

Capital Income Taxation, Depreciation Allowances and Economic Growth: A Perfect-Foresight General Equilibrium Model

By

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1. Introduction

In a previous article (Sinn 1980) the author studied the role of capital income taxation in a perfect-foresight general equilibrium model of economic growth, in which the behaviour of households and firms is derived from intertemporal optimization¹. Generating the neoclassical optimal-growth path in the case of *laissez faire*, the model demonstrates the distortions caused by the introduction of a capital income tax with lump-sum redistribution: the current level of consumption rises at the expense of the steady state level².

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¹ The paper was presented at the 1979 annual meeting of the German economic association and, in addition to capital income taxation, covers a range of further taxes. An English version, prepared for a seminar at the University of Western Ontario in spring 1979, and a German discussion paper including existence proofs (Mannheim, October 1979, No. 132-79) are available from the author on request.

² There are several articles that address the problem of capital income taxation in the context of a decentralized model of economic growth, but in none of these is household behaviour derived from preferences compatible with those assumed in the optimal-growth literature. Since this aspect implies that the market allocation is inferior to the neoclassical optimal growth path even in the absence of taxation, the analyses given in these articles are of limited use for a welfare evaluation of the distortions introduced through taxation. See Krzyzaniak (1966), Sato (1967), Diamond (1970), Feldstein (1974 a and b) and Atkinson/Sandmo

The income tax considered was however of a very idealized form. No divergence between economic and tax depreciation was allowed. This paper considers the more general case where depreciation allowances may be incomplete.

There is a large number of careful investigations into the economic consequences of incomplete depreciation allowances. (See, e. g., Boadway/Bruce 1979; Feldstein/Green/Sheshinsky 1978; Hall/Jorgenson 1971; King 1974; Samuelson 1964; Sandmo 1974; Smith 1963; Stiglitz 1976). But it seems that no attempt has been made to find out what the resulting distortions in the growth path of the economy are, let alone any attempt to address this problem in the context of a full intertemporal equilibrium framework. This lack is the stimulus for the present paper.

2. Individual Optimization under the Influence of Taxation

Consider first the firm sector. The representative firm produces one commodity which it can either sell to the household sector for consumption or use for investment in its own capital stock K . The firm's net output is $f(K, L) - \delta K$, where f is a well-behaved linear homogeneous production function, δ is the economic depreciation rate and L is a labour input which is measured in efficiency units so as to allow for the possibility of labour augmenting technological progress. In finding its optimal production plan the firm has to take into account that government taxes all kinds of capital income at the rate τ , $0 \leq \tau < 1$, and allows the share α , $0 \leq \alpha \leq 1$, of the economic depreciation to be deducted³. Accordingly, with the wage rate per efficient worker w and the planned levels of consumption

(1980). Partial exceptions are Arrow/Kurz (1970) and Schenone (1975). Arrow and Kurz address the problem of controllability of a market economy through a capital income tax, but they do not derive the allocation path in the presence of such a tax. Schenone studies an (ideal) capital income tax in a two-sector growth model and is able to demonstrate that this tax might induce distortions away from the optimal neoclassical growth path. Like Arrow and Kurz, however, Schenone does not explicitly depict a decentralized economy where private behaviour is derived from intertemporal optimization. Instead it is assumed that the market behaves as if it were steered by a central planner who maximizes welfare under some restrictions about marginal conditions.

³ This is the simple form of imperfect depreciation allowances used, e. g., by Sandmo (1974). A more sophisticated form of imperfection would be to introduce an accounting capital stock in addition to the physical capital stock. For the purpose of this paper this complication is avoided.

supply C^s and labour demand L^d , the firm calculates its after-tax cash flow as $R = C^s - wL^d - \tau [f(K, L^d) - \alpha \delta K - wL^d]$. Taking the time paths of the market rate of interest $\{r\}$ and the wage rate $\{w\}$ as given, it chooses the paths $\{C^s\}$ and $\{L^d\}$ such that its market value is maximized⁴:

$$\max_{\{C^s, L^d\}} J(0) = \int_0^{\infty} \exp \left[-\int_0^t r(s) (1-\tau) ds \right] R(t) dt \quad (1)$$

$$\text{s. t. } \dot{K} = f(K, L^d) - \delta K - C^s$$

and $K(0) = K_0$, $L^d \geq 0$, $K > 0$ ⁵. From the Hamiltonian $H_F = \exp \left[-\int_0^t r(s) (1-\tau) ds \right] [R + \lambda_F \dot{K}]$ one can easily derive two important necessary conditions for an interior optimum:

$$w = f_L, \quad (2)$$

and

$$r = (f_K - \delta) - \delta \frac{\tau}{1-\tau} (1-\alpha). \quad (3)$$

Eq. (3) reflects the result achieved by Samuelson (1964) and Sandmo (1974) that, given the market rate of interest, capital income taxation does not distort the firm's investment decision if, and only if, economic depreciation is fully tax-deductible.

Next, consider the household sector. The representative household plans not only for its current members, but also for its possibly increasing number of descendants⁶. At a point in time the num-

⁴ The formulation is equivalent to the case of debt financing with full tax deductibility of interest payments if J is defined as the firm's market value including the value of debt. Since there is a uniform tax rate on all kinds of capital income the firm is indifferent between debt and equity financing just as in the absence of taxes. Let $A \equiv$ outstanding debt. Then, adding $\dot{A} - r(1-\tau)A$ to the expression for R and taking \dot{A} as an additional control and A as an additional state variable, one can easily show that the marginal conditions (2) and (3) still have to hold.

⁵ We do not assume $C^s \geq 0$ since the representative firm might plan to buy goods in the market for its own investment. However, since in market equilibrium $C^s > 0$, this aspect is irrelevant for our results.

⁶ We assume that the household plans for an infinite time period. Because of Bellman's principle of optimality it is always possible to formulate an equivalent planning problem with a finite horizon by appropriately choosing the evaluation function for the terminal state variable(s).

ber of heads is N , $N > 0$, and, since a single person offers a quantity of efficient labour E , $E > 0$, the total supply of efficient labour is $L^s = NE$. It is assumed that this supply does not depend on the wage rate and that $\dot{N}/N \equiv n = \text{const.} \geq 0$ and $\dot{E}/E \equiv \varepsilon = \text{const.} \geq 0$. The state variable in the household's decision problem is the level of wealth X . In principle wealth consists of four parts. (1) Some share of the representative firm's market value. (2) The present value of lump-sum transfers it receives from the government which, because of the government budget constraint, equals the present value of the tax revenue. (3) The present value of its labour income stream. (4) The net claim against other households. Of course, in market equilibrium the representative household must plan to own all shares of the representative firm and to have a zero net claim against others. Nevertheless it is assumed that it believes it could sell its shares, borrow against the income from items (2) and (3), and save as much as it wants. Since the household takes the price paths $\{r\}$ and $\{w\}$ as given, as well as the corresponding policy of the representative firm, it can manipulate its wealth path only through its saving-consumption decision. In the absence of consumption, wealth increases according to the after-tax interest income it generates: $\dot{X} = r(1 - \tau)X$. With consumption the increase is correspondingly less. The household's preferences are modeled in line with the island argument of Arrow/Kurz (1970, p. 13 f.). The preference functional is the integral of felicity weighted with a discount factor $\exp(-\rho t)$, $\rho > 0$, and the number of heads, where felicity is a rising and strictly concave function of per capita consumption. Thus the problem to be solved is:

$$\max_{\{C^d\}} \int_0^{\infty} e^{-\rho t} N(t) U[C^d(t)/N(t)] dt \quad (4)$$

$$\text{s. t. } \dot{X} = r(1 - \tau)X - C^d$$

and $X(0) = X_0$, $C^d \geq 0$, $X \geq 0$. The Hamiltonian of this problem is $H_H = \exp(-\rho t) [NU(C^d/N) + \lambda_H \dot{X}]$. Assuming an isoelastic marginal felicity function where $\eta = -(C^d/N) U''/U' = \text{const.} \geq 0$ one can easily show that this Hamiltonian gives

$$\frac{\dot{C}^d}{C^d} = n + \frac{r(1 - \tau) - \rho}{\eta} \quad (5)$$

as a necessary condition for an optimal consumption path.

3. The Intertemporal Market Equilibrium

In a market equilibrium the price paths $\{r\}$ and $\{w\}$ are chosen such that $C^d = C^S \equiv C$ and $L^d = L^S \equiv L$ for all points in time⁷. In order to study the nature of equilibrium it is useful to redefine consumption, capital and output as quantities per efficiency unit of labour:

$$c \equiv C/L, k \equiv K/L, \varphi(k) \equiv f(k, 1) = f(K, L)/L. \tag{6}$$

Because of $\dot{c}/c = \dot{C}/C - \dot{L}/L$, $\dot{L}/L = n + \varepsilon$, and $f_K = \varphi'$ we obtain from (3) and (5):

$$\dot{c} = \frac{c}{\eta} [(\varphi' - \delta)(1 - \tau) - \delta\tau(1 - \alpha) - \rho - \eta\varepsilon]. \tag{7}$$

Moreover, dividing the firm's equation of motion from (1) by L and noting that $\dot{K}/L = \dot{k}K/K = k(\dot{k}/k + n + \varepsilon)$ we have

$$\dot{k} = \varphi(k) - (\delta + n + \varepsilon)k - c. \tag{8}$$

The differential eqs. (7) and (8) reflect some of the necessary conditions for a market equilibrium and define a set of paths in a

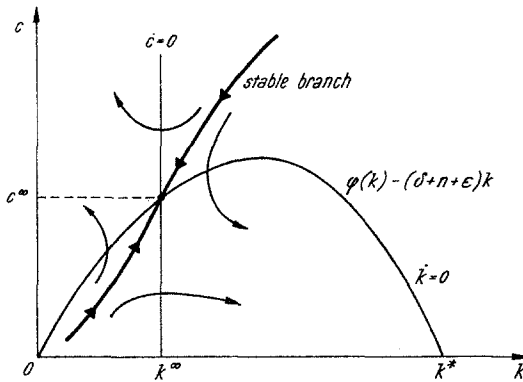


Fig. 1. The intertemporal market equilibrium

(c, k) plane, the slopes of which are $dc/dk = \dot{c}/\dot{k}$. This is shown in Fig. 1. This figure is well known from central planning growth models and hence does not need elaborate explanation⁸.

⁷ By Walras' law the capital market is then also in equilibrium.

⁸ Cf., e. g., Intriligator (1971, ch. 16).

Among the possible paths there is a positively sloped *stable branch* leading eventually to the steady state equilibrium (c^∞, k^∞) , defined such that

$$[\varphi'(k^\infty) - \delta](1 - \tau) - \delta\tau(1 - \alpha) = \rho + \eta\varepsilon, \quad (9)$$

$$c^\infty = \varphi(k^\infty) - (\delta + n + \varepsilon)k^\infty. \quad (10)$$

In the appendix it is shown that only this stable branch is compatible with further conditions that have to hold in an intertemporal market equilibrium. Moreover it is shown that the stable branch is consistent with individual optimization if, and only if, the steady state rate of time preference exceeds the natural rate of growth⁹: $\rho + \eta\varepsilon > n + \varepsilon$. Since (9) gives $\varphi' - \delta \geq \rho + \eta\varepsilon$, this inequality implies that $\varphi' - \delta - n - \varepsilon > 0$ and hence the steady state solution point must be to the left of the maximum of the $\dot{k} = 0$ curve in Fig. 1.

4. The Role of the Tax Law

To understand the nature of market equilibrium consider first the case $\tau = 0$. Here Eqs. (7)—(10) give the same stable branch as the central planning model by Arrow and Kurz (1970, pp. 64—73). In particular, (9) reduces to the *golden utility rule* in the presence of labour augmenting technological progress (ε):

$$\varphi'(k^\infty) - \delta = \rho + \eta\varepsilon. \quad (11)$$

Hence, *laissez faire* brings about the same allocation as that which a central planner would choose if he tried to maximize the representative household's utility.

When the *laissez-faire* allocation is taken as a benchmark, Eqs. (7)—(10) indicate significant welfare losses if a capital income tax with possibly incomplete depreciation allowances is introduced. An overview of the distortions in the economy's growth path is given in Table 1.

The reason for the two zeros in the first row of Table 1 is that k is a state variable that can only change continuously over time. The information in the first two fields of the second row is gained from implicit differentiation of (9). The remaining fields of that

⁹ The condition equals the optimum condition in the central planning model. Cf. Arrow/Kurz (1970, pp. 70—72). While this is not surprising it is also not trivial, since we are considering a decentralized model and distorting taxation.

row reflect (10). A quantitative assessment of the reaction coefficients for the last two fields in the first row is a difficult task, but qualitative information on their signs can easily be given by inspection of Fig. 1 and Eqs. (7) and (8). Consider an *increase* in τ or, alternatively, a *decrease* in α . From $dk^\infty/d\tau < 0$ and $dk^\infty/d\alpha > 0$ we know that such parametric changes reduce k^∞ , say from k_1^∞ to k_2^∞ .

Table 1. Immediate and Steady State Reactions of Capital and Consumption to a Change in the Tax Law*

Time perspective	Reaction coefficients			
	$\frac{dk}{d\tau}$	$\frac{dk}{d\alpha}$	$\frac{dc}{d\tau}$	$\frac{dc}{d\alpha}$
Short run	0	0	> 0	< 0
Steady state	$\frac{\varphi' - \alpha\delta}{(1-\tau)\varphi''} < 0$	$-\frac{\delta\tau}{(1-\tau)\varphi''} > 0$	$\frac{[\varphi' - (\delta + \eta + \epsilon)]}{dk/d\tau} < 0$	$\frac{[\varphi' - (\delta + n + \epsilon)]}{dk/d\alpha} > 0$

* For the indicated signs of the coefficients it is assumed that $\tau > 0$.

From the general properties of the solution it is clear that the stable branch shifts up in the range $k_2^\infty \leq k \leq k_1^\infty$, but it is not obvious how its position changes elsewhere. Suppose it does not shift up for all levels of k below k_2^∞ . Then there must exist at least one $k < k_2^\infty$, where the new stable branch coincides with the old one and has a slope \dot{c}/\dot{k} that is equal to or greater than that of the old one. Since $\dot{k} > 0$ for $k < k_2^\infty$, this requires $d\dot{c}/d\tau \geq 0$ or $d\dot{c}/d\alpha \leq 0$, respectively. Eq. (7), however, shows that these conditions are not satisfied. Suppose, alternatively, the stable branch does not shift up for all k above k_1^∞ . Then it is necessary that there be some value $k > k_1^\infty$ where the new stable branch coincides with the old one and has a slope equal to or smaller than that of the old one. Since $\dot{k} < 0$ in the case at hand, this again requires $d\dot{c}/d\tau \geq 0$ and $d\dot{c}/d\alpha \leq 0$. Hence the information on the reaction parameters given in Table 1 can be established throughout the whole range $k > 0$.

Another interesting aspect of the equilibrium growth path entails the reaction of interest rates to a change in tax parameters incorporated in Eqs. (3), (5) (implicitly) and (9). The general rule is

$$z \leq r \leq \varphi' - \delta, \quad r - z = \tau r, \quad \varphi' - \delta - r = \delta(1 - \alpha)\tau / (1 - \tau) \quad (12)$$

where $z \equiv \eta(\dot{C}/C - n) + \rho$ is the rate of time preference. (12) shows

how the rate of time preference, the market rate of interest and the marginal productivity of capital are related to one another and indicates that tax laws are able to drive wedges between them. By straightforward calculations it is possible to derive from (12) the reaction coefficients given in Table 2, if we take into account the

Table 2. Immediate and Steady State Reactions of Interest Rates to a Change in the Tax Law*

Time perspective	Reaction coefficients					
	$\frac{dz}{d\tau}$	$\frac{dr}{d\tau}$	$\frac{d(\varphi' - \delta)}{d\tau}$	$\frac{dz}{d\alpha}$	$\frac{dr}{d\alpha}$	$\frac{d(\varphi' - \delta)}{d\alpha}$
Short run	$-\frac{z + \delta(1 - \alpha)}{(1 - \tau)^2} < 0$	$-\frac{\delta(1 - \alpha)}{(1 - \tau)^2} \leq 0$	0	$\delta\tau > 0$	$\frac{\delta\tau}{1 - \tau} > 0$	0
Steady state	0	$\frac{r}{1 - \tau} > 0$	$\frac{z + r(1 - \alpha)}{(1 - \tau)^2} > 0$	0	0	$-\frac{\delta\tau}{1 - \tau} < 0$

* For the indicated signs of the coefficients it is assumed that $\tau > 0$.

fact that in the short run $\varphi' - \delta$ is fixed by the historically given capital stock, while in the long run z is fixed at the level $\rho + \eta\varepsilon$ [cf. (9)].

5. Conclusions

The results indicated in Table 1 show that incomplete depreciation allowances reinforce the distortions in the equilibrium growth path brought about by an ideal capital income tax. A reduction in the deductible share of economic depreciation, like an increase in the tax rate, raises the current level of consumption, but reduces the steady state levels of consumption and capital per efficiency unit of labour.

The reason for these distortions is that the tax law is able to drive wedges both between the rate of time preference and the market rate of interest, and between the latter and the marginal productivity of capital. The first wedge is created through capital income taxation as such and its size is directly related to the tax rate. The second wedge is created by the incomplete deductibility of depreciation. Its size is directly related to the tax rate and inversely to the deductible share of depreciation. For the distortion in the growth path of the economy it is the sum of the two wedges

that counts. Therefore it is plausible that incomplete depreciation allowances reinforce the effects of capital income taxation.

Knowing the determinants of the two wedges one can easily derive the influence of a tax reform on the marginal productivity of capital, the market rate of interest and the rate of time preference (cf. Table 2). In the short run, the system of these three interest rates is anchored by the marginal productivity of capital, and hence any measure that widens a wedge is translated into a reduction in the rate or those rates below the wedge. In the long run the system is anchored by the rate of time preference and an increase in the width of a wedge is translated into an increase in those rates or that rate above this wedge.

Appendix

The appendix sketches the proof 1. that only the stable branch may be a market equilibrium and 2. that on the stable branch the two representative agents have optimized their plans if, and only if, $\varrho + \eta\varepsilon > n + \varepsilon$. For the proof we need the transversality conditions of the representative agents. Since $\dot{K}/K = \dot{k}/k + (n + \varepsilon)$ and $\lambda_F = 1 \forall t$, the firm's transversality condition is $\lim_{t \rightarrow \infty} \{ \exp [- \int_0^t (r(s)(1 - \tau) - n - \varepsilon) ds] k(t) \} = 0$. Provided that $0 < \lim_{t \rightarrow \infty} k(t) < \infty$, by continuity the condition is satisfied if, and only if,

$$\lim_{s \rightarrow \infty} r(s)(1 - \tau) > n + \varepsilon. \tag{A 1}$$

The household's transversality condition is

$$\lim_{t \rightarrow \infty} \{ \exp (-\varrho t) U' [E(t) c(t)] X(t) \} = 0.$$

Provided that $0 < \lim_{t \rightarrow \infty} c(t) < \infty$ it can also be written as

$$\lim_{t \rightarrow \infty} \{ \exp [-t(\varrho + \eta\varepsilon)] X(t) \} = 0 \tag{A 2}$$

since the assumption of isoelastic marginal utility implies

$$U' [E(t) c(t)] = E^{-\eta}(t) U' [c(t)] = E^{-\eta}(0) \exp (-t\varepsilon\eta) U' [c(t)]$$

where $E^{-\eta}(0) > 0$.

1. Paths other than the Stable Branch

1.1. Paths above the stable branch imply that there is some $u < 0$ such that after a finite period of time $\dot{k} = \varphi(k) - (\delta + n + \epsilon)k - c < u$. Hence $k = 0$ in finite time and the paths become infeasible.

1.2. Paths below the stable branch approach the ordinate at $k = k^* > 0$ in Fig. 1, where $\varphi'(k) - (\delta + n + \epsilon) < 0$. Because of (3) and $f_k = \varphi'$ the latter inequality is equivalent to $r(1 - \tau) + \delta\tau(1 - \alpha) - (n + \epsilon)(1 - \tau) < 0$. This clearly violates (A 1).

2. The Stable Branch

2.1. On the stable branch $\lim_{t \rightarrow \infty} k(t) = k^\infty > 0$ and, from (3) and (9), $\lim_{t \rightarrow \infty} r(t)(1 - \tau) = \rho + \epsilon\eta$. Hence the firm's transversality condition (A1) is satisfied if, and only if, $\rho + \epsilon\eta > n + \epsilon$.

2.2. According to the definition of the household's wealth given in the body of the paper, in market equilibrium wealth is the present value of the representative firm's gross revenue: $X(t) = \int_t^\infty \exp[-\int_t^{t^*} r(s)(1 - \tau) ds] C(t^*) dt^* = N(0) E(0) \exp[t(n + \epsilon)] b(t)$ where $b(t) \equiv \int_t^\infty \exp[-\int_t^{t^*} (r(s)(1 - \tau) - n - \epsilon) ds] c(t^*) dt^*$. Dropping the constants $N(0)$ and $E(0)$ one can therefore write the transversality condition (A 2) as $\lim_{t \rightarrow \infty} a(t) b(t)$ with $a(t) \equiv \exp[-t(\rho + \eta\epsilon - n - \epsilon)]$. Since (10) implies $\lim_{t^* \rightarrow \infty} c(t^*) = c^\infty$ and since (3) and (9) give $\lim_{s \rightarrow \infty} r(s)(1 - \tau) = \rho + \epsilon\eta$, we find $\lim_{t \rightarrow \infty} b(t) = \lim_{t \rightarrow \infty} \int_t^\infty \exp[-(t^* - t)(\rho + \eta\epsilon - n - \epsilon)] c^\infty dt^* = c^\infty / (\rho + \eta\epsilon - n - \epsilon) < \infty$ if, and only if, $\rho + \eta\epsilon > n + \epsilon$. Now, obviously, the same condition ensures $\lim_{t \rightarrow \infty} a(t) = 0$. Hence (A 2) is satisfied if, and only if, $\rho + \eta\epsilon > n + \epsilon$.

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