this purpose it is useful to refer to the *laissez-faire* model of the firm formulated in Chapter 2.3. Unless otherwise indicated, all assumptions and definitions made there are maintained. The following remarks concentrate on a discussion of those aspects of the model that will alter under the influence of taxation.

3.2.1. *The Market Value of Shares*

As in the *laissez-faire* model, Fisher's separation theorem again implies that the firm tries to maximize the wealth of its representative shareholder and

hence the market value of its equity independently of the shareholder's specific preferences.²⁹ Differently from before, however, various taxes have now to be taken into account.

First, those taxes must be considered that reduce the net flow of funds to the household. Because of (3.13) this net flow has the magnitude

$$\Pi_n^d - Q - \tau_c(\dot{M} - Q),$$

where Π_n^d indicates net dividends.³⁰ Net dividends are defined as dividends after all taxes including the personal taxes paid by the representative shareholder. How these taxes affect the size of Π_n^d will be explained in the next section.

In addition to the taxes that reduce the firm's profit, taxes on alternative investment opportunities are of crucial importance for its decision problem. According to (3.7), the interest income which the representative household earns from a capital market investment is subject to a marginal personal tax rate of size τ_p . Thus the household's possibilities for an intertemporal transfer of consumption are described by the net-of-tax market rate of interest $\theta_p r$. Hence this rate of interest is the discount rate for calculating the present value of the funds flowing from the firm to the representative household. It has to be stressed that the net market rate of interest is *not* defined with regard to that tax rate which applies to a capital market investment of the *firm*.³¹ It is true that the tax rate on retained profits will indeed enter the marginal condition for optimal investment, i.e., that it will affect the discount rate which the firm uses for evaluating its investment projects. *This* discount rate, however, is yet to be derived from the optimization problem of the firm. Thus, instead of the laissez-faire equation, (2.3), we get the following expression for the market value of shares:

$$M(t) \equiv \int_t^\infty \{ \Pi_n^d(v) - Q(v) - \tau_c[\dot{M}(v) - Q(v)] \} \left[\exp \int_t^v -\theta_p r(s) ds \right] dv. \quad (3.19)$$

²⁹Cf. Footnote 12, Chapter 2.

³⁰In contrast to Chapter 2, for the time being control variables of the firm will not be indicated by a superscript "u". Only with the analysis of the intertemporal general equilibrium in Chapter 8 will this custom be taken up again to distinguish between the firm's and the household's control variables.

³¹Some theoretical studies that do not distinguish between different kinds of marginal tax rate seem to be intended to apply to the marginal tax rate on the returns from capital market investments that the *firm* receives. An example of a study where this is explicitly assumed is the paper of Hall and Jorgenson (1971, p. 16). The problem is related to the discussion about the so-called *gross* or *net* interest assumption in the finance literature. Cf. e.g. Wöhe (1965, pp. 198–213). Buchner (1971, pp. 672–674), or Strobel (1970, pp. 382–384).

In order to justify this expression on economic terms and to transform it into an equivalent expression that is easier to use, it can be differentiated with regard to time. The result is

$$\dot{M} = -\Pi_n^d + Q + \tau_c(\dot{M} - Q) + \theta_p rM \quad (3.20)$$

or, with consideration of the relationship $\dot{M} = \dot{m}z + \dot{z}m$:

$$\Pi_n^d + \dot{m}z\theta_c + (\dot{z}m - Q)\theta_c = \theta_p rM. \quad (3.21)$$

This equation describes a requirement that the development of the market value has to satisfy if there is to be an arbitrage equilibrium between investment in shares and bonds. It could have been used instead of (3.19) as a starting point for deriving an expression for market valuation. In order to make wealth owners indifferent between retaining shares at a value of M or exchanging these shares for bonds, the current net return on shares has to outweigh the potential net returns $\theta_p rM$ from holding bonds. The net return on shares is represented by the lefthand side of the equation and consists of three components. The first is the current net dividend paid out to shareholders, Π_n^d . The second is the capital gain $\dot{m}z\theta_c$ from the existing stock of shares net of the capital gains tax. The third is the net-of-tax capital gain $(\dot{z}m - Q)\theta_c$ from issuing new shares at a price below their market value.

This last component is not necessarily important in countries like the United States since existing shareholders will object to a policy of diluting their assets and will require that $\dot{z}m = Q$. The situation is different, however, in other countries – an example is West Germany – where new shares cannot be issued without providing the existing shareholders with tradable purchasing options. If purchasing options are distributed, existing shareholders will not necessarily object to selling the shares at a price below the stock market value since any difference creates an option value of equal size that they can realize in the market place without even buying the new shares themselves. The value of the purchasing options is $\dot{z}m - Q$, and, in principle, it is subject to the capital gains tax. Clearly the net-of-tax value of the flow of purchasing options provided to the existing shareholders contributes to the return from holding shares and has to be included in the arbitrage condition (3.21).³²

The market value function (3.19) from which the arbitrage condition (3.21) was derived is quite clumsy for certain mathematical operations since

³²Cf. Aktiengesetz (1955, §186) for further institutional details of the regulation in West Germany.

within the integral on the righthand side the derivative of the same integral, \dot{M} , appears. This problem can be avoided if (3.20) is solved for \dot{M} and the resulting differential equation

$$\dot{M} = -\frac{\Pi_n^d}{\theta_c} + Q + \frac{\theta_p}{\theta_c} rM \quad (3.22)$$

is integrated, an operation that is just the reverse of the step from (3.19) to (3.20). Under the existence requirement

$$\lim_{t \rightarrow \infty} \left[\left(\frac{\Pi_n^d(t)}{\theta_c} - Q(t) \right) \exp \int_0^t -\frac{\theta_p}{\theta_c} r(s) ds \right] = 0 \quad (3.23)$$

one obtains³³

$$M(t) = \int_t^\infty \left(\frac{\Pi_n^d(v)}{\theta_c} - Q(v) \right) \left(\exp \int_t^v -\frac{\theta_p}{\theta_c} r(s) ds \right) dv. \quad (3.24)$$

This formula for the market value of shares is completely equivalent to Equation (3.19). It shows that there is an alternative to deducting the capital gains tax from the current flow of funds from the firm to its shareholders. This alternative is to divide the net dividends and the net market rate of interest, *but not the value of the current flow of new shares*, by the capital gains tax factor θ_c .³⁴ Because of its analytical simplicity, only (3.24) will be used in the analysis.

³³ It is assumed that the market value is zero if the firm never issues new shares and never pays out any dividends. Hence the integration constant is set equal to zero. Note that allowing for a non-zero integration constant would not affect the solution of the optimization problem (3.29).

³⁴ For the case without new issues of shares ($Q = 0$), (3.24) resembles an expression derived by King (1974a, p. 23) in discrete time. Moreover, in the special case where new shares are issued at their market value that is, in the case where the value of the purchasing options given to the existing shareholders is zero ($\dot{z}m = Q$) the formula is compatible with the market value function

$$M(t) = \int_t^\infty [\Pi_n^d(v) - \tau_c \dot{m}(v)z(v)] \left\{ \exp \int_t^v -[\theta_p r(s) + \delta^*(s)] ds \right\} dv, \quad \delta^* \equiv Q/M.$$

This can be shown if this function is differentiated for t and the resulting expression $\dot{m}z + \dot{z}m = -R_n + \tau_c \dot{m}z + \theta_p rM + Q$ is transformed into a version that corresponds to (3.21). The discrete-time analog of this function is used by Auerbach (1979a) who, however, uses a debt-determined discount rate rather than the net market rate of interest. A similar expression can also be found in a paper by Poterba and Summers (1983).

3.2.2. Definitional Relationships between Taxation, Accounting Profit, and Dividends

Following these theoretical preliminaries, this section has the task of clarifying some definitional relationships that will later be needed for calculating the optimal policy of the firm.

Consider first accounting profits whose definition is of particular importance since, in all OECD countries, they are an upper limit for the dividends of corporations. In the case of true economic depreciation in the balance sheets, the accounting profit differs from the laissez-faire accounting profit of Equation (2.1) only by the capital tax and is defined by Equation (3.3).

With accelerated depreciation, however, the accounting profit may be smaller, depending on which accounting rules are used. This analysis is limited to the rules that are valid in the Anglo-Saxon countries since there, as explained above, accelerated depreciation seems to be of particular importance.

There are two distinct balance sheets, the tax balance sheet drawn up for the taxation department and the commercial balance sheet drawn up for the shareholders. Different values may be given to a firm's assets in these two balance sheets. Accelerated tax depreciation has immediate consequences only for the values appearing in the tax balance sheet. However, a potential tax on the reserves hidden from this balance sheet has to be entered in the commercial balance sheet under the category *deferred taxes*. Given the depreciation formula (3.16), the current increase in the stock of deferred taxes is $\tau_r \alpha_1 I$. Thus the current flow of net tax savings resulting from accelerated depreciation reduces the current accounting profit.³⁵ Accordingly, this accounting profit is

$$\Pi = f(K, L) - \delta K - wL - rD_f - T_k - \tau_r \alpha_1 I. \quad (3.25)$$

The accounting profit is the legal source of the dividends which the firm can pay. Consider first gross dividends Π^d . In order to calculate them, the tax on retained profits (T_r) and that part of net investment that cannot be financed through deferred taxes [$I(1 - \alpha_1 \tau_r)$] have to be subtracted from the accounting profit, and the net inflow of funds from issuing bonds, S_f , and shares, Q , has to be added:

$$\Pi^d = \Pi + S_f + Q - I(1 - \alpha_1 \tau_r) - T_r. \quad (3.26)$$

This equation differs from its laissez-faire counterpart (2.2) not only by the taxes but also by emphasizing new issues of shares as a source of finance.

³⁵Cf. Jung (1979, p. 121) or Alworth (1979).

In the laissez-faire model, no particular attention was paid to this source of finance since it was agreed to consider negative values of dividends as new issues of shares. In the case of taxation this procedure is no longer admissible since, as already follows from (3.24), tax laws do not treat new issues of shares symmetrically with dividends.

Because of (3.5), the net dividends can be calculated from Equation (3.26) by multiplying with the tax factor θ_d^* for distributed profits:

$$\Pi_n^d = \Pi^d - T_d^* = \theta_d^* \Pi^d. \quad (3.27)$$

Using (3.2), (3.18), (3.25), and (3.26), this expression can be transformed after a number of simple algebraic manipulations into the equation

$$\begin{aligned} \Pi_n^d = & \theta_d^* [f(K, L) - \delta K - wL - rD_f] + \frac{\theta_d^*}{\theta_r} [S_f + Q - I] \\ & + \tau_r \frac{\theta_d^*}{\theta_r} [\alpha_1 I + \alpha_2 rK - \alpha_3 rD_f] \\ & - \theta_d^* \tau_k K. \end{aligned} \quad (3.28)$$

The expression in the first line on the righthand side of (3.28) denotes the value of the net dividends for the case where taxes on retentions are calculated according to the initial formula (3.4) and where there are no indirect taxes. The expression in the second line shows the correcting terms that were introduced with (3.18). The expression in the third line measures the reduction in the net dividends through the tax on the capital stock.

3.2.3. The Formal Optimization Approach

Using the information given in Section 3.2.1 and 3.2.2, the structure of the optimization problem of the firm can easily be represented.

The firm faces given time paths $\{r\}$ and $\{w\}$ of the interest rate and the wage rate. It tries to choose the time paths of employment of efficiency units of labor $\{L\}$, net increase in debt $\{S_f\}$, new issues of shares $\{Q\}$, and net investment $\{I\}$ so that the market value of its shares is maximized:

$$\max_{\{L, S_f, Q, I\}} M(0). \quad (3.29)$$

The state variables of the optimization problem are the capital stock (K) and the stock of debt (D_f). As in the laissez-faire model, the following

equations of motion hold for these state variables:

$$\dot{K} = I, \quad (3.30)$$

$$\dot{D}_r = S_r. \quad (3.31)$$

Further conditions for the optimization problem are, as before,

$$K(0) = K_0 > 0 \quad \text{and} \quad L, K \geq 0, \quad (3.32)$$

but, unlike the laissez-faire model, it is assumed for the initial stock of debt that

$$D_r(0) = D_{r0} < P_K K_0. \quad (3.33)$$

Here, $P_K > 0$ is a parameter (it will later be called the effective price of capital) that is chosen such that with $D_{r0} = P_K K_0$ the market value of shares just takes on the value of zero. The magnitude of P_K will be calculated explicitly in Chapter 6 with Equation (6.3).

In practice, there are various institutional constraints on the financial decisions of the firm that have to be taken into account in the optimization approach. An explicit formulation of these constraints will be given at the beginning of the next chapter. For the time being, it suffices to state that the amounts of new debt, newly issued shares, and net investment have to be chosen out of a certain non-empty set of control policies U :

$$(S_r, Q, I) \in U. \quad (3.34)$$

The current-value Hamiltonian for this approach is, similarly to (2.8),

$$\mathcal{H}^u = \frac{\Pi_n^d}{\theta_c} - Q + \lambda_K I + \lambda_D S_r, \quad (3.35)$$

where λ_K and λ_D are the shadow prices or co-state variables of the capital stock and the stock of debt. The first-order conditions for a possibly constrained maximum of the Hamiltonian with regard to the control variables as well as the condition $\dot{\lambda}_K - \lambda_K r \theta_p / \theta_c = -\partial \mathcal{H}^u / \partial K$ are among the necessary conditions for an intertemporal optimum. In the following Section 3.3 and in Chapters 4 and 5 the information these conditions yield with regard to tax-induced substitution effects will be examined. Other conditions that have to hold in the optimum of the firm are the transversality conditions for the state variables K and D_r . Analogously to (2.17) and (2.18) they are now

$$\lim_{t \rightarrow \infty} \left[\hat{K}(t) + \hat{\lambda}_K(t) - \frac{\theta_p}{\theta_c} r(t) \right] < 0 \quad (3.36)$$

and

$$\lim_{t \rightarrow \infty} \left[\hat{D}_I(t) + \hat{\lambda}_D(t) - \frac{\theta_p}{\theta_c} r(t) \right] < 0, \quad (3.37)$$

where, however, for the time being it cannot be taken for granted that $\lim_{t \rightarrow \infty} \hat{\lambda}_K(t) = \lim_{t \rightarrow \infty} \hat{\lambda}_D(t) = 0$.

3.3. Taxation and Optimal Employment of Labor

The impact of taxation on the process of capital formation is at the center of this study, and separate sections are devoted to the analysis of the firm's financial and real investment decisions. However, the question of the optimal employment of labor under the influence of taxation should not be completely neglected.

From (3.19), (3.28), and $\partial \mathcal{H}^u / \partial L = 0$ it can be calculated without difficulty that³⁶

$$w = f_L. \quad (3.38)$$

Formally this condition does not differ from the equivalent laissez-faire condition (2.10). It should not be forgotten, though, that, in the case of a value-added tax ($\tau_v > 0$), f_L denotes the marginal product net of this tax. The gross marginal product is $f_L(1 + \tau_v)$ and hence *exceeds* the wage rate. Thus (3.38) says that the employment decision of the firm is not affected if the wage rate in terms of the consumption good *before* the value-added tax is kept constant. This condition is equivalent to the condition that, given the money wage rate, the value-added tax is completely shifted or that, given the gross commodity price, the wage rate falls in proportion with $1/(1 + \tau_v)$. This latter possibility refers implicitly to the real wage rate, that is, the wage rate in terms of the consumption good *after* value-added tax. Define the real wage rate w^r as $w/(1 + \tau_v)$ and accordingly the real marginal value product of labor as $f_L/(1 + \tau_v)$. Then it holds that

$$w^r = f_L/(1 + \tau_v). \quad (3.39)$$

This condition is equivalent to Condition (3.38) but has the advantage that it illustrates more clearly the discrimination against employment that results from the value-added tax.³⁷

³⁶The properties of the production function ensure that an interior solution prevails with regard to L if $w > 0$.

³⁷It is worth noting that only the tax rate τ_v appears in Equation (3.39). This fact casts an interesting light on the discussion of the so-called "employment program" of the German

It seems quite plausible, and does not require any additional explanation, that the taxation of the capital stock is unimportant for labor demand when the capital stock is given. The fact that the taxes on interest income and profit do not show up in (3.39) is not quite so self-evident.

The reason for the irrelevance of the taxation of the interest income of households is that the employment decision is, in principle, a static optimization problem. It is true that the discount rate may change with the tax on interest income. However, the rate at which the marginal profit of the last worker employed at a particular point in time is discounted is completely unimportant. The present value of this marginal profit is zero in any case.

The reason for the irrelevance of the profit taxes has long been known in public finance:³⁸ Since the profit from employing the marginal worker is zero, there is no tax burden on his employment and hence – whether with discounting or without – the employment decision cannot be affected.

So much for a first simple aspect of the optimization problem of the firm. It will be seen that other aspects are not quite as obvious and cannot be treated by means of static analysis.

government in 1981/82 that was to be financed through the increase of the value-added tax. Obviously the value-added tax is the only one among all company taxes considered here that has the property of implying an increase in unemployment if trade unions defend the real wage rate.

³⁸ Cf. Mill (1865, pp. 496–498) and Häuser (1959/60).