The Economic Theory of Species Extinction: Comment on Smith¹

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A widely recognized article by Smith suggests that harvesting leading to species extinction may be socially optimal, provided that the species growth-potential is sufficiently low. It is shown that this conclusion hinges on special assumptions about harvest technology which, despite a superficial compatibility with neoclassical production theory, contradict a basic postulate of a pioneering article by Gordon. As soon as Gordon's postulate is taken into account, while all other aspects of Smith's model are maintained, it turns out that even a species with a very low growth potential should not become extinct.

In a widely recognized article Smith [8] studied the properties of intertemporally optimal harvesting plans, where optimality can either be taken to refer to the goals of a social planner or to those of a private firm operating under well-defined property rights. One of Smith's conclusions is (Proposition 1) that for a selfreplenishable natural resource, such as, e.g., fish, the optimal population path ends up with extinction if its natural growth potential is sufficiently low.

A similar result has been derived by Clark [2] and Clark and Munro [4]. However, these authors stressed that for extinction to be optimal an number of further conditions have to hold. One of these is that the price of the harvested resource will exceed the unit harvest costs even if the resource stock dwindles to zero. Smith does not explicitly mention a similar condition, but specifies harvest technology by assuming a particular type of production function. At first glance, the properties of this function seem rather plausible since they closely resemble those we are used to accepting in neoclassical production theory. Also, Smith introduces his production function merely as a simplification without expressing any reservations about its generality. Hence, the casual reader gets the impression that for a production function properly specified in line with general economic laws extinction is the natural destiny for a species that is biologically slow growing and slow maturing, more natural at least than the analysis of Clark and Munro suggests.

In fact, however, Smith's harvest production function is not innocuous, it is not a mere simplification, and the analogy to neoclassical production theory is highly misleading. In the following I modify Smith's function by utilizing an argument advanced in the pioneering article of Gordon [5] and check the implications of this modification for the extinction question. It turns out that, given all other features of Smith's model, extinction will never be optimal.

Smith assumes that harvest production, f_2 , is a function of the existing resource stock, Q, and labor, L. Some of the properties of this function which he implicitly or

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explicitly assumes are the following:

$$f_2(L,Q) = Q \Phi\left(\frac{L}{Q}\right), \quad \text{such that} \quad \frac{\partial f_2}{\partial L} = \Phi'\left(\frac{L}{Q}\right); \quad (1)$$

$$f_2(0,Q) = f_2(L,0) = 0;$$
 (2)

$$\frac{\partial f_2}{\partial L} > 0; \tag{3}$$

$$\frac{\partial^2 f_2}{\partial L^2} < 0 \qquad [from \Phi''(\cdot) < 0]; \qquad (4)$$

$$\frac{\partial}{\partial Q} \frac{\partial f_2}{\partial L} > 0 \qquad \qquad [\text{from } \Phi''(\cdot) < 0]; \qquad (5)$$

$$\lim_{Q \to 0} \frac{\partial f_2}{\partial L} = 0, \qquad L > 0 \qquad \left[\text{from } \lim_{L/Q \to \infty} \Phi' = 0 \right]. \tag{6}$$

Assumptions (2), (3), (5), and (6) have also been used by Gordon. Among these, assumptions (2), (3), and (5) can be accepted without hesitation. Assumption (6) implies together with (2) that for any given (strictly positive) number of fishermen average productivity approaches zero as the stock of the resource vanishes. Clark's [3] study on the production technology of the fishery industry shows that this assumption is plausible for fish that are scattered over the ocean, but that some reservations are appropriate for gregarious herding animals such as the Peruvian Anchovy. Here labor productivity may stay high right down to the last catch. I do, however, not (yet) want to deviate from Smith in this respect.

The critical assumptions are (1) and (4). Gordon (pp. 138ff.) argues in particular against assumption (4), since he does not find that the usual reasons for deminishing returns as they are given in agriculture are of any relevance for the case of fishery. Indeed, it is hard to find arguments in favor of assumption (4). Why should the catch rate per fisherman fall if we increase the number of fishermen?

One argument could be that with the passage of time a higher catch effort leads to a reduction of the stock of fish in comparison to what it otherwise would have been and that the stock reduction in turn reduces the productivity of fishermen. This argument, however, is inadmissible since the question is: How does labor productivity change for a *given* stock of fish? The effect of the stock of fish on labor productivity is reflected by assumption (5) and not by (4).

Also one might argue that the decline in productivity occurs since there are fixed factors other than labor in the background. But this would be misleading, since Smith's model deals with a long-run intertemporal planning problem, where all factors should be variable. If we want to understand reality in terms of Smith's model, we have to interpret the variable L as an aggregate indicator of *all* factors employed.

The only argument for a declining marginal and average productivity of fishermen seems to be that for high scale operations crowding disadvantages may become relevant. At least for non-herding animals this argument does, however, not seem to be too important. Furthermore, the crowding effect could easily be outweighed by the gains of specialization that are enabled through large scale production. Thus, Gordon's assumption

$$\frac{\partial^2 f_2}{\partial L^2} = 0 \tag{7}$$

should be considered as an interesting and empirically relevant alternative to $(4)^2$.

Together with (2) and (5), assumption (7) obviously implies that the harvest production function exhibits increasing returns to scale, contrary to Smith's assumption (1). It is clearly tempting to compare the resource stock with the stock of capital in neoclassical production theory and to impose the usual assumptions on the harvest production function. But this is doing mathematics, not economics. In ordinary production theory the argument for linear homogeneity is that an identical reduplication of a factory would double the output and all factors used. Accordingly, if we double the number of fishermen, the number of fish, and the size of the ocean, it would be rather plausible that the size of the ocean is given. If we double the number of fishermen and the number of fish per square mile, then each single fisherman is likely to become more productive and thus output will more than double.³

If assumption (4) is replaced by (7) and assumption (1) is abandoned, then it turns out that extinction is never optimal. This can be shown by reference to Smith's expression 2.8:

$$v\left\{\substack{\leq\\ =\\ >}\right\}\frac{\gamma}{\frac{\partial f_2(L,Q)}{\partial L}} + \xi, \qquad \begin{cases} L=0\\ 0\leq L\leq \bar{L}\\ L=\bar{L} \end{cases}, \tag{8}$$

v = (constant) marginal utility of the harvested resource,

 ξ = marginal welfare of the unharvested resource,

 $\gamma =$ (constant) marginal productivity of labor in the nonresource sector,

 \overline{L} = total amount of labor available.

Recall that Smith assumes the marginal utility of the nonresource good to be equal to unity and regard this good as the numeraire. Then you find that expression (8) compares the consumers' price, v, of the harvested resource with its marginal costs, which is the sum of marginal extraction costs, $\gamma/(\partial f_2/\partial L)$, and the price of the unharvested resource under private ownership, ξ . This price, which is also the derivative of the welfare integral with respect to the resource stock, can never become negative. So, in any case, harvesting is not worthwhile both under private and under social optimization if the condition $\gamma/(\partial f_2/\partial L) \leq v$ is violated for the whole range $0 \leq L \leq \overline{L}$.

²Replacing assumption (4) by (7) and abolishing assumption (1) we have a production function $f_2(L,Q) = L\rho(Q)$, $\rho' > 0$, $\rho(0) = 0$, where ρ is a function relating output per fisherman to the stock of fish. Such a function is frequently used in resource economics and has been rationlized for the case of "diffuse resources" by Clark [3] who identified ρ with the "marginal stock density".

³Recently Lewis [7] showed that resource extraction stays profitable until the total stock is exhausted if harvest profit is a concave function of f_2 and Q. Now, the remarks of the preceding paragraphs indicate that f_2 is not jointly concave in Q and L. This implies that the profit function is not jointly concave in f_2 and Q if the wage rate is constant (as assumed by Smith), and hence Lewis' result is not applicable. It can easily be shown that this can never happen under Smith's assumptions, provided that there exists a resource stock at all where profitable exploitation is possible. Call such a resource stock Q^* and the amount of labor which can profitably be employed at this stock L^* . Then you find from (1), (3), and (4) that for any given Q, Q > 0,

$$\frac{\partial f_2(L,Q)}{\partial L} > \frac{\partial f_2(L^*,Q^*)}{\partial L^*} \quad \text{if } L < Q \frac{L^*}{Q^*}.$$
(9)

So, however small Q might become, under Smith's assumptions there is always some employment range, at least between zero and QL^*/Q^* , where the condition $\gamma/(\partial f_2/\partial L) \leq v$ is not violated, i.e., where immediate gains from harvesting are achievable.

Let us now regard Gordon's case, where assumption (1) is abandoned and (4) is replaced by (7). Then (6) implies that

$$\lim_{Q \to 0} \frac{\gamma}{\frac{\partial f_2(L,Q)}{\partial L}} = \infty > v, \quad 0 \le L \le \overline{L}, \tag{10}$$

i.e., a complete exhaustion would raise the marginal extraction costs to infinity for each possible employment level, far beyond the critical value v. So there must exist a size Q^{**} , $Q^{**} > 0$, below which the resource stock should never be diminished.

As a special example for a harvest production function modified in the way outlined above, consider the Cobb-Douglas version

$$f_2 = LQ^{\beta}, \qquad \beta > 0, \tag{11}$$

where the marginal productivity of labor is Q^{β} . Then, according to (8), the diminution of the resource stock terminates at the latest ($\xi = 0$) if

$$v = \gamma / Q^{\beta}. \tag{12}$$

Thus, at least the resource stock

$$Q^{**} = \left(\frac{\gamma}{v}\right)^{1/\beta} > 0 \tag{13}$$

should be preserved forever.

We have demonstrated that a slight change in Smith's assumptions towards more plausible assumptions favored by Gordon leads to a result that contradicts his findings. Hence two conclusions emerge: 1. A species with a very low growth potential is not necessarily condemned to extinction. 2. Smith's model does not give a proper explanation for the possibility of extinction.

Unfortunately for the conservationist, however, the second conclusion does not mean that there is no proper explanation. For example, if we also abandon Smith's assumption (6) and the part $f_2(L, 0) = 0$ from assumption (2), then despite a

reduction in the resource stock labor productivity might stay high enough to ensure that always

$$v > \frac{\gamma}{\frac{\partial f_2}{\partial L}},$$

i.e., that the price of the harvested resource exceeds the marginal (and unit) extraction costs even for $Q \rightarrow 0$. In this case the Clark-Munro result which was cited initially becomes applicable. Hence, extinction could be optimal from the viewpoint of the social planner or from that of the private firm operating under well defined property rights.

Another, perhaps more important, step toward a realistic theory of economic extinction is the introduction of a minimum viable population size greater than zero. If the minimum viable population size exceeds the minimum population level at which profitable fishing is possible, then extinction will occur even though marginal extraction costs approach infinity as the stock of the resource shrinks to zero. Under the assumption of free-access harvesting, where the shadow price of the unharvested resource is zero, this modification has already been studied by Gould [6] and Berck [1]. It would be interesting to check the robustness of their findings for the case of intertemporal optimization.

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