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by

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1. The Problem

The roots of modern tax theory lie in Harberger's (1962, 1966) problem of how the double taxation of corporate dividends affects the allocation of resources between the corporate and non–corporate sectors of the economy. Harberger's claim was that the double taxation of dividends discriminates against corporate investment and creates welfare losses by keeping too large a share of the economy's capital stock in the non–corporate sector. The larger the tax burden on dividends, the bigger the welfare loss that results.

This paper reconsiders Harberger's problem from an intertemporal perspective. It studies the foundation and growth of corporations in the presence of dividend taxation to find out whether, and if so, under what circumstances dividend taxes create a Harberger–type distortion. The main result is that the distortion is a transitory phenomenon and that, in an important sense, the size of the welfare loss is negatively rather than positively related to the size of the tax burden.

The traditional view of corporate taxation as formulated by Harberger has recently been questioned by holders of the so–called "new" or "trapped equity" view of corporate taxation, including, for example, King (1974a, 1974b, 1977), Bradford (1980, 1981), Auerbach (1979, 1983), King/Fullerton (1984), and Sinn (1987). Their argument is that the dividend tax is capitalized in share prices and therefore cannot affect the firm's investment decisions. The tax is simply seen as a lump sum levy on corporations. If true, tax reforms, whose aim is to remove the double taxation of dividends, would be superfluous. They would create windfall gains for the current owners of corporate shares, but would not improve the allocation of resources.

Unfortunately, however, the new view does not seem fully compatible with the empirical facts. As observed by Poterba and Summers (1983, 1985), who studied the effects of British tax reforms, changes in the statutory dividend tax rate did have adverse effects on the level of aggregate investment. Poterba and Summers attributed their findings to the fact that the trapped equity model neglects the signalling function of dividends. While this is a possible explanation for the non–neutrality of dividend taxation, there are others. The one explored in this paper is suggested by a serious shortcoming of the trapped equity model.

Existing approaches that use this model have the common characteristic that they do not explain how equity falls into the trap. Typically, it is assumed that the firm already has more than the efficient amount of equity capital at the time the investment decision is analyzed. Under these circumstances, the neutrality proposition is not especially surprising. It just means that the firm retains the efficient amount of equity and distributes the remainder. The important problems of how much equity capital shareholder may wish to inject into their firms in the first place and whether the corporate stock of capital will ever reach its efficient size are unsolved.

This paper offers a solution that is consistent with Poterba and Summers's findings. It reconsiders Harberger's problem from the viewpoint of a trapped equity model, but one that starts with the process of injecting capital into the firm. Surprisingly, no similar model seems to exist in the literature. It is true that holders of the trapped equity view typically concede that dividend taxes are distortionary to the extent that new issues of shares are a marginal source of finance. However, as far as is known, no attempt has been made to formulate an explicit intertemporal model that describes the foundation and growth of a corporation in the presence of dividend taxation.

The paper rehabilitates Harberger's view that the dividend tax discriminates against corporate investment, but, in addition, it modifies and criticizes his analysis. Harberger and many of his followers have concentrated on the general equilibrium repercussions of taxation and have placed little emphasis on microeconomic considerations such as how taxes would affect the investment decisions of the firm. Frequently they have simply assumed that the corporate firm invests until the net–of–tax marginal product of capital equals the market rate of interest. This assumption is compatible with *partial* optimization given that new share issues are the only marginal source of finance and that all profits resulting from an investment are distributed as dividends. However, there are at least two problems with this.

The first is that, instead of new share issues, the firm may choose *other sources of finance.* From an empirical point of view, both debt and retained profits are cheaper and much more important sources than new issues of shares. The holders of the new view have emphasized this and have derived investment conditions that typically imply lower distortions than those Harberger argued for.

The second problem is that, instead of using the profits from its marginal investment to pay dividends to shareholders, the firm may choose *other uses for profits.* One possibility is share repurchases. Profit financed share repurchases can be seen as a way of avoiding the double taxation of dividends and they undermine Harberger's results for obvious reasons. A second potential use for profits is internal investment. This is not only of great empirical significance in all countries, it is also suggested by theoretical considerations. In fact, it is clear that a firm would not distribute its profits immediately after it has issued new shares if, as Harberger claimed, the shares stopped being issued before the point where the marginal product of capital equals the market rate of interest. With an internal rate of return above the shareholders' discount rate, it would always pay to reinvest the profits and distribute them later. The present value of dividends, net of the dividend taxes, could be increased by postponing the distributions for as long as it takes for the process of reinvesting profits to equate the marginal product of capital and the market rate of interest. This suggests that there might be something wrong with the reasoning underlying the Harberger-type cost-of-capital formula even if it is assumed that the firm is forced by an initial shortage of retainable profits to issue new shares at the margin, cannot borrow, and cannot escape the dividend tax by repurchasing its shares.

The reinvestment of the profits generated by marginal investment projects is incompatible not only with Harberger's formula, but renders some of the formulae provided by holders of the new view also inapplicable. For example, the popular cost of capital formula of Fullerton and King (1984), which is a weighted average of the costs of the three alternative sources of finance, assumes that the profits from marginal investment projects are distributed. King's (1977) expressions for the cost of new share issues and retained profits, which enter this formula, are not applicable when the returns from marginal investment projects are reinvested at a rate of return above the market rate of interest. Section 2.5 will discuss the relationship to the cost of capital formulae provided by the holders of the new view in more detail.

Except for the exclusion of debt financing and profit financed share repurchases, the present paper makes no assumptions about the firm's source of funds for, and the use of the profits generated by, marginal investment projects. Instead, the financial and real investment decisions are endogenously derived from the firm's optimization approach. The available sources of funds are new share issues and retained profits, and the possible uses for profits are dividends and internal investment. The exclusion of debt financing and profit financed share repurchases is motivated by the attempt to treat one difficulty at a time and to follow Harberger's analysis as closely as possible. Including these possibilities would weaken his case and imply a criticism of his other results than the one made here.[1]

Essentially, the paper is a reconsideration of Harberger's problem, asking his questions and using his assumptions. It formulates an intertemporal variant of his two sector model, which is based on microeconomic optimization rather than arbitrarily postulated marginal conditions. This variant is slightly more complicated than the original model but it nevertheless reflects the attempt to be as simple as possible without giving up the rigor necessary to make the point. The economy has a given stock of capital for which two representative firms compete in a perfect capital market. The firms produce the same commodity, use only equity capital, and please their far-sighted owners by choosing the investment policies which maximize their market values. One of the firms is corporate, the other non-corporate. The trapped equity property is modeled by the assumption that the government does not contribute to funds shareholders inject into the firm, but taxes all payments to shareholders that result from current or previous profits. A tax exempt return of the original capital is allowed.[2]

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[1]A critical discussion of the Harberger approach that allows for debt financing and other modifications of his assumptions can be found in Sinn (1987, ch. 6).

[2]The paper should not be seen as an attempt to solve the dividend puzzle. See Poterba (1987) for an excellent discussion of this puzzle.

2. Dividend Taxation and the Growth of the Corporation

Before the Harberger problem can be meaningfully discussed, a model of the firm is needed that explains how a corporate firm is set up and how the equity capital falls into the trap. This section provides one. The next section will add an untaxed firm and analyze the properties of capital market equilibrium.

2.1. *A Model of the Firm*

The firm's policy is determined by its shareholders who, in line with Fisher's separation theorem, unanimously agree to maximize the initial market value of shares net of the original capital injected. Let t_1 be the point in time t at which the firm's planning problem starts. Shareholders are price takers, look through the corporate veil, and are endowed with perfect foresight of all variables of the model. They can borrow and lend at the going market rate of interest r, $r > 0$, whose time path they take as exogenously given. The market value of shares is therefore implicitly determined by the following arbitrage condition that requires shareholders to be indifferent between keeping their wealth in the form of bonds or shares:

$$(1) \qquad rM = \Theta D + \dot{m}z + (\dot{z}m - Q) \qquad \text{for } t > t_1 .$$

The lefthand side of (1) is the return from selling the existing stock of shares at its market value M and investing the funds received in bonds which yield the market rate of interest r. The righthand side of (1) measures the current return from continued shareholding. D is the gross dividend and $\Theta \equiv 1 - \tau$ is one minus the dividend tax rate. It is assumed that $0 < \Theta < 1$ and that Θ is a constant.[3] The next term, $\dot{m}z$, is the capital

[3]See Howitt and Sinn (1986) for an analysis of anticipated changes of the dividend tax rate in a trapped equity model with debt financing.

gain from existing shares where m is the price of a share and z the number of outstanding shares. The term in round brackets is the value of purchasing options for new shares issued to the existing shareholders. It is the difference between the market value of the new shares, $\dot{z}m$, and the flow of funds Q that must be paid to the firm in exchange for the shares. When there are no purchasing options (as in the U.S., for example) it can be assumed that $\dot{z}m = Q$ as existing shareholders will not vote for a policy of diluting their shares. In general, however, there is no need to assume that $\dot{z}m$ and Q are linked to one another.

Noting that $\dot{m}z + \dot{z}m = \dot{M}$, (1) can be transformed to $\dot{M} = -\Theta D + Q - rM$ which, upon integration, gives

$$(2) \qquad M(t) = \int_t^{\infty} [\Theta D(v) - Q(v)] \exp\int_t^{v} - r(u)du\, dv \qquad \text{for } t \geq t_1$$

plus some arbitrary constant. The constant is taken to be zero to ensure that the market value of a firm that promises never to issue new shares and never to distribute any dividends is zero. It is assumed for the derivation of (2) that the integral exisits which requires that $\lim_{t^* \to \infty}[\Theta D(t^*) - Q(t^*)] \exp\int_t^{t^*} - r(u)du = 0$.

Following Harberger, it is assumed that the firm produces its output only with equity capital K. Moreover, with only small losses in generality, all commodity prices are assumed constant and normalized to unity. Under these assumptions, the firm's revenue, profit, and output can all be described by the function $f(K)$ satisfying the usual properties $f' > 0$, $f'' < 0$, $f'(0) = \infty$, $f'(\infty) = 0$. The dividend the firm can pay is

$$(3) \qquad D = f(K) - \dot{K} + Q ,$$

where $\dot{K}$ is the firm's net investment. Let K_0 be the stock of capital available from the firm's past history and K_1 the stock of equity capital reached at t_1 after the initial issue of shares.

Then, the firm's problem can be expressed as

$$\text{(4)} \qquad \max_{\{\dot{K},Q,K_1\}} \quad M(t_1) - K_1$$
$$s.t.$$
$$K_1 \geq K_0 = 0$$
$$D \geq 0,$$
$$Q \geq 0,$$

where K is the state variable and $\dot{K}$, Q, and K_1 are the controls. The three constraints implicitly capture the trapped equity assumption that the government participates in corporate distributions but not in what shareholders inject into their corporation. For the time being, the trapped equity assumption is made in the extreme form that it is impossible for the firm to pay cash to its shareholders that is not taxed as dividends; i.e., that $Q \geq 0$ for all $K \geq 0$. This assumption will be relaxed in section 4 to allow the firm to return its original share capital. A similar remark applies to the assumption $K_0 = 0$.

2.2. *The Optimality Conditions*

The problem of the firm can be solved by using Pontryagin's Maximum Principle. Using (3) and associating a co-state variable q (Tobin's q) with K and Kuhn–Tucker multipliers μ_D and μ_Q with the flow constraints, the current value Hamiltonian of problem (4) can be written as:

$$\mathcal{H} = (\Theta + \mu_D)[f(K) - \dot{K} + Q] + q\dot{K} - Q(1 - \mu_Q).$$

From $\partial \mathcal{H}/\partial \dot{K} = 0$ it follows that

(5) $$q = \Theta + \mu_D$$

and from $\partial \mathscr{H}/\partial Q = 0$ that

(6) $$\mu_D + \mu_Q = \tau\ .$$

Both equations together imply:

(7) $$q = 1 - \mu_Q\ .$$

Because of (5), the canonical equation $\dot{q} - rq = - \partial \mathscr{H}/\partial K$ can be transformed to

(8) $$f'(K) + \frac{\dot{q}}{q} = r\ .$$

The Kuhn–Tucker conditions of the problem are

(9) $$\mu_Q Q = 0,\ \ \mu_Q \geq 0,\ \ Q \geq 0,$$

(10) $$\mu_D D = 0,\ \ \mu_D \geq 0,\ \ D \geq 0\ .$$

The firm's starting condition is $\partial M(t_1)/\partial K_1 - 1 = 0$ which, as $\partial M(t)/\partial K(t) \equiv q(t)$ holds by definition, implies that

(11) $$q(t_1) = 1\ .$$

Finally, the transversality condition is

(12) $$\lim_{t \to \infty} q(t)K(t) \exp \int_{t_1}^{t} - r(v)dv = 0\ .$$

Notice that, because of (6), (9), and (10), the firm cannot simultaneously issue new shares and pay dividends. Instead, at any point in time after t_1, it must either be the case that $(Q > 0;\ D = \mu_Q = 0)$, that $(Q = D = 0;\ \mu_Q,\ \mu_D \geq 0)$, or that $(Q = \mu_D = 0;\ D > 0)$. Together with the initial condition, this implies that the following activity phases are available. The names of these phases anticipate properties yet to be derived.

Phase Ia $(K_1 \geq 0;\ t = t_1)$

Phase Ia refers to the starting point where the original stock of equity K_1 may be injected. According to (11), this phase is characterized by

$$q(t_1) = 1 \,.$$

Phase Ib $(Q > 0;\ D = \mu_Q = 0;\ t > t_1)$

Phase Ib is a phase of continuing equity injections after the time of foundation. During this phase, $\mu_Q = 0$ and hence (7) implies that

$$q = 1, \quad \dot{q} = 0 \,.$$

It therefore follows from (8) that

$$r = f'(K) \,. \tag{13}$$

Phase II $(Q = D = 0;\ \mu_Q,\ \mu_D \geq 0;\ t > t_1)$

If the firm neither issues new shares nor pays out any dividends, then from (8):

$$\dot{q} = q\,[r - f'(K)] \,, \tag{14}$$

and, from (3):

(15) $$\dot{K} = f(K) .$$

Phase III $(D > 0;\ Q = \mu_D = 0;\ t > t_1)$

For a firm that pays dividends, (5) and $\mu_D = 0$ indicate that

$$q = \Theta, \quad \dot{q} = 0$$

which, because of (8), implies that

(16) $$f'(K) = r .$$

2.3. *The Optimal Growth Path*

The optimal growth path of the firm is a combination of the four phases that satisfies the the transversality condition (12) and the Maximum Principle's general requirement that there be no jumps in the state and co–state variables. Assuming for the time being that the market rate of interest is a positive constant[4] for $t \geq t_1$, the growth path can be uniquely determined in (q,K) space as illustrated in Figure 1.

The position of the vertical line in this diagram characterizes the equity level K_2 implicitly defined by the laissez–faire condition $f'(K_2) = r$. During Phase III, the firm is on this vertical line at $q = \Theta$. It distributes all its profits since $\dot{K} = Q = 0$ and it can stay in this phase forever since the transversality condition (12) is satisfied.

Phase III is the phase on which the new view of corporate taxation concentrates. The shadow price of equity, q, is Θ rather than one, since Θ is the shareholders' net dividend foregone if the firm decides to retain and invest one additional unit of profit. The low value of the shadow price just compensates for the tax on the

[4]In the next section, this assumption will be relaxed and the time path of r will be endogenously determined by the conditions of a market equilibrium.

returns which an additional unit of equity capital will generate and explains why, despite the tax, the firm follows the laissez–faire investment rule $f'(K) = r$. Let t_2 be the point in time at which Phase III begins.

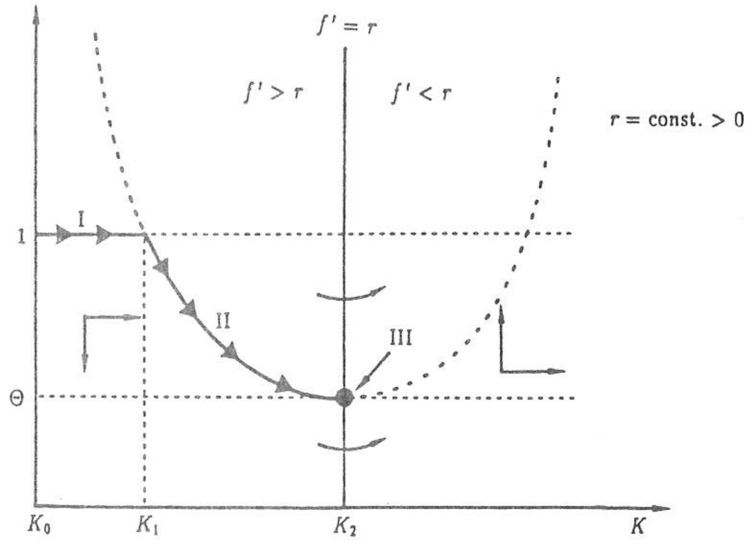

Figure 1: *The optimal growth path under the dividend tax*

Clearly, Phase III cannot be a starting phase. The firm first has to raise enough equity capital to get there. A potential candidate for explaining how to reach Phase III is Phase II, for the two differential equations (14) and (15) define a set of possible paths in (q,K) space one of which intersects the vertical at $q = \Theta$. The slope of these paths is given by

$$\text{(17)} \qquad \mathrm{d}q/\mathrm{d}K = \dot{q}/\dot{K} = q\,[r - f'(K)]/f(K) \qquad \text{(Phase II)}\,.$$

As $f\,'' < 0$ and $\dot{K} > 0$ for $K > 0$, the slope is negative in the region to the left of the vertical, zero on the vertical, and positive in the region to the right of it. The arrows in Figure 1 indicate the possible movements. Assuming that the production elasticity of K

is bounded away from unity, it is shown in the appendix that the ordinate is an asymptote for all paths:

$$q(K) \rightarrow \infty \text{ for } K \rightarrow 0 \qquad \text{(Phase II)} .$$

This property ensures that the path leading to the steady state point (Θ, K_2) intersects the horizontal line of height $q = 1$ at some strictly positive value of K; call this value $K_1{}^*$.

Notice that Phase II cannot be a final phase. If it were, the firm would never again pay dividends. Yet it is clear from (2) that this cannot be an optimum since, at any point in time t^* during this phase and given the then available stock of equity $K(t^*)$, the firm could increase its market value $M(t^*)$ by paying out all future profits as dividends and keeping the stock of equity constant. The existence of such a possibility would violate the Bellman Principle on which the Maximum Principle is built.

Neither can Phase II be an initial phase. Starting with Phase II means starting with an infinite value of q. Given the possibility of injecting equity capital at a price of unity from outside the firm, this cannot be optimal.

Potential candidates for an initial phase are Phases Ia and Ib. It can easily be seen, however, that Phase Ib does not exist. During Phase Ib, $q = 1$ and the firm issues new shares without distributing any dividends. In the diagram, this means that there is a horizontal movement to the right ($\dot{K} > 0$) with q being kept constant at the level of unity. Such a horizontal movement clearly implies that condition (13) cannot be met, for this condition and the assumption $\dot{r} = 0$ imply that $\dot{K} = \dot{r}/f'' = 0$. The firm would have to satisfy the laissez faire condition $f' = r$, if it continued to issue new shares, but it cannot.

Thus, only Phase Ia remains. Like Phase Ib, Phase Ia requires that $q = 1$. However, since this phase merely refers to the starting point $t = t_1$, the flow condition (13) is not required. The firm issues a sufficient number of shares to reach the Phase–II path in one step when it is founded: $K_1 = K_1{}^*$. Because of the non–existence of Phase Ib, the subscript "a" is dropped in the remainder of this paper and all references to Phase I are meant to refer to Phase Ia.

With Phase I as the necessary starting phase and Phase III as the only available final one, the question is whether Phase II is needed at all. It might be tempting to believe that the optimal strategy involves issuing enough shares to reach K_2 during Phase I and then to start with Phase III ($t_2 = t_1$). However, such a direct move between the two phases would require a forbidden jump in the co–state variable q from 1 to Θ. Phase II therefore is a necessary link between Phase I and Phase III, and the only continuous transition between the phases is one that satisfies the following pattern. During Phase I, $q = 1$ and new shares are issued until the desired stock of original capital K_1 is reached. Then the firm drifts along the curved Phase–II path towards the steady state point ($q = \Theta$, $K = K_2$) accumulating a surplus reserve of amount $K_2 - K_1$. Once there ($t = t_2$), it is in Phase III and stays there forever.

It is important to realize that, unlike many other intertemporal models, the steady state is reached in finite time. As $Q = D = 0$ during Phase II, it follows from (3) that the increase in K per period is positive and bounded away from zero, and, in fact, even the speed of increase is increasing:

$$\dot{K}(t) = f\,[K(t)] \geq f\,(K_2) > 0\ ,$$
$$\ddot{K}(t) = f'[K(t)]\,\dot{K}(t) > 0 \text{ for all } t_1 \leq t \leq t_2 \qquad \text{(Phase II)}\ .$$

This clearly implies that $t_2 < \infty$; i.e., that the steady state stock of surplus reserves $K_2 - K_1$ is accumulated in finite time.

The following proposition summarizes these findings. It describes the optimal growth path of a corporation that is subject to dividend taxation.

Proposition 1: *When the firm is founded, new shares are issued to generate some equity to start with. The starting stock is smaller than the one at which the marginal product of capital equals the market rate of interest. After the foundation, a phase of internal growth follows during which the firm neither issues new shares nor distributes any profits. This phase terminates in finite time when sufficient surplus reserves have been accumulated to equate the marginal product of capital with the market rate of interest. The firm will then stop growing, issue no shares, and distribute all its profits.*

While Proposition 1 refers to the qualitative aspects of the growth path when there is a dividend tax, it does not clarify the role of the dividend tax itself. To understand this role, note that the size of the dividend tax rate neither affects the initial value of q, nor the set of Period–II paths compatible with (14) and (15), nor the steady state value of equity, K_2. The only thing that is affected is the steady state value of q, $q(t_2) = \Theta$. This value singles out the optimal path during Period II and determines both the length of this period and the size of the original capital K_1. Obviously, the higher τ, the lower Θ and q, the lower K_1, and the longer the time span that must elapse before the missing amount of surplus reserves $K_2 - K_1$ is accumulated. When there is no dividend tax, then $q(t_2) = q(t_1) = 1$, and Phase II is not needed to avoid a jump in q. Shareholders inject a sufficient amount of original capital to reach the steady state value K_2 in one step.

Again, this is summarized in a proposition.

Proposition 2: *The phase of internal growth is longer and the starting stock of equity smaller the higher the tax rate on dividends. Without the dividend tax, there is no such phase. All equity is then generated through share issues when the firm is founded.*

2.4. *The Nucleus Theory of the Corporation*

While Propositions 1 and 2 are meant to prepare the ground for a discussion of the Harberger problem, they may be interesting in their own right. They show that the policy of maximizing the rate of internal growth and minimizing dividend payments that has been so graphically described by Penrose (1959) and others does not have to be explained by a divergence between manager and shareholder interests. The high burden of dividend taxes can also be an explanation. It is particularly reaffirming in this context to hear what Barlow/Wender (1955, ch. 11) and Penrose (1956, pp. 227–229) say about the growth of foreign affiliates of U.S. corporations. The typical pattern of growth these authors observed was that, when founded, the affiliates were given only a nucleus of equity capital and then had to grow by themselves. Only mature affiliates that had reached their desired size were expected to distribute profits. In the light of the fact that border crossing profit distributions are frequently subject to international double and triple taxation, this observation is not at all surprising.[5]

A further observation involves the relatively infrequent occurrence of new share issues that has been observed by many authors. For example, in the period from 1980 to 1985, on average 67.8 % of gross investment by U.S. non-financial corporations was internally financed and 31.0 % was debt financed, but only 1.2 % was financed by share issues.[6] The actual figures may be somewhat different for other countries, but their tendency clearly describes a general empirical phenomenon. In developed economies, corporations are self-perpetuating enterprises that rarely rely on equity injections from the household sector but generate most of their equity capital internally.

2.5. *Relationship to the New View*

Propositions 1 and 2 complement the new view of corporate taxation. They confirm this view for Phase III and they show that this phase will indeed be reached. In Phase III, profit retentions are a *potential* marginal source of finance and the profit from marginal investment projects is paid out as dividends. As predicted by holders of the new view, the marginal product of capital equals the market rate of interest.

There is less agreement, though, for Phases I and II. As is well-known, the King-Fullerton formula says that, when there is only a dividend tax, the cost of capital is r for a firm that relies exclusively on retained profits, and r/Θ for one that relies exclusively on new issues of shares, to finance its marginal investment projects. By way of contrast, in Phase II, where retained profits are the only marginal source of finance,

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[5]Cf. Sinn (1987, pp. 197–200) for short review of tax rules applying to international capital income flows.

[6]Calculated from *Survey of Current Business*, Volumes 57 (July 1977, p. 24 n.), 61 (1981, special supplement, p. 10), 63 (July 1983, p. 30), 66 (July 1986, p. 33); and *Federal Reserve Bulletin*, Volumes 55 (November 1969, p. A 71.4), 60 (October 1974, p. A 59.4), 64 (June 1978, p. 433), 65 (December 1979, p. A 44).

the cost of capital exceeds r. Moreover, as shown in the appendix, the cost of capital is not, in general, equal to r/Θ when new share issues are the only marginal source of finance (Phase I). In fact, under mild conditions, it can be proved to be higher than this value. It therefore seems that the King–Fullerton formula underestimates the cost of equity finance for immature firms which are in Phases I or II.

As mentioned in the introduction, the reason for this divergence is the reinvestment of profits generated by marginal investment projects. The King–Fullerton formula is based on the assumption that marginal profits are distributed to shareholders regardless of the marginal source of finance. This assumption is harmless for mature firms whose marginal product equals the market rate of interest. Even if these firms retain their marginal profits, the cost of capital can be calculated as if the profits were distributed since the shareholders are indifferent between dividends and retentions. Retentions generate a present value of future dividends that just equals the value of the dividends foregone. Things are different, though, for young firms that have investment projects yielding a gross rate of return above the market rate of interest. For these firms retentions dominate dividends strictly and so it does make a difference whether the returns from marginal investment projects are retained or distributed. In fact, the reinvestment of profits reduces q, the marginal value of equity, and this reduction is a capital loss which increases the firm's cost of capital beyond the value which simple arbitrage conditions are able to predict.[7]

A useful study in the firm's cost of equity capital that also allows for a change in q is that of Edwards and Keen (1984). These authors calculate cost of capital formulae for situations where adjacent periods are characterized by different combinations of new share issues and retained profits. However, they do not allow for a phase of internal growth, and, except for Phase III, their formulae are not applicable to

[7]This does not mean that the value of equity itself is being reduced. It follows from (1) that, during Phase II where $D = Q = 0$, $(\dot{m}z+\dot{z}m)/M = \dot{M}/M = r$. Thus, the market value of equity grows at a rate that equals the market rate of interest. The co–state variable q is the slope of a concave curve in (M, K) space that depicts the market value of the firm as a function of its stock of equity capital. During Phase II, there is an upward movement along this curve that comes to a halt where the slope equals Θ.