A SIMPLE MODEL OF PRIVATELY PROFITABLE BUT SOCIALY USELESS SPECULATION*

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The paper presents a general equilibrium model of a pure exchange economy with stochastic endowments, in which speculation in the forward market is profitable and stabilizes prices but is useless from a welfare point of view. Reconciling the Siegel paradox with the theory of incomplete markets, we show that banning speculation by closing the forward market may increase social welfare. We also show that the addition of a market might reduce the gains from international trade for all participating countries.

JEL Classification Numbers: F10, F11, F19.

1. Introduction

Traditionally, economists have emphasized the beneficial role of commodity speculation. Following Lerner (1944) and Friedman (1953), they have argued that profitable speculation stabilizes prices and creates welfare gains by narrowing the gaps between marginal costs and benefits at different points in time or at different locations. In a recent paper, in which he allows for heterogeneous information among market participants, Stein (1987) also concludes that price stabilization is welfare-improving.

In the present paper, on the other hand, we take up the popular view to the contrary: that speculation is a dubious economic activity which can be very profitable for individuals but harmful from the viewpoint of society as a whole.

The paper offers a very simple general equilibrium model of pure sequential exchange with stochastic endowments and demonstrates that, for that model, speculation in the forward market is (privately) profitable and stabilizes prices, but has no effect on the allocation of resources to consumption and therefore fails to generate (social) welfare gains.\(^1\) It is shown further (indeed, it follows almost immediately) that, if the model is extended to accommodate transaction costs in the forward market, then speculative profits and stabilized prices may coexist with welfare losses. Speculators gain from the mechanics of Jensen’s inequality but create negative externalities which, from a social perspective, may outweigh the private gains. In short, it is shown that there are circumstances in which profitable and price-stabilizing speculation is worse than useless. In those circumstances, the closing down of the forward market would increase social welfare.

The model lacks a full set of markets; in fact, it contains only a single forward

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* We thank Syed Ahsan, William Baumol, Dominique Demougin, Martin Hellwig, Ngo Van Long and Edmund Malinvaud for useful comments. The research was supported by the Woodrow Wilson School, Princeton University, while Hans-Werner Sinn was Olin Visiting Professor.

1) The model has little in common with the earlier literature on the Lerner--Friedman proposition, which clarified why destabilizing speculation can be profitable but did not focus on the welfare implications. See Baumol (1957), Kemp (1963), Stein (1961) and Telser (1959).
market and a single spot market. Speculators buy non-preferred commodities on the forward market to resell on the future spot market. Thus, the model links the emerging theory of incomplete markets (Radner, 1972; Hart, 1975; Geanakoplos, 1990 and Elul, 1995) to the Siegel paradox of the finance literature (Siegel, 1972, 1975; Roper, 1975; Black, 1990). In particular, it provides a natural and transparent illustration of the possibility, first noted by Hart, that the addition of a market may impoverish everyone. Moreover, it reveals the hitherto unnoticed possibility that the addition of a market might reduce the gains from international trade for all participating countries.

The Siegel paradox has provoked mixed reactions among economists. Some authors have dismissed it as a “partial analytic illusion” that vanishes once the necessity of “settling all accounts” is considered (McCulloch, 1975), or as a purely nominal phenomenon with no significance for the real allocation of resources (Boyer, 1977; Beenstock, 1985); or “a trivial mathematical inconvenience without economic or empirical significance” (Adler and Dumas, 1983, p. 955, n. 60). Others have taken a more positive view, seeing the paradox as an incentive to hold perverse international portfolios or as an explanation of the observed bias between forward and spot exchange rates (Krugman, 1981; Sibert, 1989; Sinn, 1989). Always, however, the discussion has been couched in partial equilibrium terms. Moreover, the welfare implications of the paradox remain largely unexplored.

In our model, risk-averse speculators are induced to stake substantial proportions of their endowments to capture profits of Siegel type. (On this point, see also Sinn, 1989.) There is no illusion, monetary or otherwise; indeed, the model contains no money. Moreover, the model is general equilibrium in scope, so that all “accounts” are “settled”. It is a distinguishing feature of the model that randomness relates to relative prices and that those prices are endogenously determined. In common with earlier contributors, we distinguish groups of speculators with disparate preferences. For concreteness, the model is given an open-economy interpretation; however, it is straightforward to apply the results in a closed economy setting. Finally, it will be shown that our conclusions are robust in the sense that, if speculation is price-stabilizing but welfare-reducing, then it remains so after sufficiently small perturbations of at least some parameters.

The paper has six sections. Section 2 sets out a general equilibrium model which incorporates speculative behaviour of Siegel type. Section 3 develops an example that may help clarify the nature of the equilibrium and the magnitude of the effects described. Section 4 discusses the welfare implications of the model. Section 5 establishes the robustness of our conclusions, and Section 6 offers some final remarks.

2. A simple general equilibrium model

Consider a pure-exchange economy with random endowments. There are two countries (Australia and Germany), two commodities (lamb and kraut), and two

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2) In a recent contribution, Cole and Obstfeld (1991) constructed a simple example in which everyone is worse off after the addition of a market. As the authors note, however, their example is fragile; that is, it is not robust even to small changes in the values of its parameters.

3) Of course, it is possible to interpret the endowments as outputs from fixed factor inputs in alternative country-specific states of nature.
periods (the present and the future). Each country has non-negative future endowments of both commodities, but Australians eat only lamb and Germans only kraut. All endowments of a particular commodity in a particular country are perfectly correlated, but there is stochastic independence across commodities and countries. Within each country, all households have the same preference and endowments. It will be convenient to assume that Australian and German households are equal in number.

There is a spot market and a one-period forward market on which kraut and lamb can be traded. The forward market opens in the present, the spot market in the future. The forward market allows an exchange of unconditional delivery promises that do not depend on the state of nature (no Arrow–Debreu markets) and must be fulfilled once the spot market has opened. Let \( p \) and \( p^f \) denote the spot and forward prices of kraut in terms of lamb.

A typical Australian household seeks to maximize its expected utility from lamb consumption. Essentially, its problem is to determine the amount of lamb to be traded against kraut in the forward market. Together with the future domestic endowment of kraut, the kraut received from the forward contract determines the total amount of kraut that can be traded against lamb in the future spot market. Let \( c \) be the amount of lamb consumed by the representative Australian household \( (c \geq 0) \), \( s \) its forward sales of lamb \( (s \geq 0) \), \( b \) its endowment of lamb \( (b \geq 0) \), \( k \) its endowment of kraut \( (k \geq 0) \), and \( U, U', U'' < 0 \), its Neumann–Morgenstern utility function. Then the household's present problem is to find

\[
\max_s E[U(c)] \quad \text{s.t.} \quad c = b - s + [(s/p^f) + k]p. \tag{1}
\]

The first-order condition for an interior solution is

\[
E[U'(c)r] = 0, \tag{2}
\]

where

\[
r \equiv (p/p^f) - 1 \tag{3}
\]

is the rate of return on a short position in lamb. The second-order condition \( E[U''(c)r^2] < 0 \) is satisfied because \( U'' < 0 \).

Note that problem (1) is algebraic. It allows not only the strategy of selling a positive amount of lamb in the forward market \( (s > 0) \), but also that of buying lamb and selling kraut \( (s < 0) \). The net amount of kraut that remains for sale in the spot market is \( (s/p^f) + k \), and the amount of lamb received in exchange is \( [(s/p^f) + k]p \). This amount, plus the domestic endowment of lamb, \( b \), plus the lamb bought in the forward market, \( -s \), is the total amount of lamb available for consumption. In the optimum described by equation (2), \( s \) is chosen so that the expected increase in utility resulting from a marginal increase in \( s \), i.e. the expectation of marginal utility weighted by the rate of return on a short position in lamb, is zero.

The problem of the typical German household is analogous to that of its Australian counterpart. Let us distinguish German variables by asterisks, so that \( s^* \) denotes the German forward sales of kraut \( (s^* \geq 0) \), \( k^* \) the German endowment of kraut \( (k^* \geq 0) \), \( b^* \) the German endowment of lamb \( (b^* \geq 0) \), \( c^* \) the German consumption of kraut \( (c^* \geq 0) \) and \( U^* \) the German utility, with \( U'^* > 0, U''^* < 0 \). Then we have, instead of (1),

max \[ E[U^*(c^*)] \] s.t. \[ c^* = k^* - s^* + (s^*p^f + b^*)/p, \]

and the optimality condition for the typical German household is

\[ E[U^*(c^*)r^*] = 0, \]

where

\[ r^* \equiv (p^f/p) - 1 \]

is the rate of return on a short position in kraut. Again, the second-order condition \( E[U^**(c)r^{*2}] < 0 \) is satisfied.

In market equilibrium, the plans of Australian and German households are compatible. Equilibrium in the forward market requires equality between the Australian forward demand for, and the German forward supply of, kraut:

\[ s/p^f = s^*. \]

The condition for equilibrium in the future spot market is equality between the Australian spot demand for, and the German spot supply of, lamb:

\[ p[(s/p^f) + k] = s^*p^f + b^*. \]

Condition (7) implies that

\[ p^f = s/s^*, \]

which, together with (8), gives

\[ p = (s + b^*)/(s^* + k). \]

Because of (1), (4), (7) and (10), market-clearing also implies that, eventually, Australian households consume all the lamb,\(^4\)

\[ c = b + b^*, \]

and German households all the kraut,

\[ c^* = k^* + k. \]

Substitution of (11) and (12) into the first-order conditions (2) and (5) gives

\[ E[U'(b + b^*)r] = 0 = E[U^{**}(k + k^*)r^*], \]

where, from (3), (6), (9) and (10),

\[ r = \frac{s^*(s + b^*)}{s(s^* + k)} - 1, \quad r^* = \frac{s(s^* + k)}{s^*(s + b^*)} - 1. \]

\(^4\) Equations (11) and (12) show that in (1) and (4) it is not necessary to restrict the size of \( s \) or \( s^* \) in order to avoid bankruptcy problems: it is sufficient to assume that the endowments are non-negative. In equilibrium, traders would even be able to satisfy a forward commitment to deliver more of the preferred commodity than they "produce"; they can always buy the required quantities in the spot market, which will be open when the forward contract must be fulfilled. The only restriction imposed on an individual's choice of \( s \) is that, in each future state of nature, the value of his assets \( b + [(s/p^f) + k]p \) must be not less than his liability \( s \). This is the same as requiring that, in each future state, his income be non-negative.
Equations (13) and (14) are to be solved for the equilibrium values of \( s \) and \( s^* \).

Let us simplify the problem by assuming that Australian and German utility functions are identical and that domestic endowments of the preferred commodities (resp. non-preferred commodities) are identically and independently distributed (i.i.d.):

\[
U'(c) = U^*(c^*) \quad \text{for all } c = c^* \geq 0,
\]

\( b, k^* \text{ i.i.d.}, \)

\( k, b^* \text{ i.i.d.} \).

Under these symmetric assumptions, \( s = s^* \), and (14) reduces to

\[
r = \frac{s + b^*}{s + k} - 1, \quad r^* = \frac{s + k}{s + b^*} - 1. \quad (15)
\]

Since \( b^* \) and \( k \) are stochastically independent, but identically distributed, it is clear from these expressions that, in a market equilibrium,

\[
E(r) = \frac{E(s + b^*)}{H(s + k)} - 1 > 0 \quad (16)
\]

and

\[
E(r^*) = \frac{E(s + k)}{H(s + b^*)} - 1 > 0, \quad (17)
\]

where \( H(x) \equiv 1/E(1/x) \) is the harmonic mean of a random variable \( x \).\(^5\) From Jensen's inequality, \( E(x) > H(x) \) whenever \( x \) is positive and has a positive variance.

Equations (16) and (17) reflect the Siegel paradox. Obviously, price randomness per se can be profitable for both parties. The forward exchange rate between lamb and kraut, given by (9), is unity, and both the expected spot price of kraut, \( E(p) \), and the expected spot price of lamb, \( E(1/p) \), are greater than one. There is scope for mutually attractive speculative commitments that consist in taking a short position in the preferred commodity (lamb for Australians, kraut for Germans).\(^6\)

Whether the expected profits are large enough to imply a speculative equilibrium with \( s > 0 \) depends on the degree of risk aversion. Since a short position in the preferred commodity entails risks that are positively correlated with the existing exchange risk for the non-preferred commodity, we cannot rule out the possibility that \( s \leq 0 \) for sufficiently high risk aversion. However, the interesting question is whether equilibria with \( s > 0 \) are possible at all.

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5) Note that the stochastic independence of \( b^* \) and \( k \) implies stochastic independence of \( s + b^* \) and \( 1/(s + k) \), which makes it possible to write \( E(r) = E((s + b^*)/(s + k)) - 1 = [E(s + b^*)]E(1/(s + k)) - 1 = [E(s + b^*)]/H(s + k) - 1 \). Analogous reasoning applies to \( E(r^*) \).

6) The Siegel paradox that drives our results should not be confused with the paradox of Waugh (1944) and Oi (1961). The latter results from the convexity of the indirect utility and profit functions, implying that both firms and households prefer price variations around a given mean. The former results from the fact that price variations increase the mean, not from a preference for such variations. The Waugh–Oi paradox is a normative proposition which Samuelson (1972) showed to be incompatible with the requirements of general equilibrium. The Siegel paradox is a positive result which this paper shows to be compatible with such requirements.
To answer this question, note that, from the definition of a covariance, 
\( E(x \cdot y) = \text{cov}(x, y) + E(x)E(y) \), and \( \text{cov}(x, y + z) = \text{cov}(x, y) \) when \( \text{cov}(x, z) = 0 \).

Recalling (15), and normalizing the utility function so that \( E(\text{U}'(b + b^*)) = E[U'(k + k^*)] = 1 \), we can then write (13) in the form

\[
\text{cov}[\text{U}'(b + b^*), b^*/(s + k)] + E(r) = 0 = \text{cov}[\text{U}'(k + k^*), k^*/(s + b^*)] + E(r^*).
\]

(18)

The covariance terms in this expression are negative, their exact magnitudes depending on, among other things, the degree of concavity of the utility function \( \text{U}(\cdot) \). With any given value \( s > 0 \), and hence a given positive value of the expected returns \( E(r) \) and \( E(r^*) \) (cf. (16) and (17)), it is clearly possible to find a degree of concavity that makes the covariance terms sufficiently strongly negative to satisfy (18). Moreover, given the required degree of concavity, no other value of \( s \) can satisfy (18). This shows that there is a large class of utility functions for which the Siegel paradox harmonizes with the conditions of a unique market equilibrium.

An interesting limiting case is that of vanishing risk aversion, where \( \text{U}''(c) = 0 \) for all \( c \). As in this case the covariances are zero, it is necessary that the expected rates of return \( E(r) \) and \( E(r^*) \) are also zero. In view of (16) and (17), it is obvious that this in turn occurs when \( s \) equals infinity. Thus, without risk aversion, there is an incentive to hold infinitely large short positions in the preferred commodities, and there is no equilibrium.

We summarize this section with the following proposition.

**Proposition 1**: In the symmetrical model specified above, with strictly concave utility functions, there exists a class of speculative equilibria in which each party sells its preferred commodity in the forward market in order to participate in Siegel profits.

3. An example

It may be useful to construct a special example to further illustrate the nature of the solution.\(^7\) Suppose that

(i) \( b, k^* = A > 0 \), non-stochastic;

(ii) \( k, b^* \) are i.i.d. with equal probability of \( k = 1, 2 \); and

(iii) \( \text{U}'(c) = c^{-\varepsilon}, \text{U}'(c^*) = c^{* - \varepsilon}, \varepsilon > 0 \).

Assumption (i) says that in each country the endowment of the preferred commodity is non-stochastic. Assumption (ii) reduces the number of states of nature to four: the endowment vector \( (b^*, k) \) attains one of the four variates \((1, 1), (1, 2), (2, 1) \) and \((2, 2) \) with probability \( 1/4 \) each.\(^8\) Assumption (iii) requires the utility functions to display constant relative risk aversion, where \( \varepsilon \) is the degree of relative risk aversion according

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7) A non-symmetrical example where (i) \( (b, b^*) = A > 0 \), non-stochastic, and (ii) \( (k, k^*) = (1, 1), (1, 2), (2, 1) \) and \((2, 2) \) with probability \( 1/4 \) each was communicated to us by Syed Ahsan in a comment on this paper.

8) Since there are two commodities and four states of nature, an Arrow–Debreu model would have seven contingency markets under these circumstances.
to the Pratt–Arrow definition. Given these simplifying assumptions, it is again true that \( s = s^* \), and it follows from (13) and (14) that

\[
(A + 1)^{-1} - \frac{1}{s + 1} + (A + 1)^{-1} - \frac{1}{s + 2} + (A + 2)^{-1} - \frac{1}{s + 1} + (A + 2)^{-1} - \frac{2}{s + 2} = 0
\]
or

\[
\left( \frac{A + 2}{A + 1} \right)^{\varepsilon} = \frac{s + 2}{s + 1}.
\]  \hspace{1cm} (19)

Figure 1 helps to interpret equation (19). The heavy curve depicts the values of \( (s + 2)/(s + 1) \) for alternative values of \( s \); the two other curves depict \( [(A + 2)/(A + 1)]^{\varepsilon} \) for alternative values of \( A \) where the upper curve is typical for \( \varepsilon > 1 \) and the lower one for \( \varepsilon < 1 \). Given \( A \) (the domestic output of the preferred good), the corresponding optimal value of \( s \) can be found in the manner illustrated for the two values \( A = A_1 \) and \( A = A_2 \). It is obvious that

\[
S_{\text{opt}} \{ \geq \} A \Leftrightarrow \varepsilon \{ \leq \} 1.
\]

The following proposition expresses the finding in words.

**Proposition 2:** With relative risk aversion less than or equal to one, and under the conditions of the example specified above, people sell short at least their endowment of the preferred commodity. When relative risk aversion exceeds unity, forward sales fall short of the endowment but remain positive provided the excess of relative risk aversion over unity is not too large.

4. Welfare implications

In our simple model, speculation does generate private gains. In a speculative equilibrium with, say, \( s > 0 \), each individual household attains a higher level of expected utility than

\[\text{Figure 1. Relative risk aversion (}\varepsilon\text{) and the optimal forward supply of the preferred commodity (}\text{s}_1\text{)}\]

if it had chosen a lower speculative commitment or had decided not to speculate at all. This follows from the first-order conditions (2) and (5) and the fact that expected utilities are strictly concave in $s$ and $s^*$, that is, $E[U(c)] < 0$ and $E[U'(c^*)] < 0$. Nevertheless, every household ends up with the same future consumption as when only spot contracts are allowed. In both cases, Australian households consume $b + b^*$ and German households $k + k^*$. The introduction of a forward market induces speculative activities but leaves unchanged the expected and realized utilities of both countries. Speculation is individually profitable but socially useless.

While speculation is neither beneficial nor harmful in the above model, it becomes harmful when transaction costs are introduced. Suppose that activity in the forward market incurs transaction costs $\delta(|s|)$ and $\delta^*(|s^*|)$, in terms of lamb for Australians and kraut for Germans, and payable in the future. Suppose further that the transaction technology is convex and, in particular, that $\delta(0) = 0 = \delta^*(0)$, and that $\delta(|s|)$ and $\delta^*(|s^*|)$ are positive, $\delta^*(|s|)$ and $\delta^*(|s^*|)$ non-negative. Suppose finally that the transaction costs are not so great that in equilibrium households choose not to speculate. Then, in equilibrium, Australians must consume less than $b + b^*$ and Germans less than $k + k^*$.

Clearly, in this case policy measures directed against speculation, including the closure of the forward market, are welfare-enhancing for both countries. Moreover, such measures must increase the gains from trade for both countries.

It is widely believed that profitable and price-stabilizing speculation is socially beneficial. One therefore might suspect that in the extended model the welfare loss can be understood in terms of the destabilizing role of speculation. However, it is easy to see that this suspicion is unfounded. In view of (10) and the properties of the symmetrical equilibrium, the spot price of kraut in units of lamb is endogenously determined as

$$p = \frac{(s + b^*)}{(s + k)}.$$

This equation shows that speculation is indeed stabilizing. Speculation reduces the price variance because it adds to both sides of the market non-random supplies of the non-preferred commodities received from the forward contracts:

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9) It has been suggested to us that, whenever transactions in the forward market are costly, that market will be idle in equilibrium. That this is not so can be shown by extending the worked example of Section 3 to accommodate linear transaction costs $\delta(|s|)$ and $\delta(|s^*|)$, $\delta > 0$. Then (19) takes the more general form

$$\begin{aligned}
\left( \frac{A + 2}{A + 1} \right)^c &= \frac{s + 2}{s + 1} \frac{1 - 2\delta(s + 1)}{1 + 2\delta(s + 2)}.
\end{aligned}$$

The right-hand side of (19') goes to $-1$ as $s$ goes to infinity. However, for sufficiently small $\delta$ the locus of the right-hand side cuts the vertical axis of Figure 1 just below 2 and therefore lies between the other two loci for all sufficiently small and positive $s$. It is therefore possible to choose $A$ so that the optimal $s$ is positive.

10) The example can be further extended to accommodate costs of transacting on both the spot and forward markets. In particular, it can be extended to allow for constant average costs, the same for both markets; for then individuals who sell their preferred commodity short incur a second round of costs when they buy it back spot. Allen and Gale (1990) have constructed a model in which the establishment or non-establishment of an options exchange is determined endogenously. They show that there are situations in which an options exchange is set up in equilibrium even though everybody could be better off if an exchange were not set up. In their model, which is a good deal more complicated than ours, a monopolist owner of the exchange charges a fixed entry fee for the privilege of transacting.
\[
\frac{\partial \text{var}(p)}{\partial s} = \frac{\partial \text{var}(1/p)}{\partial s} < 0.
\]

To understand the role of speculation in reducing welfare, one must recall that our model is one of incomplete markets. With complete markets, pecuniary externalities (like those generated by our speculators) are Pareto-irrelevant; that is, they cannot make everyone worse off. With incomplete markets or other distortions, however, such externalities may be welfare-reducing, as Scitovsky (1954) observed long ago.

**Proposition 3:** In the model of this paper, with transaction costs accommodated, speculation is individually profitable and stabilizes the spot price but nevertheless may lower social welfare.

5. The robustness of our conclusions

In Section 3 we relied on a very special assumption about preferences: each individual, whatever his country of residence, derives utility from his consumption of just one commodity. It has been suggested to us that this assumption is crucial—that if some Australians care even a little for kraut and if some Germans care even a little for lamb, our welfare conclusions are unattainable; for, the reasoning goes, the forward market enables those individuals who care for both goods to spread their risks and thus makes possible a better social allocation of risk. However, the argument does not survive close examination. Let us denote by \(c_b\) and \(c_k\) the Australian consumption of lamb and kraut, respectively, and let us define

\[
u(1 + c_b) \equiv U(c_b).
\]

Evidently, \(u(1 + c_b)\) is obtained as a special case of

\[
u[(1 + c_b)^a(1 + c_k)^{1-a}]
\]

by setting \(a = 1\). Similarly, \(u(1 + c_k^*) \equiv U(c_k^*)\) is obtained as a special case of

\[
u[(1 + c_b^*)^{1-a}(1 + c_k^*)^a]
\]

by setting \(a = 1\). Let us suppose that, when \(a = 1\), the equilibrium values of \(s\) and \(s^*\) are positive. Now let \(a\) jump from 1 to \(1 - \Delta, \Delta > 0\). The Australian marginal rate of substitution of kraut for lamb, at the old equilibrium quantities, is

\[
\frac{dc_k}{dc_b} \bigg|_U = \frac{1 - \Delta}{\Delta} \frac{1 + c_k}{1 + c_b} = \frac{1 - \Delta}{\Delta} \frac{1}{1 + b + b^*}.
\]

This expression can be made as large as need be by choosing \(\Delta\) sufficiently small. It follows that, for sufficiently small \(\Delta\), the new equilibrium coincides with the old.

6. Conclusions

We have shown that the traditional view of economists that profitable and price-stabilizing speculation is a socially useful activity needs qualification, at least in a general equilibrium context.
To understand the potentially harmful role of speculation, one must look not to illusion, monetary or otherwise, nor to incompatible expectations: instead, one must look to the possibly harmful role of pecuniary externalities in a context of incomplete markets.

Final version accepted 18 February 1998.

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