

## The Theory of Exhaustible Resources

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(Received February 13, 1981)

*In economic textbooks one cannot find much of use about the phenomenon of depletable resources.* It was shortly before the time the economic editor of the leading German news magazine wrote this sentence<sup>1</sup> when two new and important books on natural resources, one by Murray C. Kemp and Ngo V. Long (KL), the other by Partha S. Dasgupta and Geoffrey M. Heal (DH), appeared in the book stores.

In the last decade resource economics has occupied the attention of cohorts of professional economists and by now a well established theory is available. DH have written an excellent text book reporting the state of the art and also adding some new results. KL have edited a collection of high-quality articles that generalize, extend and correct the present theory of exhaustible resources.

DH's book is rather voluminous. On 500 pages a large number of interesting problems in resource economics is discussed<sup>2</sup>. The

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\* The paper is a discussion of P. S. Dasgupta and G. M. Heal, *Economic Theory and Exhaustible Resources*, Cambridge: University Press, 1979, and M. C. Kemp and N. V. Long (eds.), *Exhaustible Resources, Optimality, and Trade*, Amsterdam, New York, Oxford: North Holland Publishing Company, 1980. It was written in association with the Sonderforschungsbereich 5, project II/B.

<sup>1</sup> Wolfgang Kaden, "Ein Glück, daß es die Opcc gibt", *Der Spiegel*, June 16, 1980, p. 60. The sentence is translated from German.

<sup>2</sup> A kind of summary of some of the major problems is given in a paper that Heal presented to the 1979 annual meeting of the German economic association. See G. M. Heal: *Intertemporal Allocation and Intergenerational Equity*, in: H. Siebert (ed.): *Erschöpfbare Ressourcen (Arbeitstagung des Vereins für Socialpolitik, Mannheim 1979)*, Berlin 1980, pp. 499—528.

analysis is mainly theoretical, but includes some case studies and also reports empirical research. Since the authors avoided dynamic optimization techniques the book is readable by students. The first fifth of the book does not address the problem of exhaustible resources and is designed as an introduction into microeconomic theory as such. This part might have been abbreviated, in particular since problems like, e. g., the allocation of non-rival goods are addressed that practically are not taken up in the remainder of the book. The book is, however, well written and will please the careful reader.

KL's book is about half as thick as that of DH. It contains 19 separate articles covering many of the major topics in resource economics. One article was written by Akira Takayama, three by Carl Chiarella, the rest jointly by the editors. The book has three parts, one about the theory of the mining firm, another containing aggregate models of a closed economy, the third about international trade. Each part is introduced by a brief but informative overview over the problems discussed. Due to their concise and technical lay-out the papers are designed for the advanced student or the professional. The reader acquainted with the basic results in the theory of exhaustible resources and the standard techniques of dynamic optimization will appreciate the elegant way the authors typically go right into the hearts of their problems and find the solutions.

Although DH, KL and co-workers study related problems and did so in the past their mutual citation index is relatively low. DH cite 29 publications of DH, one of Kemp and none of his co-workers. Kemp et al., in turn, cite 19 of their own publications, but only two of Dasgupta and Heal. There seems to be an unexploited potential for gains from trade.

Facing the impossibility of discussing even a considerable share of the problems taken up in the two books I constrain my attention to a selection of those that, I feel, merit comment.

### Optimal Order of Extraction

Suppose, a firm owns several deposits of ore that differ in their (constant) unit extraction costs. In which order should it extract? The basic result of Herfindahl and Solow/Wan, also illustrated by DH (pp. 172—175), is that under a positive rate of interest it should start with the lowest-cost deposit and then proceed gradually to the higher-cost ores.

KL (essays 3 and 12) study a similar but more general problem where the unit extraction costs are time dependent. Suppose in a Ricardian fashion that a unit extraction cost is the product of a labour coefficient (characterizing the quality of ore) and a wage rate that may vary over time. If the wage rate is constant, the Herfindahl result implies that the order of extraction is given by the order of labour coefficients where the deposit with the lowest labour coefficient comes first. But what if the wage rate is rising? KL study this question a) within a complete model of the single mining firm and also b) in a general equilibrium framework. They conclude for case a) that the order of extraction is maintained if the firm is a competitive seller and the price of the extracted resource rises at a rate less than the rate of interest (Prop. 1, p. 34) and for b) that there is a general equilibrium where the order of extraction is indeterminate and the wage rate rises exponentially at the rate of interest (Lemma 3, p. 158, and the proposition on p. 159).

These are interesting results. The only problem is that they hide to some extent the clear statement that, *independently of market structures, price changes etc.*, the order of extraction is *solely* determined by the time path of the discounted wage rate<sup>3</sup>. If the discounted wage rate is constant, the order of extraction is a matter of indifference. If it rises over time, the deposit with the highest labour coefficient is extracted first, if it falls, the one with the lowest coefficient comes first, the others, respectively, following in strict sequence.

The proof is simple. Assume for example that the discounted wage rate rises over time. Consider a firm that (whatever the other market data might be) has optimized its extraction path and suppose it violates the rule given above. Then it is possible to find at least two units of ore, each from a different deposit, such that the unit from the deposit with the lower coefficient of labour is extracted first. Obviously, interchanging the time order of extraction for the two units does not affect the path of extracted resources the firm sells in the market, but reduces the present value of extraction costs. So, contrary to the above assumption, the firm cannot have optimized its extraction path. The proof can easily be extended to the cases of a constant and falling discounted wage rate.

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<sup>3</sup> The reader is advised to consult another paper of the authors which is more translucent and explicitly derives the result following in the text. See M. C. Kemp and N. V. Long: On the Optimal Order of Exploitation of Deposits of an Exhaustible Resource: The Case of Uncertainty, in: H. Siebert, op. cit., pp. 301—317.

### Technological Progress and Substitution Possibilities

Since the works of Solow and Stiglitz we know that a constant level of consumption can be maintained permanently despite resource extraction if the elasticity of substitution between capital and natural resources is greater than one (for the Cobb-Douglas case: if the partial production elasticity of capital exceeds that of the resource) or if there is resource-augmenting technological progress.

DH and KL offer some ideas about what they think the elasticity of substitution will be in the relevant range for low levels of output and resource use. On the basis of the familiar Inada conditions KL (Appendix) try to defend the Cobb-Douglas case of a unitary elasticity. DH (pp. 208—212) instead infer from elementary thermodynamic considerations the more pessimistic possibility that the elasticity be less than unity. I do not see how Inada can withstand thermodynamics.

Since an elasticity below unity implies that technological progress is essential for the survival of a resource-consuming economy the reader will appreciate the interesting microeconomic theory of the development of a substitute (like nuclear fusion) that DH develop on pages 175—190 and their analysis of the production of information on pages 418—427.

Instead of a microeconomic theory of technological progress, in KL's book Chiarella and Takayama (essays 7 and 8) study macroeconomic planning models with Cobb-Douglas technologies and endogenous technological progress. While both authors consider the possibility that progress affects output in a multiplicative way, Takayama in addition studies technological progress that augments the resource stock.

This would have been a nice endeavour were it not for the Cobb-Douglas function where, as is well known, there is no meaningful difference between factor-augmenting and output-augmenting progress. Takayama contends that in the case of resource-augmenting technological progress the Solow/Stiglitz condition for intertemporal efficiency has to be replaced by another condition which requires that the growth rate of the marginal productivity of the resource plus the rate of factor augmentation be equal to the rate of return on capital [Eq. (14), p. 99]. Although formally correct, this contention is misleading for in fact the "marginal productivity" appearing in his formula is the marginal productivity of an efficiency unit. Since the marginal productivity of a physical unit grows at the rate of factor augmentation plus the rate of growth of the marginal

productivity of an efficiency unit, Takayama's formula does not alter the Solow/Stiglitz condition, but reaffirms it<sup>4</sup>.

### Taxation

At various places in KL's book taxation to restore optimality is considered. The analysis is constrained to commodity taxation and the authors assume that the time paths of the tax rates can be freely chosen. DH (ch. 12) instead offer a more comprehensive discussion of taxation that includes inter alia capital gains, interest income and sales taxes, all with constant tax rates. The analysis is elegant and straightforward, but the loss from being partial analytical is greater than usual.

In fact, DH's results do not carry over to a general equilibrium world in the cases where there is a positive interest income tax. For example the authors contend (p. 366) that an interest income tax by itself leads to a deceleration in the speed of resource extraction and hence should be corrected by a capital gains tax at the same rate to restore the pre-tax extraction path. The argument uses the conditions of a portfolio equilibrium and runs as follows. Given the gross rate of interest a taxation of interest income requires the growth rate of the resource price to fall, unless a capital gains tax is introduced. Given the resource demand curves for each point in time, the decline in this growth rate, in turn, demands a more even extraction path.

So far so good. But why should the rate of interest and, in particular, the demand curves be given? From all that we know a taxation of interest incomes by itself slows down the rate of growth in the economy. So the demand curves shift inwards, the more the further ahead the date. Given the path of resource extraction this effect by itself leads to the desired decline in the growth rate of the resource price, possibly enough to render superfluous any capital gains taxation<sup>5</sup>.

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<sup>4</sup> The only way to legitimize Takayama's formula would be to assume that, unlike the usual specification, technological progress is not factor augmenting in terms of efficiency units, but in terms of physical units. But then a mere reinterpretation of the output-augmenting type of technological progress would have sufficed.

<sup>5</sup> The point was discussed in a German paper which was presented to the 1979 annual conference of the German economic association: H.-W. Sinn: Besteuerung, Wachstum und Ressourcenabbau. Ein allgemeiner

## Social Optimality and the Performance of Market Economies

Probably the most important problem in resource economics is whether and in what sense market economies can be expected to exploit natural resources with the socially optimal speed. Is the Club of Rome right in accusing the present generation of over-extraction?

It is clear that even under ideal circumstances the market mechanism ensures the well being of future generations only to the extent that the present generation has an altruistic concern for its heirs (see DH, pp. 255—260). Whether this fact is considered alarming is a matter of value judgement. I do not want to go into this<sup>6</sup>. A more limited question is whether the existing market economies can at least be expected to optimize welfare as viewed by the current generation. DH and KL offer a number of interesting thoughts on this problem. Some of them are taken up in the following sections.

### Imperfect Competition

As opposed to static allocation problems a monopolization of resource supply is *not* an important source of suboptimality. Extending in various ways the basic findings of Weinstein/Zeckhauser and Stiglitz, DH (ch. 11) and KL (essays 4, 6 and 17) show that in many cases monopolistic markets perform equally well as competitive markets and, surprisingly, may even perform better (KL, essay 4). Furthermore, in those cases where monopolies involve welfare losses they are more likely to overconserve than to over-extract. So the Club of Rome cannot blame them.

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Gleichgewichtsansatz, in: H. Siebert, op. cit., pp. 499—528. A somewhat extended version is available under the number 132-79, October 1979, in the Mannheim discussion paper series. The paper, written independently of DH, gives a perfect foresight general equilibrium approach to an economy with capital accumulation and resource extraction where all private agents are intertemporal optimizers. It considers taxation of profit and interest income, capital gains, produced goods and resource extraction. Due to suitable separability assumptions in this paper the “automatic” decline in the growth rate of the resource price is just enough to leave unchanged the resource extraction path when the capital gains tax rate is set equal to zero.

<sup>6</sup> A detailed discussion of the problem can be found in T. Page: Conservation and Economic Efficiency. An Approach to Materials Policy, Baltimore and London 1977.

### Intertemporal Wealth Transfers and Overlapping Generations

Another argument for overconservation is given by the overlapping generations model à la Samuelson that KL present in their essay 9. If there is no other store of value than a natural resource it may happen that this resource is completely withdrawn from consumption to serve only the purpose of intertemporal wealth transfer from working years to the retirement period. So the sheiks sit on their oil to sell it to their children in exchange for an old-age pension. These, however, plan to do the same and so the reserves are never actually used up. KL themselves show that this implausible implication vanishes as soon as sufficiently large quantities of other stores of value like bonds are introduced into the economy.

One cannot deny that the theory has some relevance for very precious resources like gold, silver or diamonds, but even without bonds I find it hard to see that in the foreseeable future it might become relevant for those resources essential for the survival of man.

### Expectations

A more important real problem seems to be the question to what extent expectations are able to fill the gap left by the incompleteness of future markets. DH (pp. 107—111, 161—163 and ch. 8) address this problem extensively. Although they analyse in detail the cases of imperfect myopic, perfect myopic and perfect long-run foresight<sup>7</sup>, they seem to favour (pp. 163, 239) a hypothesis that is near to perfect myopic foresight, but slightly more optimistic. DH presume that people not only correctly predict the current rate of change in the resource price, but in addition cast an eye on the resource stock. Thus, if resources tend to become exhausted too

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<sup>7</sup> The analysis of perfect long-run foresight, however, has its drawbacks since in a Keynesian manner consumption is assumed to be solely a function of current income. While this hypothesis is defensible under certain types of imperfect foresight, it seems hardly appropriate under perfect long-run foresight. In the latter case one cannot reasonably avoid deriving the consumption behaviour from an intertemporal optimization problem, except possibly if attention is restricted to steady states. It must however be admitted that an intertemporal general equilibrium model where agents are intertemporal optimizers might be analytically too difficult for a textbook. Examples of such a model can be found in KL's book (essays 12 and 16—19). See in particular essay 19 by Chiarella which studies the interaction between capital accumulation and resource extraction. Cf. also my own paper cited in footnote 5.

soon, they will recognize this in time. As a reaction there is a speculative reduction in supply raising the price and transferring the economy to a lower path of resource consumption. If, however, the level of the extraction path is too low people will never be able to detect that conservation is excessive.

This is a neat and simple idea. However, it does not imply that there cannot be a substantial overextraction for a long period of time. Just the opposite may be true. If we interpret the last two oil crises in its light, then we do not know whether the current rates of extraction are too high or too low, but we can infer that there was an overextraction all the time before.

### Contingency Markets and Resource Stock Uncertainty

While it is easily understandable that a market economy does not work properly if there is a lack of markets, it is surprising for the theorist to hear that it may even malfunction if a complete set of future markets in commodity flows is available. In essay 5, which contains the most puzzling piece of theory one can find in the two books to be reviewed here, KL derive this result. They consider a pure exchange economy under uncertainty. There are two groups of identical people. Individuals in the first group are each endowed with a given, but unknown stock of a non-replenishable resource. Each individual in the second group is endowed with a constant and known flow of another commodity. Similar to the Arrow/Debreu model there is a complete set of forward markets for a trade in the two goods, where a contract is contingent on time and the ability of the resource owner to deliver. With each unit of extraction the veil of ignorance is gradually drawn away from the resource stock, but it is only known with certainty after extraction has turned out to be infeasible. All trading partners are price takers and have the same access to the available information<sup>8</sup>.

If the probability of delivery were exogenous the model would be exactly of the Arrow/Debreu type and hence ensure Pareto optimality. In fact, however, the probability of delivery can be manipulated by the resource owners with the choice of their extraction paths. KL assume that resource owners do not care about the intertemporal variance of their consumption profiles (i. e. they are

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<sup>8</sup> On pages 427—436 DH develop an uncertainty model that is related to the KL model. This also shows that the competitive outcome is sub-optimal, but the reason is that, at the cost of sinking a well, each individual can find out how large the stock size is and then engage in fraudulent contingency contracts.

risk neutral in a certain sense), but that the other party has a preference against such a variance. Due to this difference in preferences resource owners can exploit their power to manipulate the probabilities and extract faster than is socially optimal.

It is worth noting that the welfare loss does not imply that the economy does not reach a social optimum in the Arrow/Debreu sense. *Given* the extraction path planned in equilibrium and given the corresponding probabilities for delivery the allocation is clearly a Pareto optimum. The point is that there is an infinite number of such *constrained* Pareto optima in the Arrow/Debreu sense, and only a subset of these represents *unconstrained* Pareto optima in the sense of Kemp and Long.

KL (pp. 65—68) demonstrate that such an unconstrained optimum can be achieved if an additional kind of market is introduced where a bargaining about the probabilities of delivery themselves (i. e. the speed of extraction) is enabled. So the suboptimality in fact *is* due to a lack of markets. But it is a lack of markets that seem neither to be definable in terms of the Arrow/Debreu framework nor to be observable in reality.

### Common Access

In chapter 5 DH discuss the problem of common pool fishery. Assuming a positive minimum viable population size and free entry they convincingly demonstrate the possibility of overexploitation and species extinction. The subsequent normative part of their analysis where they assume present value maximization could have been enriched by a discussion of the frightening possibility that on purely economic grounds extinction may even be socially optimal<sup>9</sup>.

DH (pp. 372—375) and KL (essay 10) also study the Khalatbari problem of firms extracting from a common pool of oil and selling to a common market. In comparison with the fishery problem it involves the additional difficulty that, for natural reasons, there is no free entry and so a positive immediate profit that can be earned from a unit of extraction does not necessarily imply that this unit *is* extracted. Nevertheless one should expect overextraction and indeed this is the outcome of Khalatbari's model as well as of the somewhat simplified version reported in DH.

<sup>9</sup> See C. W. Clark: Profit Maximization and the Extinction of Animal Species, *The Journal of Political Economy* 81 (1973), pp. 950—961; V. L. Smith: Control Theory Applied to Natural and Environmental Resources. An Exposition, *Journal of Environmental Economics and Management* 4 (1977), pp. 1—24.

The matter is however not as obvious as it might seem at first glance. KL show that Khalatbari's result hinges on a particular inconsistency on the part of the extracting firm. A la Cournot, Khalatbari assumes the firm to optimize under the presumption that it does not induce its competitors to change their *extraction or sales paths*. He also assumes it to believe that a unit of oil saved in the ground will gradually seep away and be lost forever. In fact, however, both assumptions are mutually exclusive. With the other firms maintaining their extraction paths the oil that seeps away must gradually return if an additional unit of oil is extracted in a later period of time. If the firm does not lose anything by not extracting it clearly has no incentive to overextract.

Although KL's solution of the inconsistency is brilliant and formally correct, it represents only one of the possibilities. Another equally simple type of Cournotesque assumption would be that the single firm optimizes under the presumption that its competitors try to maintain the time *path of the stock of oil* under their field rather than the path of their rate of extraction. In this case the single firm believes that a unit of oil that seeps away would never return but would be extracted and sold by others as soon as they become aware of it. This assumption, which to me seems to be nearer to reality, is likely to restore Khalatbari's result.

### Uncertainty About Future Property Rights

A final point that should neither be missing in this note nor in DH's textbook concerns the intertemporal guarantee of property rights. If the present owner of a resource stock feels uncertain whether in the future he or his heirs will still be able to profit from his mine, there will be a strong incentive for him to overextract<sup>10</sup>. With regard to its importance for the real world we live in this obvious effect may well compete with any of those mentioned above. There are still too many Shahs fearing their Khomeinis (or the other way round).

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<sup>10</sup> See N. V. Long: Resource Extraction under Uncertainty about Possible Nationalization, *Journal of Economic Theory* 10 (1975), pp. 42—53.