Surprise Price Shifts, Tax Changes and the Supply Behaviour of Resource Extracting Firms

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SURPRISE PRICE SHIFTS, TAX CHANGES AND THE SUPPLY BEHAVIOUR OF RESOURCE EXTRACTING FIRMS*

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I. Introduction

Recent discussions about the macro-economic effects of a "surprise" increase in the price of oil typically assume that such a surprise price rise will stimulate the extraction of oil[1]. The purpose of this paper is to explore the micro-economic foundation (or the lack of it) for such assumptions. While it is well known that the output of oil depends not only on its current price but also on the time profile of future prices, we believe it is useful to derive sufficient conditions for a positive increase or decrease in current output of oil in response to a surprise perturbation of the price path, brought about by changes in taxes or subsidies, or by exogenous international forces. We will restrict our attention to the case of a competitive resource extracting firm (or a small oil producing economy) and refrain from discussing industry-wide (or world-wide) repercussions of changes in taxation.[2]

II. The Model

In this section we study the optimization problem of a resource extracting firm facing an exogenous net price path. The resource stock and capital stock owned by the firm are denoted by $X(t)$ and $K(t)$, respectively. Let $q(t)$, $I(t)$ and δ denote the rate of resource extraction, the gross rate of investment and the exponential rate of capital depreciation, respectively. Then

$$\dot{X}(t) = -q(t) \tag{1}$$

$$\dot{K}(t) = I(t) - \delta K(t), \tag{2}$$

where the dot denotes the derivative with respect to time.

The cost of extraction is a function of t, q, X, K and $\dot{K}$,

$$C = C(t, q, X, K, \dot{K}) \tag{3}$$

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[1]See, for example, Gregory (1976), Corden and Neary (1982).

[2]In this respect, our analysis is close to that of Burness (1976); however, we do not restrict our attention to the case of a constant pre-tax price path. Other studies of the effects of taxes on the extraction path typically do not take the price path as exogenously given; see for example, Dasgupta and Heal (1979, pp. 361-375), Kemp and Long (1979, pp. 265-267), Sinn (1980, esp. p. 517 f.) and Dasgupta, Heal and Stiglitz (1980). In all these studies neither stock-dependent extraction costs nor capital accumulation by the resource extracting firm are allowed.

In writing equations (1) and (3), we assume the existence of a resource aggregate. This is a standard assumption in the literature on natural resources. This assumption is justifiable if, as a matter of technological necessity, lower cost layers of the resource must be taken away before the higher cost layers can be reached. However, if there is a continuum of deposits and the order of exploitation is a matter of economic choice, then (1) and (3) cannot always be justified. In fact Kemp and Long (1984, p.44) showed that if for each grade there are variable unit costs of production depending upon the intensity of exploitation then in general more than one grade will be produced at a time. In section IV of this paper we will show that if unit costs are constant then (1) and (3) are justifiable.

For simplicity, we assume that the resource, once extracted, is not storable, or alternatively, that storage cost is so high that it is economically inefficient to store the extracted resource.

The objective of the firm is to maximise the present value of the stream of cash flow:

$$\max_{\{I,q,T\}} \int_0^T \pi(t)e^{-rt}dt \tag{4}$$

where (assuming that the price of the investment good is normalized at unity and that investment costs are additively separable from adjustment costs and other costs)

$$\pi(t) = p(t)q(t) - C(t,q,X,K,I-\delta K) - I(t), \tag{5}$$

and where T is to be determined endogenously. The terminal constraints are

$$X(T) \geqq 0, \tag{6}$$

$$K(T) \geqq 0. \tag{7}$$

We also require that

$$I(t) \geqq 0, \tag{8}$$

$$q(t) \geqq 0. \tag{9}$$

The Hamiltonian for problem (4) is

$$H = [p(t)q(t)-C(t,q,X,K,I-\delta K)-I(t)]e^{-rt} \tag{10}$$

$$- \mu(t)q(t) + \lambda(t)[I(t)-\delta K(t)],$$

where $\mu(t)$ and $\lambda(t)$ are the undiscounted co-state variables associated with the resource stock and the capital stock respectively.

The necessary conditions are

$$[p(t)-C_q]e^{-rt} - \mu(t) \leqq 0 \quad (= 0 \text{ if } q(t) > 0), \tag{11}$$

$$[C_{\dot{K}} - 1]e^{-rt} - \lambda(t) \leqq 0 \qquad (= 0 \text{ if } I(t) > 0), \tag{12}$$

$$\dot{\mu}(t) = e^{-rt} C_X, \tag{13}$$

$$\dot{\lambda}(t) = e^{-rt}[C_K - \delta C_{\dot{K}}] + \lambda(t)\delta, \tag{14}$$

and the transversality conditions are

$$H(T) = 0, \tag{15}$$

$$\mu(T) \geqq 0, \quad X(T) \geqq 0, \quad \mu(T)X(T) = 0, \tag{16}$$

$$\lambda(T) \geqq 0, \quad K(T) \geqq 0, \quad \lambda(T)K(T) = 0. \tag{17}$$

(If the optimal terminal time T turns out to be infinite then conditions (15), (16) and (17) are assumed to hold in the limiting sense.)

Let $p_1(t)$ denote the price path which the firm expects (with certainty) to prevail. Given this price path, the firm forms an optimal investment and extraction program $(I_1(t), q_1(t))$. We wish to consider the effects of a surprise shock which leads the firm to revise its expectation of the price path. Let $p_2(t)$ denote the new price path. The difference between the two price paths is

$$D(t) = p_2(t) - p_1(t). \tag{18a}$$

Thus $D(t)$ is the deviation, at time t, of $p_2(t)$ from $p_1(t)$. At this stage, we do not restrict the form of $D(t)$. For example, $D(t)$ may be a constant, *i.e.* $D(t) = \bar{D}$, or it may be a constant proportion of $p_1(t)$, *i.e.* $D(t) = bp_1(t)$. For any given path $D(t)$, we define $\theta(t)$ by

$$\theta(t) = D(t)/p_1(t). \tag{18b}$$

This definition enables us to write

$$p_2(t) = [1+\theta(t)]p_1(t). \tag{18c}$$

One may interpret p_1 as the world price, p_2 as the domestic price, with θ as the tariff rate on resource imports, or alternatively p_1 as the world price (and consumers' price), p_2 as the net price received by the firm, with $-\theta$ as the royalty rate or production tax expressed as a percentage of the consumers' price. Our "comparative dynamics" question may therefore be interpreted as follows: suppose that initially the "tariff rate" $\theta(t)$ is expected to be equal to zero for all time t, and the firm's optimal plan is $(I_1(t), q_1(t))$; suddenly the government announces a time path of tariff rate $\theta(t) \neq 0$. The time path may be constant, $\theta(t)=b>0$ say, or it may be increasing or decreasing over time, with the rate of increase $\hat{\theta}(t) = [1/\theta(t)]d\theta(t)/dt$, which itself may be constant, increasing or decreasing with time. Thus our "thought experiment" is quite general: it is consistent with a constant, once and for all increase in the tariff rate ($\theta(t) = b$, $\hat{\theta} = 0$), or with the phasing in and/or phasing out of protection and/or taxation. Other interpretations are possible: an unexpected event (resource discovery in a foreign country, war, cartelisation of a sub-group of foreign producers) causes the firm to revise its expected price path from $p_1(t)$ to $p_2(t)$, with $\theta(t)$ representing the proportional deviation of p_2 from p_1. (The surprise shock would not be possible if there existed a complete set of futures markets.)

The remaining sections of this paper are devoted to the study of the effects of a surprise shock on the investment and extraction plans of the firm. In what follows, the following notational convention will be adopted: we shall use: $\{q_i(t), I_i(t), X_i(t), K_i(t), \mu_i(t), \lambda_i(t), T_i\}$ to denote the solution for problem (4) when $p(t) = p_i(t)$, $i = 1,2$, and use the notation $\hat{Z} = \dot{Z}/Z$.

III. An Invariance Theorem

In this section we will show that if prior to the shock the firm plans to exhaust (eventually) its resource stock, then a surprise shock (with $\theta(t) \neq 0$) does not necessarily cause the firm to alter its plan. In fact we will show that if $\hat{\theta}(t) = r - \hat{p}_1(t)$ (that is, if $D(t) \equiv \theta(t)p_1(t)$ rises at the rate of interest, so that $D(t) = D(0)\exp(rt)$) then the firm will not alter its plan, as long as $D(0)\exp(rT_1)$, if it is negative, does not exceed $\mu_1(T)$ in absolute value. Upon reflection, this result is intuitively obvious, because the price change is in this case equivalent to a per unit subsidy (if $D(t) > 0$, or a tax, if $D(t) < 0$) whose present value is a constant, $D(0)$. Such a tax or subsidy is therefore non-distorting, being equivalent to a tax on pure rent, provided that the tax $(-D(t))$ is not so heavy as to make extraction unprofitable.

On the other hand if prior to the price shock the firm plans not to exhaust its stock (perhaps because the extraction cost becomes very high when the stock is nearly depleted), then any $\theta \neq 0$ will affect the extraction profile. This second result is well known; see for example Levhari and Liviatan (1977).

We now state the invariance theorem:

Proposition 1: The surprise shock *does not* affect the *extraction* and *investment* paths if and only if

(i) $\hat{\theta} = r - \hat{p}_1$ and

(ii) $\theta(0)p_1(0) + \mu_1(T_1) \geqq 0,$

$$[\theta(0)p_1(0) + \mu_1(T_1)]X_1(T_1) = 0.$$

Proof[3]:

(a) *Sufficiency*:

It is easily verified that $\{q_1(t) I_1(t), X_1(t), K_1(t), \mu_2(t), \lambda_1(t), T_1\}$ is a solution to problem (4) when $p(t) = p_2(t)$, provided

$$\mu_2(t) = \mu_1(t) + \theta(t)p_1(t)e^{-rt}, \tag{19}$$

[3]In what follows we shall assume for simplicity that $q(t) > 0$ for all $t < T$. Relaxation of this assumption does not affect the results.

$$\dot{\mu}_2(t) = e^{-rt} C_{X_1} \tag{20}$$

$$\mu_2(T_1)X_1(T_1) = 0, \quad \mu_2(T_1) \geqq 0. \tag{21}$$

With the help of (13), it is easy to see that conditions (19) and (20) are mutually consistent if and only if

$$\theta(t)p_1(t)e^{-rt} = \text{constant} = \theta(0)p_1(0). \tag{22}$$

Conditions (i) and (ii) are equivalent to (19) – (22).

(b) *Necessity*:

If $q_1(t) = q_2(t) > 0$ (for all t), then

$$\mu_2(t) = \mu_1(t) + \theta(t)p_1(t)e^{-rt},$$

$\dot{\mu}_2(t)$ must equal $\dot{\mu}_1(t)$ by (13). Thus $\theta(t)p_1(t)e^{-rt}$ must be a constant. This implies (i). Condition (ii) is then simply the transversality condition that

$$\mu_2(T_2) \geqq 0, \qquad \mu_2(T_2)X_2(T_2) = 0. \tag{23}$$

(End of proof.)

Remark: If $X_1(T_1) > 0$ then (i) and (ii) are satisfied if and only if $\theta(t) = \theta$ for all t, because $X_1(T_1) > 0$ implies $\mu_1(T_1) = 0$.

IV. Reaction To A Price Shift

Our next step is to investigate the direction of intertemporal bias when the invariance conditions (i) and (ii) above do not hold. We shall restrict our attention to two special cases: (a) extraction cost is stock-independent and is strictly convex in q, and (b) extraction cost is stock-dependent and is linear in q. In both cases we shall assume that there is no capital accumulation or decumulation.

Case (a): Assuming that

$$C = C(t,q),\ C_q \geq 0,\ C_{qq} > 0 \tag{24}$$

we can prove the following proposition:

Proposition 2: Assume that $q_i(t) > 0$ prior to exhaustion, that the resource stock is eventually exhausted, that $q_i(t)$ tends to zero continuously (but not necessarily monotonically), then path $q_2(t)$ cuts path $q_1(t)$ only once and from below, provided that

$$\theta(t)p_1(t)(\hat{\theta} + \hat{p}_1 - \mathrm{r}) > 0. \tag{25}$$

If the reverse of (25) holds, then path $q_2(t)$ cuts path $q_1(t)$ only once and from above.

Proof:

The hypotheses that $q_i(t)$ tends to zero continuously and that the resource stock is eventually exhausted imply that the two paths $q_1(t)$ and $q_2(t)$ must intersect at least once. Let t^* be a point of intersection of the two paths. The cross-over at t^* occurs because the iso-perimetric constraint is binding in this case: since it is optimal to deplete the stock, if along one path more is extracted in earlier periods (compared with the other path), then less must be extracted during the later periods.

We wish to compare $\dot{q}_1(t)$ and $\dot{q}_2(t)$ at $t=t^*$. Differentiating (11) with respect to time, we obtain (after substitution using (13), and some rearrangement of terms),

$$C_{qq}\dot{q} = \dot{p} - r(p - C_q) - C_{qt} - C_{qX}\dot{X} - C_X \tag{26}$$

From (26) and (18), and noting that $C_X = C_{qX} = 0$, we obtain the result that, at $t = t^*$,

$$(C_{qq})(\dot{q}_2 - \dot{q}_1) = \theta\dot{p}_1 + \dot{\theta}p_1 - \mathrm{r}\theta p_1 = \theta(t)p_1(t)(\hat{\theta} + \hat{p}_1 - r). \tag{27}$$

Since $C_{qq} > 0$, proposition 2 follows immediately.

(End of proof.)

Proposition 2 implies that if $\theta(t) > 0$ (a subsidy) and yet the rate of subsidy increases at a rate exceeding $r - \hat{p}_1$, then less output will be produced in some initial time interval. Similarly, if $\theta(t) < 0$ (e.g. an advalorem export tax is imposed) and $\hat{\theta} > r - \hat{p}_1$ then more output will be produced in some initial time interval. Perhaps the consideration of some special cases will highlight the scope of applications of our result. Suppose the home country is a net importer of oil. The international price of oil is exogenously given and rises at the rate $\hat{p}_1$. A tariff at the rate $\theta(t)$ raises the domestic price of oil to $p_2(t) = [1 + \theta(t)]p_1(t)$. Initially, the tariff rate is zero. A surprise increase in tariff rate from zero to a positive constant $\theta(t) = b$ (for all t) will cause domestic firms to change their extraction plans if and only if $\hat{p}_1 \neq r$. If $\hat{p}_1 > r$, domestic firms will extract less in earlier periods (as compared with their extraction plan under free trade) and consequently more in later periods. (Note that $\hat{p}_1 > r$ is consistent with market equilibrium provided that storage cost is high; it is also consistent with Hotelling's rule provided that extraction cost rises with time.) Conversely, if $\hat{p}_1 < r$, a surprise change in tariff rate from zero to $b > 0$ will cause domestic firms to extract more in earlier periods. However if the tariff rate is expected to be rising over time at a rate exceeding $r - \hat{p}_1$ then firms will extract less in earlier periods.

Consider an alternative scenario. Suppose the home country is a net exporter of oil and the world price of oil, $p_1(t)$, is exogenously given. A surprise announcement of an export tax (expressed as a proportion of international price) at rate $u > 0$ (so that $\theta(t) = -u < 0$) causes a fall in the net price received by domestic firms. The net price is $p_2(t) = [1+\theta(t)]p_1(t)$. If the tax rate is constant ($\hat{\theta} = 0$) and if $\hat{p}_1 > r$, domestic firms will extract more in earlier periods.

The reverse will happen if $\hat{p}_1 < r$. However, if the tax rate is expected to rise at a rate exceeding $r - \hat{p}_1$, then more will be extracted in earlier periods.

Case (b):

Consider now the case in which the cost function takes the form

$$C = w(t)g(X)q, \qquad w(t) > 0. \tag{28}$$

where $w(t)$ is the wage rate in the extraction industry and $g(X)$ is the stock-dependent coefficient of labour inputs in extraction. This cost function is dual to the production function

$$q = L/g(X).$$

This case is indeed very special. It is chosen because of its relative simplicity.

Before studying the effect of a surprise price change, it is necessary to determine the sign of the derivative $g'(X)$. If one assumes that, due to some technological reason, the lower cost layers of the resource must be taken away before the higher cost layers can be reached, then $g'(X) < 0$. The more interesting case is that in which there is a continuum of deposits and the order of exploitation is a matter of economic choice. To be more precise, let β be the labour coefficient required for the deposit with index β, where $a \leq \beta \leq b$. Let $f(\beta)$ be the density function defined over the continuum of deposit $[a,b]$. If the firm adopts the Herfindahl order of exploitation (*i.e.* extracting the deposits in *strict* sequence, beginning with the *lowest cost* deposit), then $\beta'(t) > 0$ and the stock remaining at time t is

$$X(t) = \int_{\beta(t)}^{b} f(\beta)d\beta. \tag{29}$$

Now, from definition,

$$g(X(t)) = \beta(t). \tag{30}$$

Hence

$$g'(X)(dX/dt) = \beta'(t). \tag{31}$$

But $\beta'(t) > 0$ because the firm adopts the Herfindahl order of exploitation.

Therefore

$$g'(X) < 0. \tag{32}$$

In fact, we can calculate $g'(X)$. From (31),

$$g'(X) = \beta'(t) \ / \ (dX/dt), \tag{33}$$

and from (29),

$$dX/dt=-f(\beta(t))\beta'(t), \tag{34}$$

hence

$$g'(X)=-1/f(\beta(\mathrm{t}))<0. \tag{35}$$

Similarly, if the firm adopts the anti-Herfindahl order of exploitation[4] (*i.e.* extracting the deposits in *strict* sequence, beginning with the highest cost deposit), then $\beta'(t)>O$ and the stock remaining at time t is

$$X(t) = \int_a^{\beta(t)} f(\beta)d\beta' \tag{36}$$

in which case

$$dX/dt=f(\mathrm{t}))\beta'(t), \tag{37}$$

and

$$g'(X) = 1/f(\beta(t))>0. \tag{38}$$

Assuming the existence of a non-degenerate optimal extraction path (in the sense that extraction rates are finite), Kemp and Long (1980b) and (1984) have shown that, over the time interval of non-degenerate extraction, the Herfindahl order of exploitation is optimal (and any other order of exploitation sub-optimal) if $\hat{w} < r$, and the anti-Herfindahl order of exploitation is optimal (and any other order of exploitation is sub-optimal) if $\hat{w} > r$ (*i.e.* the discounted wage rate rises over time[5]). This result will be stated formally below, as Lemma 1.

Lemma 1: Assuming the existence of a non-degenerate optimal extraction path over $[t_1,t_2]$. Then

$$\text{(i)} \quad (\hat{w} < r) \rightarrow \mathrm{H} \tag{39}$$

$$\text{(ii)} \quad (\hat{w} > r) \rightarrow \mathrm{AH} \tag{40}$$

over the interval $[t_1,t_2]$, where H (resp. AH) signifies that "the Herfindahl (the anti-Herfindahl) order of exploitation is optimal, and any other order of exploitation is sub-optimal".

[4]For an example in which the anti-Herfindahl order of exploitation is optimal, see Kemp and Long (1980a, p. 36).

[5]For an intuitive explanation of this result, see Sinn (1981, p. 185), or Kemp and Long (1984, p. 48).

Given our assumption that the firm is a price-taker, we can state an additional result:

Lemma 2: A necessary condition for the existence of a non-degenerate optimal extraction path over $[t_1,t_2]$ is that over that time interval either $\hat{w}-r=\hat{p}-r$, or the ratio $(\hat{p}-r)/(\hat{w}-r)$ is positive and less than unity. In symbols,

$$\text{ND} \rightarrow \begin{cases} \text{either } \hat{w}-r=\hat{p}-r, \text{ or} \\ 1 > (\hat{p}-r)/(\hat{w}-r) > 0, \end{cases} \tag{41}$$

where ND stands for "non-degeneracy" (degeneracy means that q is on the "skin" on the admissable control set).

Proof:

Assume that $q(t)$ is positive and finite over some time interval $[t_1,t_2]$.

Then from (1), (26) and (28),

$$\dot{p} - r[p-wg(X)] - \dot{w}g(X) = 0, \tag{42}$$

which is equivalent to

$$wg(X)/p = (\hat{p}-r)/(\hat{w}-r). \tag{43}$$

But the profitability of extraction implies

$$0 < wg(X)/p \leqq 1. \tag{44}$$

(End of Proof.)

Lemma 2 can be given an intuitive explanation. If the discounted price rises $(\hat{p} > r)$ while discounted wage falls $(\hat{w} < r)$, it will be profitable to postpone extraction; similarly if the discounted price falls $(\hat{p} < r)$ while the discounted wage rises $(\hat{w} > r)$, the firm should extract all profitable deposits at the first instant of time. Therefore non-degeneracy implies that

$$\text{Sign } (\hat{p}-r) = \text{Sign } (\hat{w}-r). \tag{45}$$

If $\hat{p}-r > 0$ and $\hat{w}-r > 0$, then it is necessary for non-degeneracy that $\hat{w} > \hat{p}$ $(> r)$, for otherwise it would not be profitable to extract now. Similarly if $\hat{p}-r < 0$ and $\hat{w}-r < 0$, then a necessary condition for non-degeneracy is that $\hat{w} < \hat{p}(< r)$, for otherwise all profitable deposits should be extracted at the first instant.

We are now ready to study the effect of a change in the price path on the path of accumulated extraction. Assume that $q(t) > 0$ and finite over $[0,T_1]$. From (18) and (43),

$$g(X) = (\dot{p}_2 - rp_2)/(\dot{w} - rw)$$

$$= (\dot{p}_1 - rp_1 + L)/(\dot{w} - rw) \tag{46}$$

where

$$L \equiv \theta p_1(\hat{p}_1 + \theta - r). \tag{47}$$

From Proposition 1, if $L = 0$ the presence of $\theta(t)$ has no effects on extraction. Differentiating (46) with respect to L gives the reaction of the desired stock of the resource to a perturbation in the price path as indicated by L:

$$dX/dL = 1/[g'(X)(\dot{w} - rw)] \tag{48}$$

Thus, if for technological reasons $g'(X) < 0$, an increase in L induces a more conservationist extraction policy in the case $\hat{w} < r$ and a less conservationist policy in the case $\hat{w} > r$. If, however, the assumptions underlying Lemma 1 apply, then $g'(X) < 0$ if $\hat{w} < r$ and $g'(X) > 0$ if $\hat{w} > r$. Hence we can state *Proposition 3*: If $C = w(t)g(X)q$ and the order of exploitation is a matter of economic choice, then, given a non-degenerate extraction path, a small price perturbation which satisfies

$$\theta(t)p_1(t)(\hat{\theta} + \hat{p}_1 - r) > 0 \tag{49}$$

will cause extraction to be more conservationist in the sense that at each point in time firms wish to hold a lower stock.

So far we have concentrated on the interpretation that the whole subsidy path $\theta(t)$ is suddenly announced at the time $t = 0$. An alternative interpretation is possible. Suppose that originally the price path is expected to be $p_1(t)$. At time $t = 0$, a proportional change $\theta(0)$ is observed. Producers then form expectations about $\theta(t)$. Consider the hypothesis that the expectation elasticity

$$\eta(t) \equiv \frac{\triangle p(t)}{\triangle p(0)} \quad \frac{p(0)}{p(t)}$$

$$= \theta(t)/\theta(0) \tag{50}$$

is changing at a constant relative rate:

$$\hat{\eta} = \hat{\theta} = \text{constant}. \tag{51}$$

If $\hat{\theta} > 0$, i.e., $\eta(t) > 1$ for $t > 0$, this hypothesis implies that producers have unstable expectations in the sense that a currently observable relative change in price makes them expect an even higher relative change in the future. Similarly if $\hat{\theta} < 0$, ($\eta(t) < 1$, for $t > 0$) it implies stable expectations in the sense that the expected future relative change in p falls short of change observed at $t = 0$. If $\hat{\theta} = 0$, i.e., $\eta = 1$ for all $t > 0$, the intermediate case prevails where producers revise their expected prices for all $t > 0$ by the same percentage by which the current price is changing.

Assume that the originally planned extraction path is non-degenerate.

Then, from Lemma 2,

$$\text{Sign } (\hat{p}_1 - r) = \text{sign } (\hat{w} - r)$$

so that

(i) If $\hat{w} > r$, then:

$\eta(t) \geqq 1$ for all $t > 0$(or $\hat{\theta} \geqq 0$) implies that (49) holds.

(ii) If $\hat{w} < r$, then:

$\eta(t) \leqq 1$ for all $t > 0$ (or $\hat{\theta} \leqq 0$) implies that

$\theta(t)p_1(\text{t})(\hat{\theta} + \hat{p}_1 - r) < 0.$

(iii) If $\hat{w} = r$, then:

$\eta(t) \gtreqless 1$ for $t > 0$ (or $\hat{\theta} \gtreqless 0$)

if and only if

$$\theta(t)p_1(t)(\hat{\theta} + \hat{p}_1 - r) \gtreqless 0.$$

Thus, using (i), (ii) and (iii) and Proposition 3, the following table can be derived which indicates the reaction of *current* extraction to an increase in the *current* price of the resource, under the expectational hypothesis (51):

TABLE I

Reaction of Current Extraction to an Increase in Current Price

	$\hat{w} > r$	$\hat{w} = r$	$\hat{w} < r$
$\eta > 1$	< 0	< 0	?
$\eta = 1$	< 0	0	> 0
$\eta < 1$	?	> 0	> 0

V. Conclusion

We have concluded this paper by showing that the response of resource extracting firms to a sudden change in the current price depends on how future prices are expected to change. Earlier, we were able to prove an invariance proposition and two propositions concerning the direction of bias. Even under a very simple expectation hypothesis, as specified by equation (51), a variety of possible responses can be deduced. Table I clearly shows that the usual assumptions about the supply behaviour of resource extracting firms may well be fallacious. That firms react to a sudden rise in the price level by increasing their supply is only one of several possibilities. Perhaps the most surprising result is that even under a unitary expectation elasticity, which in some sense is the best guess for a simple and plausible expectational hypothesis, firms may abnormally react to a price change; necessary conditions for an abnormal reaction are that the wage rate in the extracting industry rises at a rate higher than the rate of interest and that the order of exploitation of different deposits is a matter of economic choice.

We have restricted our attention to the case of an exogenous price path. It would be interesting to study the case in which the price path is determined endogenously through the interaction of consumers and producers.

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