Common Property Resources, Storage Facilities and Ownership Structures: A Cournot Model of the Oil Market

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During the last decade economists have tried hard to convince Meadows and his followers that market forces do not tend to exhaust our natural resources too rapidly as compared with the requirements of an intertemporal Pareto optimum. It is well known now that, under well-defined property rights and correct price expectations, a competitive extraction industry brings about a Pareto-optimal depletion path, and it has been shown that oligopolistic and monopolistic market structures may even produce a bias towards conservation.\(^1\) It is still an unresolved issue, however, which extraction path will result if property rights are imperfectly defined. Following the pioneering work of Gordon (1954) there has been a large body of literature demonstrating that, in the extreme case of free entry and myopic profit maximization, there are strong reasons for excessive extraction.\(^2\) But this literature hardly yields reliable conclusions for the perhaps more realistic case where firms derive their behaviour from intertemporal optimization and where there is a common ownership, but limited access to the resource.

The basic intuition behind the common-pool problem is that the individual agent chooses an extraction policy other than the one he would choose under well-defined property rights since he is afraid that part of the resource he leaves underground will be extracted and sold by others.\(^3\) Despite the simplicity of this intuition, there does not seem to be any publication that analyses it within an intertemporal equilibrium model with rational and far-sighted agents. This is the motivation for the present approach.

The approach studies the common-pool problem in the context of an oligopolistic world market for oil. It posits a given number of identical firms extracting oil from holdings between some of which there is mutual seepage. The firms sell the extracted oil in the same market or they store it in privately owned, seepage-proof tanks. Each firm derives its behaviour from intertemporal optimization under perfect foresight of the time paths of the model variables, but Cournotesque conjectures about the reactions of its competitors.

Except for the assumption of storage facilities, this specification is closely related to models previously studied by Khalatbari (1977) and Kemp and Long (1980, essay 10).\(^4\) The two models are very similar but they yield strikingly different conclusions. While Kemp and Long find that the extraction path chosen by the firms is Pareto-optimal, i.e. that the price of the resource rises at a rate given by the market rate of interest, Khalatbari contends that there is over-extraction. Kemp and Long convincingly demonstrate that Khalatbari’s model suffers from an inconsistency in the way the individual firm expects its rivals to respond to its own actions, and they argue that this inconsistency is the reason for the divergence in results. Thus the casual reader gets the
impression that, with consistent expectations on the part of the individual firm, there is no reason to be afraid that common pools will be extracted too quickly. *Individual* rationality seems to be sufficient for avoiding the "tragedy of the commons".

Unfortunately, however, this would be too hasty a conclusion, since the Kemp/Long result—that there is no over-extraction in spite of the common-pool problem—follows from a particular conjectural hypothesis which fails to represent the basic intuition concerning the common-pool problem. The precise nature of this shortcoming is explained below, and it is shown that the approach taken by Kemp and Long is not the only way to avoid Khalatbari's inconsistency. On the one hand, it is possible to alter Khalatbari's model in a way that ensures consistency but preserves his results. On the other hand, there is a natural conjectural hypothesis that represents the above-mentioned intuition and does not seem to be inferior to either that of the Kemp/Long model or that of the corrected Khalatbari model. This hypothesis, by itself, brings about an intertemporal allocation that is even more pessimistic regarding the market's ability to prevent over-extraction than is suggested by Khalatbari's work.

Usually, models with exhaustible resources do not allow for storage facilities since the single firm has no incentive for using them. This is not true for the present model. Since over-extraction means that the price of the resource rises at a rate greater than the market rate of interest, there is clearly a potential for speculative gains from storing. A firm that extracts the oil and stores it in seepage-proof tanks will enjoy a higher discounted revenue than a firm that sells the oil immediately. Ideally, one could well imagine storing and intertemporal arbitrage eliminating any divergence between the rate of increase in the oil price and the market rate of interest, thus forcing the path of resource consumption to coincide with the Pareto-optimal path. However, the question is how much validity this argument retains in the presence of storage costs. This question will be examined in detail.

Imperfectly defined property rights may imply not only a divergence from a Pareto-optimal allocation but also a divergence from the extraction policy which maximizes the discounted profit of the resource-extracting industry as a whole. Thus there are strong incentives for mergers between firms and for attempts of consolidating the holdings. The implications of various changes in the structure of property rights are examined below with particular emphasis being placed on the possibility of mitigating the problem of misallocation.

I. The Basic Structure of the Model

Of central importance for the present model is a description of the seepage pattern. Let \( n \geq 2 \), \( S_i \geq 0 \), \( R_i \geq 0 \), and \( \alpha > 0 \) be, respectively, the number of firms, the stock of oil the \( i \)th firm possesses, the rate of market supply released from this stock, and a seepage parameter. Moreover, let \( S_i^u \geq 0 \) be the part of firm \( i \)'s stock of oil kept underground and \( S_i - S_i^u \) be the part kept above ground in seepage-proof tanks, where \( 0 \leq S_i^u \leq S_i \). Then

\[
\dot{S}_i(t) = -R_i(t) - \alpha \left[ S_i^u(t) - \left( \sum_{j \neq i} S_j^u(t)/(n-1) \right) \right], \quad i = 1, \ldots, n
\]
where \( t \) is the time index and \( \sum_{j \neq i} S_j^n/(n-1) \) the average stock of oil under the holdings of the rivals of firm \( i \). With this formulation it is not necessarily assumed that there is only one oil field from which all firms extract. In general, there may be any number \( f \geq 1 \) of separated oil fields, but it is assumed that each field is equally shared by all firms and that the geologically determined seepage parameter is the same for the holdings in all fields. Section VI takes up the seepage problem in greater detail.

The aim of the \( i \)th firm is to

\[
(2) \quad \max_{(R_i, S_i^n)} \int_0^\infty \left( P[R(t)] R_i(t) - C\left[\frac{[S_i(t) - S_i^n(t)]}{n}\right]\right) e^{-rt} dt
\]

subject to \( R_i, S_i \geq 0 \) where \( r > 0 \) is the market rate of interest;

\[
(3) \quad R(t) = \sum_{j} R_j(t)
\]

the aggregate rate of extraction; and \( P, P > 0, P' < 0 \), an inverse market demand function with an absolute price elasticity greater than \( 1/n \). The function \( C, C'(0) = C(0) = 0, C'' > 0 \), is an aggregate storage cost function in the sense that, if each firm decided to store the amount \( X/n \) in tanks, the total storage cost would be \( C(X) \). Implicit in this formulation is the assumption that merging cannot directly affect the size of storage costs. The functions \( C \) and \( P \) are assumed to be twice differentiable.\(^5\)

In finding its optimal decision the single firm \( i \) is assumed to know not only the functional form of (1) including \( \alpha \), the demand and cost functions, the market rate of interest and its own initial stock of oil,

\[
(4) \quad S_i(0) = S_{i0} > 0
\]

but also the full intertemporal plans of all other agents as represented by \( R_j(t), S_j(t) \) and \( S_j^n(t) \) for all \( t \) and all \( j \neq i \). Hence, the equilibrium where all agents have optimized their plans is of the perfect-foresight type: since the single firm correctly anticipates the time paths of its rivals’ control and state variables, there is no need for binding contracts and no incentive to recontract.

II. On Cournotesque Reaction Hypotheses

In the model described there are two ways by which a firm’s actions affect its rivals: through oil seepage and by the usual oligopolistic market interference. For both, the firm has to formulate hypotheses as to how the rivals will react. The attention of this paper is confined to Cournotesque reaction conjectures.

Market interference

Where market interference is concerned there is no ambiguity about what a Cournotesque conjecture is. If the firm decides to sell an additional unit of oil (from its store), it expects that there will be no quantity reactions on the part of its rivals. The rival’s time paths of market sales \( R_j \) of oil stored in tanks \( S_j - S_j^n \), and of oil stored underground \( S_j^n, j \neq i \), are all expected to remain unchanged.
Seepage interference

For the seepage interference, however, a similarly clear-cut formulation of a Cournotesque reaction hypothesis is not available. Suppose the firm decides to leave an additional quantity of oil underground, a part of which then gradually seeps away to the holdings of its rivals. Obviously, the firm cannot reasonably assume that none of the three types of path will be altered. A decision therefore has to be made concerning the firm’s conjectures about the fate of the additional oil appearing under the holdings of its rivals. Basically, there are three possibilities.

The first is that the firm expects its neighbours never to extract the additional oil they find underground. This possibility is chosen by Kemp and Long. It implies that the firm considers the time paths of the rivals’ market sales and stocks above ground to be exogenous to its own decision problem, but not the time paths of the stocks underground. Despite seepage, the firm is confident that it will not suffer a permanent loss. It expects to be able to recoup the temporary loss of oil in later periods by simply extracting more and thus reducing its own stock below what it otherwise would have been. Thus, conservation is not punished and there is no incentive to over-extract.

The second possibility is that the firm conjectures that its neighbours will extract the additional oil seeping over to their holdings and will permanently store it in tanks. This possibility can be associated with Khalatbari. He assumes that the firm optimizes under the presumption that it can neither influence the time paths of its rivals’ rates of extraction nor the paths of their stocks underground. Strictly speaking, this means that the firm expects the oil seeping away to vanish into the air; this is the inconsistency correctly pointed out by Kemp and Long. However, rather than assuming that the seepage loss augments the rival’s stocks underground, as Kemp and Long suggested, it is possible to alter Khalatbari’s model such that the firm takes the rival’s rates of market sales as exogenous, while considering the rates of extraction to be endogenous. In this case the implications of the model can be maintained provided, however, that, contrary to its expectations about its competitors, the single firm itself has no storage facilities.

Neither the first nor the second possibility seems attractive. They both imply that the firm does not expect its rivals to be alert enough to make any profitable use of the additional oil they inadvertently gain. This suggests the third possibility that is considered in the present paper. The firm expects its rivals to keep unchanged the time paths of their stocks of oil, both above ground and underground, and hence to extract and sell each additional unit of oil they find under their holdings. This reaction conjecture also has its drawbacks from an ultra-rational point of view. However, among the set of simple Cournot hypotheses it seems to be the one that best fits the basic intuition of the common-pool problem mentioned in the introduction.

III. Analysis of Intertemporal Equilibrium

Optimality conditions of the single firm

For the $i$th firm maximizing its market value according to (2), subject to the constraints (1) and (4) and taking $S_j^a(t)$ and $S_j(t)$ as exogenous for all $j \neq i$
and all \( t \geq 0 \), the Hamiltonian is

\[
H_t = e^{-\tau_t} \left( P(R_t)R_t - \frac{C((S_t - S^u_t)n)}{n} + \lambda_t \left[ -R_t - \alpha \left\{ S^u_t - \sum_{j \neq i} S^u_j / (n-1) \right\} \right] \right)
\]

where, because of (1) and (3), the total rate of market sales can be written as

\[
R = R_t + \sum_{k \neq i} \left[ -\dot{S}_k - \alpha \left\{ S^u_k - \sum_{j \neq k} S^u_j / (n-1) \right\} \right].
\]

It is assumed that the Hamiltonian is a strictly concave function of \( R_i \) and \( S^u_i \) in the range \( R_i \geq 0, 0 \leq S^u_i \leq S_n \), and has an interior maximum with respect to the constraint \( R_i \geq 0 \).

From \( \frac{dH_t}{dR_i} = 0 \) we have \( \lambda_i = P'(R)R_i + P(R) \), which is the usual equality of marginal cost and marginal revenue, where the marginal cost is the costate variable or shadow price of the resource stock. If \( \eta \) denotes the absolute price elasticity of demand, this condition becomes

\[
P(R) = \lambda_i - P'(R)R_i = \lambda_i \left( 1 + \frac{1}{\eta R_i / R_i - 1} \right).
\]

The derivative of the Hamiltonian with respect to the second control variable is \( \frac{\partial H_t}{\partial S^u_i} = e^{-\tau_t} \{ C' - \alpha (\lambda_i - P'R_i) \} \). Here, \( \alpha(\cdot) \) is the seepage loss and \( C' \) the saving of storage cost from an additional unit of oil kept underground. The seepage loss consists of two parts, the direct loss of oil evaluated at the shadow price, \( \alpha \lambda_i \), and the indirect loss of revenue, \( -\alpha P'(R)R_n \), as the oil seeping away is sold by others and reduces the market price. From (7), the sum of these two parts amounts to \( \alpha P(R) \): since a change in the market price occurs regardless of who sells the oil, the marginal loss must be evaluated at the market price, not at the shadow price or marginal revenue. So the derivative becomes

\[
\frac{\partial H_t}{\partial S^u_i} = e^{-\tau_t} \{ C'(S_i - S^u_i)n \} - \alpha P(R).
\]

Since \( C'(0) = 0, \alpha P > 0 \), and \( 0 \leq S^u_i \leq S_n \), this gives

\[
C'(nS_i) \left\{ \begin{array}{l} \leq \alpha P(R) \Rightarrow \left[ 0 \left\{ \begin{array}{l} \leq \right\} S^u_i < S_n \right. \end{array} \left. C'(n(S_i - S^u_i)) \left\{ \begin{array}{l} \leq \right\} \alpha P(R) \right. \end{array} \right. \right. \end{array}
\]

for \( S_i > 0 \). In general, the stocks held above ground and underground are strictly positive and are chosen so as to balance the marginal storage cost with the marginal seepage loss. If, however, the marginal storage cost falls short of the marginal seepage loss, even when the total stock is stored in tanks, then everything is stored in tanks.
Utilizing the notation $\dot{X} = \dot{X}/X$, we obtain a third optimality condition from $\delta(e^{-\gamma t} \lambda_i)/\delta t = -(\delta H_i/\delta S_i)$:

$$
\dot{\lambda}_i = r + \frac{C'_i \{ (S_i - S^u_i) n \}}{\lambda_i}.
$$

This condition states that the stock of oil has been such that an additional value unit of oil not put on the market creates capital gains equal to the sum of its interest and storage costs.

The transversality condition for the single firm's optimization problem is

$$
\lim_{t \to \infty} e^{-\gamma t} \lambda_i(t) S_i(t) = 0.
$$

Since (9) says in connection with (8) that $\dot{\lambda}_i > r$ for $S_i > 0$, (10) obviously requires

$$
\lim_{t \to \infty} S_i(t) = 0.
$$

Together with the starting condition (4), conditions (7), (8), (9) and (11) determine firm $i$'s intertemporal plan, given $S_j(t)$ and $S^u_j(t)$ for all $t \geq 0$ and $j \neq i$.

**Conditions for a market equilibrium**

In a perfect-foresight equilibrium the extraction policies of all firms are determined in this way, and the paths of the different stocks of oil chosen by the firms are compatible with those assumed exogenous in each firm's optimization problem. To investigate the properties of the equilibrium, let us follow a common practice and add two simplifying assumptions: (1) all firms start with the same resource stock so that $R/R_i = n$; (2) the market demand function is isoelastic.

Because of (7), these assumptions imply $\dot{\lambda}_i = \dot{P} \forall i$, and in connection with (8) and (9) they give

$$
\dot{S}^u \left\{ \begin{array}{ll} > 0 & \text{if } C'(S) \left\{ \begin{array}{ll} > 0 & \alpha P(R) \\
\end{array} \right. \\
\end{array} \right.
$$

$$
\dot{P} = r + \alpha \left( 1 + \frac{1}{\eta n - 1} \right) \text{ if } S^u > 0
$$

$$
\dot{P} = r + \frac{C'(S - S^u)}{P(R)} \left( 1 + \frac{1}{\eta n - 1} \right) \text{ if } S^u \geq 0
$$

where $S^u = \sum S^u_i$ and $S = \sum S_i$. By the definition of $\eta$, the corresponding relative change in the aggregate rate of market sales, or resource consumption, is

$$
\dot{R} = -\eta \dot{P}.
$$

By summation over all firms, we achieve from (1),

$$
\dot{S} = -R,
$$

from (4),

$$
S(0) = S_0
$$

where $S_0 = \sum S_{i0}$; and from (11),

$$
\lim_{t \to \infty} S(t) = 0.
$$
The equilibrium path

Equations (12)-(18) describe the intertemporal market/seepage equilibrium. Since none of the variables depends on calendar time, the properties of this equilibrium can be meaningfully studied by trying to establish a function of the type

\[ R = \bar{R}(S, \alpha, n) \]  

which holds for given \( \eta, r \), and given functions \( C'(.) \) and \( P(.) \). A discussion of the roles of \( \alpha \) and \( n \) is postponed to Section VI. For the time being, the analysis will be concerned solely with the relationship between \( R \) and \( S \). This relationship will be illustrated by the resulting equilibrium path in a \((R, S)\) diagram. It is useful, therefore, to note that (15) and (16) imply the equation

\[ \bar{R}_S = \frac{\dot{R}}{\dot{S}} = \eta \hat{P} \]  

which relates the slope of the equilibrium path to the relative rate of change in the resource price. An example of the path \( \bar{R}(S, \alpha, n) \) satisfying the properties yet to be derived is shown in Figure 1.

![Figure 1. The intertemporal equilibrium in the common-pool market.](image)

Depending on the size of \( C'(S) \) relative to that of \( \alpha P(R) \), expressions (12), (13) and (14) require two alternative types of differential equation determining \( \hat{P} \) and hence \( \bar{R}_s \). Consider the equation \( C'(S) = \alpha P(R) \). Since \( \alpha > 0 \), \( C'(0) = 0 \), \( C'' > 0 \), \( P > 0 \) for \( R > 0 \), \( P \to \infty \) as \( R \to 0 \), \( P \to 0 \) as \( R \to \infty \), and \( P' < 0 \), the solution
of this equation for $R$ gives a function of the type

\begin{equation}
R = \tilde{R}(S, \alpha)
\end{equation}

\[
\tilde{R} > 0 \quad \text{for } S > 0, \quad \tilde{R} \to \infty \quad \text{as } S \to 0
\]

\[R_s < 0, \quad \tilde{R}_\alpha > 0.\]

(A possible graph of this function is depicted in Figure 1.) Obviously,

\begin{equation}
\tilde{R}(S, \alpha, n) \begin{cases} > 0 & \text{if } \tilde{R}(S, \alpha) > \tilde{R}(S, \alpha) \\
< 0 & \text{if } \tilde{R}(S, \alpha) < \tilde{R}(S, \alpha) \end{cases}
\end{equation}

\[
\Rightarrow \tilde{R}(S, \alpha) \Rightarrow C'(S) \begin{cases} > 0 & \text{if } \alpha P(\tilde{R}(S, \alpha, n)) \\
< 0 & \text{if } \alpha P(\tilde{R}(S, \alpha, n)) \end{cases}.
\]

Thus, from (12) and (13),

\begin{equation}
\tilde{R}_s = \eta \left\{ r + \alpha \left( 1 + \frac{1}{n\eta - 1} \right) \right\}, \quad S'' > 0
\end{equation}

if $\tilde{R}(S, \alpha, n) > \tilde{R}(S, \alpha)$. And, from (12) and (14),

\begin{equation}
\tilde{R}_s = \eta \left[ r + \frac{C'(S)}{P(\tilde{R}(S, \alpha, n))} \left( 1 + \frac{1}{n\eta - 1} \right) \right], \quad S'' = 0
\end{equation}

if $\tilde{R}(S, \alpha, n) \approx \tilde{R}(S, \alpha)$.

In the Appendix it is shown that, as usual,

\begin{equation}
\tilde{R}(S, \alpha, n) \begin{cases} > 0 & \text{if } S > 0 \\
= 0 & \text{if } S = 0 \end{cases}
\end{equation}

and that $\tilde{R}(S, \alpha, n)$ is unique. It is also shown that $\tilde{R}(S, \alpha, n)$ is continuous and differentiable in $S$; so the equilibrium path in the $(R, S)$ plane does not "jump" and does not have kinks. To ensure that on the equilibrium path each agent has optimized his plan, it is demonstrated that a complete exhaustion does not occur in finite time and that the transversality condition (10) of the single firm's optimization problem is satisfied. This information allows us to concentrate on the salient properties of the equilibrium path.

Both (23) and (24) indicate that

\begin{equation}
\tilde{R}_s > \eta r \quad \text{for } S > 0
\end{equation}

and hence $\dot{P} > r$ for $t < \infty$. Together with (16), this also implies that $R$ and $S$ are continuously declining over time.

Provided that the initial stock of oil, $S_0$, is large enough, there are two phases of development with rather distinct characteristics.

In Phase I, $\tilde{R}(S, \alpha, n) > \tilde{R}(S, \alpha)$, $S'' > 0$, $R_s(S, \alpha, n) = \text{const.} > \eta r$, and, equivalently, $\dot{P} = \text{const.} > r$. Since $\dot{P} = \text{const.} > r$, $P' < 0$, $C'' > 0$, and $C'(0) = 0$, (14) implies that the amount of oil stored in tanks $(S - S'')$ is strictly positive and is gradually increasing over time. Since $\tilde{R}(S, \alpha) > 0$ for $S > 0$, Phase I terminates in finite time with an exhaustion of the stock held underground. At the point of exhaustion $\alpha P(R) = C'(S)$; i.e. $\tilde{R}(S, \alpha, n) = \tilde{R}(S, \alpha)$.

In Phase II, $\tilde{R}(S, \alpha, n) \approx \tilde{R}(S, \alpha)$ and $S'' = 0$. The stock of oil is fully stored in tanks and is gradually released on to the market. Differentiating (24), we find

\begin{equation}
\tilde{R}_{ss} = [C''(S) P(\tilde{R}(S, \alpha, n)) - P'(\tilde{R}(S, \alpha, n)) \tilde{R}_s(S, \alpha, n) C'(S)] b > 0
\end{equation}

for $S'' = 0$, $S > 0$, where $b = \text{const.} > 0$ and the indicated sign follows from $C''$. 

Thus, in Phase II the equilibrium path $\tilde{R}(S, \alpha, n)$ is strictly convex. Because $\tilde{R}_S/\eta = \hat{P}$ and $\dot{S} < 0$, this also implies that the rate of increase in the resource price is gradually falling with the passage of time. Now, $C' \to 0$ as $S \to 0$ and $P \to \infty$ as $R \to 0$. Obviously, from (24), this implies that the equilibrium path enters the origin with a slope $\eta r$

$$\lim_{S \to 0} \tilde{R}_S = \eta r.$$

This is equivalent to $\lim_{t \to \infty} \hat{P} = r$.

By referring to time paths rather than to the path in the $(R, S)$ diagram, the major features of these findings can be summarized by the following proposition.

**Proposition 1.** There exists a unique equilibrium path on which the rate of resource consumption and the stock of the resource are continuously declining with the passage of time. Provided the initial stock of the resource is sufficiently large, there are two phases. In Phase I, some of the resources being extracted is used to fill up stores. The phase terminates in finite time with an exhaustion of the stock underground. During phase II the market supply is fully withdrawn from stores, with exhaustion occurring as time goes to infinity. Throughout, the resource price ($P$) as well as the rate of increase in this price ($\dot{P}$) are continuous functions of time. In Phase I, $\dot{P}$ is constant and greater than the market rate of interest. In phase II, $\dot{P}$ is gradually falling, approaching the market rate of interest as time goes to infinity.

**IV. Interpretation: The Common Property Pathology**

The main aspect of the equilibrium path is that $\dot{P} > r$ for all $t \geq 0$. So the rate of growth in the market price of oil is permanently higher than required by the Hotelling rule or the Solow-Stiglitz efficiency condition. This is a clear sign of suboptimality. The reason for violating the efficiency condition is that the price must rise fast enough to create appropriate incentives for holding the resource stock, although this stock is subject to seepage losses if held underground and is subject to storage costs if held above ground.

Storage in seepage-proof tanks is a potential tool for avoiding suboptimality, but with storage costs this tool loses much of its force. The mere fact that firms choose to bear storage costs, although in the aggregate underground "storing" is costless, indicates misallocation. Storage increases the price of oil for each level of the resource stock; but, with a linear seepage function and in the absence of extraction costs, it has no influence whatsoever on the rate of increase in this price as long as the stock underground is not yet exhausted. Eventually, after extraction has terminated, storage will reduce the gap between the rate of increase in the resource price and the market rate of interest. But even then, when the total stock is safe from seepage losses, the gap will remain as long as the marginal storage cost is strictly positive. The reason for violating the Hotelling rule is "stored" together with the oil.

A rate of growth in the resource price permanently above the rate of interest brings about an over-consumption of oil at each level of the resource stock: because of (18) and (20) the Hotelling rule $\dot{P} = r$ implies that the rate of decline
in the resource stock is $R/S = \eta r = \text{const.} > 0$. This model's result $\hat{P} > r \forall t$, on the other hand, gives $R/S > \eta r \forall t$. Figure 2 illustrates this difference where path $A$ is the equilibrium path derived above and path $B$, which according to (28) is tangent to $A$ at the origin, is the Hotelling path. Path $B$ is also the outcome of the Kemp/Long model where firms act as if their property rights were perfectly defined. In the present framework this path could be achieved only in the absence of seepage ($\alpha = 0$); for then, according to (12) and (13), $\hat{P} = r$, so that $R/S = \eta r$ for all $t$.

Figure 2 illustrates this difference where path $A$ is the equilibrium path derived above and path $B$, which according to (28) is tangent to $A$ at the origin, is the Hotelling path. Path $B$ is also the outcome of the Kemp/Long model where firms act as if their property rights were perfectly defined. In the present framework this path could be achieved only in the absence of seepage ($\alpha = 0$); for then, according to (12) and (13), $\hat{P} = r$, so that $R/S = \eta r$ for all $t$.

![Figure 2. The equilibrium path in comparison with other paths.](image)

Figure 2 also demonstrates the equilibrium path derived by Khalatbari. That path is a straight line through the origin with a slope $\eta (r + \alpha)$. Clearly, the slope is lower than that of path $A$ in the range where the stock underground is not yet exhausted (see (23)). The reason for this difference is that in Khalatbari's model the firm takes account of only one of two disadvantages it may face when deciding to leave an additional unit of oil underground: it reckons with the oil seeping away being lost, but does not conjecture that its rivals will sell the gains from seepage and thus reduce the market price. This second disadvantage vanishes if the market share of a single firm is too small to influence the resource price, but under the usual oligopolistic assumptions it should not be neglected.

By itself, the difference in slopes gives rise to an even more pessimistic view of the market's degree of over-consumption than Khalatbari's work suggests. However, the present model incorporates seepage-proof storage as a mitigating element. The possibility of storage is responsible for the strictly convex lower part of path $A$ and implies that path $A$ intersects path $B$ once from above, when seen in a temporal perspective. Hence, compared with
Khalatbari's result, this model predicts more over-consumption when the resource stock is large and less when it is small.

V. MORE ON THE SEEPAGE STRUCTURE

The next section will study the question of how the equilibrium path of oil consumption is affected by changes in the degree of competition. Since there is not only a global competition for customers, but also a local competition for the oil underground, it seems useful to consider in greater detail the problem of oil seepage as a preparation for that study.

Seepage groups

Up to now it was assumed that each of \( f \geq 1 \) identical separated oil fields was equally shared by each of the \( n \) resource firms in the market. Assume now, instead, that there are \( q \) identical seepage groups of firms, in the sense that the firms within a group are mutually connected via seepage, but that there is no seepage between the groups. Each seepage group comprises \( m = n/q \) identical firms which equally share each of \( f/q \) identical oil fields, \( 1 \leq f/q \leq f, \ 2 \leq n/q \leq n, \ n \geq 2, \ f \geq 1 \).

The model presented above is robust enough to withstand this relaxation of assumptions. Let \( S_{ij}, S_{ij}', \) and \( R_{ij} \) denote the total individual stock of oil, the stock kept underground, and the rate of market sales of the \( i \)-th firm from the \( j \)-th seepage group at a particular point in time. Then, analogous to (1), the time change in the individual stock is

\[
\dot{S}_{ij} = -R_{ij} - \alpha \left\{ S_{ij}' - \sum_{\substack{l=1 \\ l \neq i}}^{m} S_{lj}' / (m-1) \right\}, \quad i = 1, \ldots, m; \ j = 1, \ldots, q.
\]

If we take the total rate of market sales to be

\[
R = \sum_{j} \sum_{i} R_{ij}
\]

then little changes. The Hamiltonian (5) can be maintained if it is interpreted as referring to the \( i \)-th firm in a particular seepage group and if \( n \) is replaced by \( m \). Equation (6) becomes

\[
R = R_{ij} + \sum_{k=1 \ (k \neq i)}^{m} \left[ -\dot{S}_{ij} - \alpha \left\{ S_{ij}' - \sum_{l=1 \ (l \neq k)}^{m} S_{lj}' / (m-1) \right\} \right] + \sum_{k=1}^{q} \sum_{k=1 \ (k \neq j)}^{m} R_{kh}
\]

where the third term on the right-hand side contains only variables that cannot be manipulated by firm \( ij \). Although (31) looks somewhat different from (6), together with (5) it still produces equations like (7)-(9) and the other equations following from them. In particular, equations (12)-(14), which give the basic result of this approach, remain valid. Thus it turns out to be unimportant whether or not it is assumed that the oil seeping away from one firm's holdings spreads evenly over the holdings of all other firms. What matters in the optimization problem of the single firm is how much of an additional unit of oil saved underground is seeping away; who enjoys the seepage gain is irrelevant.
The seepage parameter

While the nature of the intertemporal equilibrium is not affected by the prevalence of separate seepage groups, the seepage parameter $\alpha$ that plays a crucial role in determining the equilibrium path is directly dependent on the size of these groups.

Suppose, over each of the single oil fields, there are $s \geq 2$ different wells symmetrically connected via seepage, so that there are $s/m$ wells per firm engaged in this field, $1 \leq s/m \leq s$. An additional unit of oil not extracted from a well is subject to a seepage loss of size $\beta > 0$. The corresponding inflow to a single other well is $\beta/(s-1)$. For a firm owning $s/m$ wells, this means that the net seepage loss per unit of oil saved underground is

$$\alpha = [(s-1) - (s/m - 1)]\beta/(s-1),$$

or

$$\alpha = \left(1 - \frac{1}{m}\right)\beta^*$$

where $\beta^* = \beta s/(s-1) = \text{const.} > 0$. Thus the seepage parameter is an increasing function of the number of firms per seepage group, i.e. of the degree of local competition for the oil underground.

VI. Changes in the Ownership Structure

Provided with the notion of seepage groups and with equation (32), studying the role of changes in the ownership structure is straightforward. It also seems to be useful, as altering property rights may be both a policy means to reduce the over-consumption of oil arising from the common-pool structure and a means by which private firms themselves try to increase their aggregate profits.

The method of analysis is to examine three types of "experimental" changes in the ownership structure and to see how these changes affect the equilibrium path in the $(R, S)$ diagram. In Figure 3, path 1, on which $n = n_1$ and $\alpha = \alpha_1$, is taken as a benchmark. The other paths are the outcomes of the "experiments".

Global merging

Suppose the total number $n$ of firms is reduced by merging firms from different seepage groups, so as to reduce the number $q$ of seepage groups without changing the number $m$ of firms per seepage group. Clearly, from (32) this will not change the seepage parameter $\alpha$ and hence, in the $(R, S)$ diagram, the position of the curve $\tilde{R}(S, \alpha)$ defined by (21) is unchanged. However, since (23) and (24) require a higher slope at each point in the interior of the $(R, S)$ diagram, the equilibrium path bends upward for $S > 0$; i.e. we have $\tilde{R}(S, \alpha_2, n_2) > \tilde{R}(S, \alpha_1, n_1)$ for all $S > 0$ when $\alpha_2 = \alpha_1$ and $n_2 < n_1$. This gives the following proposition:

Proposition 2. Under global merging, i.e. when the number of firms competing in the market is reduced without changing the number of firms per seepage group, the rate of resource consumption increases for all levels of the resource stock greater than zero.
The result is illustrated in Figure 3 with path 2. Obviously, global merging exacerbates the problem of over-extraction. The remedy for mitigating the market's disease therefore is, on the contrary, an increase in the degree of global competition. Unfortunately, however, (23) and (24) reveal that, even in the limit as $n \rightarrow \infty$, over-extraction cannot be totally removed: it clearly holds that $\bar{R}_S > \eta r$ for all $S > 0$, even when $1/(n\eta - 1) = 0$.

Apart from the problem of avoiding misallocation, the role of global competition is interesting in itself. Applied to the current situation in the world oil market, the present model predicts that a merging of firms not connected through oil seepage would bring about a greater supply and a lower oil price today. This prediction is in striking contrast to static price theory. It is also at variance with the results of intertemporal resource models that do not incorporate the common-pool aspect. According to these models, imperfect competition does not necessarily imply a divergence from the competitive outcome and, when its does, there seems to be a bias towards supplying less rather than more.\(^{12}\)

**Consolidating the holdings**

Consider next a reduction in the degree of local competition without changing the number $n$ of firms competing in the market. This can be achieved by a process of consolidating the holdings, so that the number $q$ of seepage groups increases while the number $m$ of firms (and the number $f/q$ of oil fields) per seepage group is reduced so as to keep $n = mq = \text{const.}$
According to (32), the process of consolidation reduces the seepage parameter $\alpha$. An inspection of (21), (23) and (24) shows that, in the $(R, S)$ diagram, the reduction in $\alpha$ leads to a downward shift of the curve $\bar{R}(S, \alpha)$, reduces the slope $\bar{R}_s$ of the equilibrium path in the range above the new $\bar{R}$ curve, but does not affect the equilibrium path at or below this curve:

$$\bar{R}(S, \alpha_3, n_3) \begin{cases} < & \bar{R}(S, \alpha_1, n_1) \text{ for } \bar{R}(S, \alpha_1, n_1) \begin{cases} > & \bar{R}(S, \alpha_3) \end{cases} \end{cases}$$

where $\alpha_3 < \alpha_1$, $n_3 = n_1$. Hence the following proposition emerges.

**Proposition 3.** A process of consolidating holdings to internalize seepage flows, i.e. a reduction in the number of firms per seepage group given the number of firms competing in the market place, decreases the rate of resource consumption for each level of the resource stock except for those levels where, after the consolidation, the total stock is stored in tanks.

The proposition is illustrated in Figure 3 with path 3. Obviously, the consolidation of properties is a suitable means of reducing the over-consumption of oil. This is a plausible result, for, in the limit when there is no field of oil from which more than one firm is extracting, we are back to the usual oligopolistic resource model with well-defined property rights. It is well known that, with isoelastic demand and in the absence of extraction costs, this model brings about a Pareto-optimal path of resource consumption.

**Local merging**

Consider, finally, the case of merging within the seepage groups. The number $n$ of firms competing in the market is reduced through a decrease in the number $m$ of firms per seepage group, while the number $q$ of seepage groups remains unchanged. Clearly, when all firms of a seepage group are united, then there is no seepage externality and hence no overconsumption.\(^{13}\)

But how will the equilibrium path be affected if the unitization is imperfect, that is, if $m$ is reduced to a number of firms not less than two?

Under the policy described, both $n$ and, from (32), $\alpha$ are declining. Expression (21) shows that the decline in $\alpha$ brings about a downward shift of the curve $\bar{R}(S, \alpha)$ in the $(R, S)$ diagram; and, according to (23), the decline in $n$ results in an upward shift of the equilibrium path below the new $\bar{R}$ curve. However, (23) and (32) indicate that it is not clear whether local merging raises the slope of the part of the equilibrium path that lies above the $\bar{R}$ curve, i.e. of the part where the stock underground is not yet exhausted. Utilizing $m = n/q$ and inserting (32) into (23), we find through elementary manipulation that

$$\bar{R}_s = \eta \left( r + \frac{n - q}{n - 1/\eta} \beta^* \right), \quad S^u > 0. \tag{33}$$

Since it holds by assumption that $\eta > 1/n$, this equation implies that

$$\bar{R}_s(S, \alpha_4, n_4) \begin{cases} < & \bar{R}_s(S, \alpha_1, n_1) \begin{cases} > & \bar{R}_s \begin{cases} > & \frac{1}{q} \end{cases} \end{cases} \end{cases}$$

and $n_4 < n_1$, $\alpha_4 = \alpha_1(1 - q/n_4)/(1 - q/n_1)$, $S^u > 0$.\(^{14}\)
If $\eta \leq 1/q$, the slope of the equilibrium path above the $\bar{R}$ curve will either rise or stay constant, and hence the new equilibrium path lies everywhere above the old one. On the other hand, if $\eta > 1/q$, the slope will fall so that the new equilibrium path intersects the old path at some $S$ sufficiently large. These pieces of information yield the following proposition.

**Proposition 4.** Under partial local merging, i.e. when the number of firms in the market is diminished (albeit not to one) through a reduction in the number of firms per seepage group, the rate of resource consumption out of a given stock of the resource may rise or fall. A rise will occur if $\eta \leq 1/q$. However, if $\eta > 1/q$, then there exists some critical level $S^* > 0$ of the resource stock such that the rate of resource consumption rises, falls or stays constant depending on whether $0 < S < S^*, S > S^*$ or $S = S^*$.

The case $\eta > 1/q$ is illustrated in Figure 3 with path 4. Because of the ambiguity in the result, a policy of partial local merging does not seem to be an attractive way of solving the misallocation problem. Instead, an increase in the degree of global competition and a consolidation of oil fields given the number of firms are the policies recommended by the present model.

**VII. Conclusions**

The basic message of this paper is that there are reasons to fear that, in a world with rational and far-sighted agents, the existence of common oil pools implies over-extraction relative to the Solow–Stiglitz or Hotelling rule. It thus confirms a frequently held belief and helps to evaluate the relevance of a counterexample provided by Kemp and Long.

Despite the common-pool aspect, there would be no misallocation with costless, seepage-proof storage facilities. However, with costly storage, the rate of increase in the market price of the resource is permanently above the rate of interest, a clear sign of suboptimality.

A possible remedy for mitigating the consequences of the common-pool problem is to reorganize the ownership structure among the resource-extracting firms. More global competition in the market and less local competition for the oil underground have been shown to be appropriate, as both of these measures tend to result in a more conservationist extraction policy. A change in the ownership structure that is often brought about through market forces themselves is a local merging of the firms extracting from the same field. Unless all firms from a field are united, this change is not desirable. As it means both less local and less global competition, it may well strengthen the incentives to over-extract.

In sum, we cannot cross our fingers and hope that the market will solve all our problems. When faced with the common-pool problem, there clearly is a case for government intervention in setting the rules of the game and determining property rights. Only then can we expect the invisible hand to do its job correctly.

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1. Continuity and differentiability. The continuity of $\bar{R}(S, \alpha, n)$ in $S$ follows from the continuity in the time paths of the state ($S_t$) and costate ($\lambda_t$) variables in the firm's optimization problem, the proportionality between $\lambda_t$ and $P$ that holds in a symmetric equilibrium, the continuity of the inverse demand function $P(R)$, and the fact that, from (16), $S < 0$ for $R > 0$. In connection with the continuity of $C'(S)$ and $P(R)$, (23) and (24) ensure that the continuity of $\bar{R}(S, \alpha, n)$ in $S$ in turn implies that $\bar{R}_S$ exists for all $S$: the equilibrium path has no kinks. Note that this result includes the point where $R(S, \alpha, n) = \bar{R}(S, \alpha)$. Calculating (24) at this point, i.e. for $\alpha P(R) = C'(S)$, we clearly find the same value as that given by (23).

2. The equilibrium path leads to the origin of the $(R, S)$ diagram. Consider a path that satisfies (12)-(14), but reaches the ordinate above the origin. Because the path is continuous, and because $R_S > 0$ from (26), there exists some $R^* > 0$ for this path such that $R > R^*$ for all $S > 0$. Hence the stock of oil is exhausted in finite time and the path becomes infeasible. Consider, alternatively a path reaching the abscissa to the right of the origin. On this path some of the resource will never be used up, an aspect that violates (18). Thus, only a path leading to the origin is admissible.

3. On the path leading to the origin, exhaustion does not occur in finite time. Note that

$$S(t) = S(0) \exp \int_0^t \bar{R}(\tau) d\tau = S(0) \exp - \int_0^t \frac{R(\tau)}{S(\tau)} d\tau$$

where $\bar{R}(\tau)$, $R(\tau)$ and $S(\tau)$ denote the time paths of the respective variables. Since, from (27), $\bar{R}_S > 0$ when $\bar{R}(S, \alpha, n) > \bar{R}(S, \alpha)$, $R/S$ is falling over time. This clearly ensures that $\int_0^t R(\tau)/S(\tau) d\tau < \infty$ if $t < \infty$ and hence $S(t) > 0$ for $t < \infty$, provided that $S(0) > 0$.

4. The function $\bar{R}(S, \alpha, n)$ is unique. Suppose, on the contrary, there are two possible values of $\bar{R}$, given $S$, $\alpha$ and $n$. Then there are two possible paths in the $(R, S)$ plane. To ensure that both lead to the origin, there must be a level of $R$, $R > 0$, where the path that is further to the right in the $(R, S)$ diagram has a lower slope. However, such a level does not exist: for the range $R > \bar{R}(S, \alpha)$, it follows from (23) that $\bar{R}_S = \text{const.} > 0$. For the range $R < \bar{R}(S, \alpha)$, it follows from $C'' > 0$ and (24) that an increase in $S$, given $\bar{R}(S, \alpha, n)$, will always increase $\bar{R}_S$.

5. Checking the transversality condition. Although (18), which is an implication of the transversality condition (10), has been used in paragraph 2 above to find the path characterizing a market equilibrium, it has not yet been shown that the transversality condition itself is satisfied on this path. Let $\hat{P}(t)$ and $\hat{S}(t)$ describe the time paths of $P$ and $S$. Because of the proportionality between market price and shadow price, and since $P\{R(0), S(0) > 0$, the transversality condition is equivalent to

$$\lim_{t \to \infty} \exp \int_0^t \{-r + \hat{P}(\tau) + \hat{S}(\tau)\} d\tau = 0$$

or

$$\lim_{t \to \infty} \int_0^t \{r - \hat{P}(\tau) + R(\tau)/S(\tau)\} d\tau = \infty.$$  

Because of (25), (26) and the continuity of $\bar{R}(S, \alpha, n)$ in $S$, it holds that $R(\tau)/S(\tau) \geq \eta r$ for all $\tau \geq 0$. Utilizing this property and substituting (14) for $\hat{P}(\tau)$, we find that condition (i) will be satisfied if

$$\int_0^t I(\tau) d\tau + \lim_{t \to \infty} \int_0^t I(\tau) d\tau = \infty$$

(i)
where
\[ I(\tau) = \eta r - C'(S(\tau) - S''(\tau)) \left( 1 + \frac{1}{\eta n - 1} \right) \]
and \( t^* \geq 0 \). Since \( \eta r = \text{const.} > 0 \), and \( P \to \infty \), \( C' \to 0 \) as \( \tau \to \infty \), this condition is clearly met: choose \( t^* \) such that \( I(\tau) > E \) for all \( \tau \geq t^* \) where \( E \) is some arbitrary constant in the range \( 0 < E < \eta r \). Then the first integral in (ii) is finite, but it is dominated by the second integral, which becomes infinite in the limit as \( t \to \infty \). Thus the transversality condition of the single firm's optimization problem is satisfied.

NOTES

1 See Weinstein and Zeckhauser (1975), Kay and Mirrlees (1975), Stiglitz (1976), Pindyck (1978), Dasgupta and Heal (1979, Ch. 11), and Kemp and Long (1980, essays 4 and 6).
2 See, e.g. Smith (1968), Weitzman (1974), Hoel (1978), Berck (1979), and Dasgupta and Heal (1979, pp. 55–78, 113–126).
3 See, in particular, McDonald (1971, pp. 84–86), who also provides a rudimentary analysis that is similar to that of Khalatbari (see the discussion below). Compare also my own remarks in Sinn (1981, p. 191n.).
4 For a concise presentation of Khalatbari's model see Dasgupta and Heal (1979, pp. 372–375).
5 For the sake of brevity and analytical clarity (see n. 8), this formulation of the firm's optimization problem disregards the role of extraction costs. Note, however, that an admittedly special kind of extraction cost is implicitly incorporated in the present approach. Suppose that, with the process of extraction, a fixed proportion of the extracted oil is lost. Then a mere redefinition of \( S''(\tau) \) as measuring the stocks underground net of this proportion is all we need to allow for this type of extraction cost.
6 A similar Cournotesque assumption is chosen by Bolle (1980), who studies the problem where each agent has immediate access to the whole stock and extracts for his own consumption rather than for selling the resource in the market place.
7 Since in the Kemp/Long model the single firm has no possibility of, and no need for, storing, it will reasonably conjecture that the time paths of its rivals' stocks above ground are identically zero.
8 Even with this first conjectural hypothesis, however, over-extraction would occur in the presence of stock-dependent extraction costs. The reason is that, when the firm decides to leave an additional quantity of oil underground, it does not internalize the effect of lowering the extraction costs on the part of its rivals. This was shown by Hartwick (1980).
9 The problem occurs on p. 412 of Khalatbari's paper (1977), where \( R^* \) denotes both the total rate of extraction (see equation above (5) and the total rate of market sales (see the expressions with \( P_i(R^*) \) or \( P_i(R^*) \)). If a new variable is introduced for the rate of market sales and is inserted as an argument into the market demand function, the interpretation given in the text emerges.
10 For a discussion of an ultra-rational conjectural hypothesis within a model where firms have immediate access to the whole stock of the resource, see McMillan and Sinn (1982). Cf. also the discussion of a 'credible' strategy as given by Eswaran and Lewis (1982).
11 In reality, immediately after a change in the ownership structure, there will certainly be some adjustment problems not captured in the present model. For example, since (13) and (14) require \( \alpha P_i(R) = C(S - S'') \) for \( S'' > 0 \), in the present case an initial rise in the stock underground will occur at the expense of the storage stock. It is technically simple, but a little bit tedious, to avoid this aspect by imposing an irreversibility constraint on \( S'' \). Such a constraint would imply that, immediately after the mergers, the total market supply is even higher than that derived above and is sustained solely through withdrawals from the storage stock until this stock has reached its desired size. Effects of this type, however, do not seem to be of great importance for long-run allocation problems of the kind studied in this paper. (Current reserves above ground are 40 days or so.)
12 Cf. the literature cited in n. 1 above. The prediction of this model is not at variance with the fact that in the OPEC negotiations of recent years Saudi Arabia, the world's largest supplier of crude oil, has always exhibited a moderating influence on the oil price.
13 The policy of perfect unitization has been frequently recommended to overcome the common-pool problem. See, e.g. McDonald (1971, Ch. 10). Cf. also pp. 213–216, where McDonald points out the obstacles to voluntary unitization.

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