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THE VANISHING HARBERGER TRIANGLE

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ABSTRACT

The paper presents a trapped equity model, but instead of studying how taxes affect corporate decisions when a sufficient amount of equity is already in the trap, it asks the question how does the equity get there. To be more specific, the paper analyzes how the double taxation of dividends affects the growth of a corporation that starts with no equity capital. One conclusion is that dividend taxes are distortionary *before* they are paid, but not *when* they are paid. Once the firm is in a stage of maturity where it pays dividends and dividend taxes, tax neutrality prevails. Thus the true intersectoral distortion resulting from corporate taxation is negatively correlated with the measured tax burden, and it is lower, the higher the distortion which estimates of Harberger type would predict. Another conclusion is that the King-Fullerton cost of capital formulae are not applicable in the case of immature firms. These formulae are based on the assumption that firms distribute their profits from marginal investment projects as dividends. However, immature firms strictly prefer a reinvestment to a distribution of profits. The reinvestment changes the cost of equity capital, and typically this cost is higher than a hasty application of the King-Fullerton formulae would predict.

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1. The Problem

The roots of modern tax theory lie in Harberger's (1962, 1966) problem of how the double taxation of corporate dividends affects the allocation of resources between the corporate and non-corporate sectors of the economy. Harberger's claim was that the double taxation of dividends discriminates against corporate investment and creates welfare losses by keeping too large a share of the economy's capital stock in the non-corporate sector. The larger the tax burden on dividends, the bigger the welfare loss that results.

This paper reconsiders Harberger's problem from an intertemporal perspective. It studies the foundation and growth of corporations in the presence of dividend taxation to find out whether, and if so, under what circumstances dividend taxes create a Harberger-type distortion. The main result is that the distortion is a transitory phenomenon and that, in an important sense, the size of the welfare loss is negatively rather than positively related to the size of the tax burden.

The traditional view of corporate taxation as formulated by Harberger has recently been questioned by holders of the so-called "new" or "trapped equity" view of corporate taxation, including, for example, King (1974a, 1974b, 1977), Bradford (1980, 1981), Auerbach (1979, 1983), King/Fullerton (1984), and Sinn (1987). Their argument is that the dividend tax is capitalized in share prices and therefore cannot affect the firm's investment decisions. The tax is simply seen as a lump sum levy on corporations. If true, tax reforms, whose aim is to remove the double taxation of dividends, would be superfluous. They would create windfall gains for the current owners of corporate shares, but would not improve the allocation of resources.

Unfortunately, however, the new view does not seem fully compatible with the empirical facts. As observed by Poterba and Summers (1983, 1985), who studied the effects of British tax reforms, changes in the statutory dividend tax rate did have adverse effects on the level of aggregate investment. Poterba and Summers attributed their findings to the fact that the trapped equity model neglects the signalling function of dividends. While this is a possible explanation for the non-neutrality of dividend taxation, there are others. The one explored in this paper is suggested by a serious shortcoming of the trapped equity model.

Existing approaches that use this model have the common characteristic that they do not explain how equity falls into the trap. Typically, it is assumed that the firm already has more than the efficient amount of equity capital at the time the investment decision is analyzed. Under these circumstances, the neutrality proposition is not especially surprising. It just means that the firm retains the efficient amount of equity and distributes the remainder. The important problems of how much equity capital shareholder may wish to inject into their firms in the first place and whether the corporate stock of capital will ever reach its efficient size are unsolved.

This paper offers a solution that is consistent with Poterba and Summers's findings. It reconsiders Harberger's problem from the viewpoint of a trapped equity model, but one that starts with the process of injecting capital into the firm. Surprisingly, no similar model seems to exist in the literature. It is true that holders of the trapped equity view typically concede that dividend taxes are distortionary to the extent that new issues of shares are a marginal source of finance. However, as far as is known, no attempt has been made to formulate an explicit intertemporal model that describes the foundation and growth of a corporation in the presence of dividend taxation.

The paper rehabilitates Harberger's view that the dividend tax discriminates against corporate investment, but, in addition, it modifies and criticizes his analysis. Harberger and many of his followers have concentrated on the general equilibrium repercussions of taxation and have placed little emphasis on microeconomic considerations such as how taxes would affect the investment decisions of the firm. Frequently they have simply assumed that the corporate firm invests until the net-of-tax marginal product of capital equals the market rate of interest. This assumption is compatible with *partial* optimization given that new share issues are the only marginal source of finance and that all profits resulting from an investment are distributed as dividends. However, there are at least two problems with this. The first is that, instead of new share issues, the firm may choose other sources of finance. From an empirical point of view, both debt and retained profits are cheaper and much more important sources than new issues of shares. The holders of the new view have emphasized this and have derived investment conditions that typically imply lower distortions than those Harberger argued for.

The second problem is that, instead of using the profits from its marginal investment to pay dividends to shareholders, the firm may choose other uses for profits. One possibility is share repurchases. Profit financed share repurchases can be seen as a way of avoiding the double taxation of dividends and they undermine Harberger's results for obvious reasons. A second potential use for profits is internal investment. This is not only of great empirical significance in all countries, it is also suggested by theoretical considerations. In fact, it is clear that a firm would not distribute its profits immediately after it has issued new shares if, as Harberger claimed, the shares stopped being issued before the point where the marginal product of capital equals the market rate of interest. With an internal rate of return above the shareholders' discount rate, it would always pay to reinvest the profits and distribute them later. The present value of dividends, net of the dividend taxes, could be increased by postponing the distributions for as long as it takes for the process of reinvesting profits to equate the marginal product of capital and the market rate of interest. This suggests that there might be something wrong with the reasoning underlying the Harberger-type cost-of-capital formula even if it is assumed that the firm is forced by an initial shortage of retainable profits to issue new shares at the margin, cannot borrow, and cannot escape the dividend tax by repurchasing its shares.

The reinvestment of the profits generated by marginal investment projects is incompatible not only with Harberger's formula, but renders some of the formulae provided by holders of the new view also inapplicable. For example, the popular cost of capital formula of Fullerton and King (1984), which is a weighted average of the costs of the three alternative sources of finance, assumes that the profits from marginal investment projects are distributed. King's (1977) expressions for the cost of new share issues and retained profits, which enter this formula, are not applicable when the returns

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from marginal investment projects are reinvested at a rate of return above the market rate of interest. Section 2.5 will discuss the relationship to the cost of capital formulae provided by the holders of the new view in more detail.

Except for the exclusion of debt financing and profit financed share repurchases, the present paper makes no assumptions about the firm's source of funds for, and the use of the profits generated by, marginal investment projects. Instead, the financial and real investment decisions are endogenously derived from the firm's optimization approach. The available sources of funds are new share issues and retained profits, and the possible uses for profits are dividends and internal investment. The exclusion of debt financing and profit financed share repurchases is motivated by the attempt to treat one difficulty at a time and to follow Harberger's analysis as closely as possible. Including these possibilities would weaken his case and imply a criticism of his other results than the one made here.¹

Essentially, the paper is a reconsideration of Harberger's problem, asking his questions and using his assumptions. It formulates an intertemporal variant of his two sector model, which is based on microeconomic optimization rather than arbitrarily postulated marginal conditions. This variant is slightly more complicated than the original model but it nevertheless reflects the attempt to be as simple as possible without giving up the rigor necessary to make the point. The economy has a given stock of capital for which two representative firms compete in a perfect capital market. The firms produce the same commodity, use only equity capital, and please their far-sighted owners by choosing the investment policies which maximize their market values. One of the firms is corporate, the other non-corporate. The trapped equity property is modeled by the assumption that the government does not contribute to funds shareholders inject into the firm, but taxes all payments to shareholders that result from current or previous profits. A tax exempt return of the original capital is allowed.²

¹A critical discussion of the Harberger approach that allows for debt financing and other modifications of his assumptions can be found in Sinn (1987, ch. 6).

²The paper should not be seen as an attempt to solve the dividend puzzle. See Poterba (1987) for an excellent discussion of this puzzle.

2. Dividend Taxation and the Growth of the Corporation

Before the Harberger problem can be meaningfully discussed, a model of the firm is needed that explains how a corporate firm is set up and how the equity capital falls into the trap. This section provides one. The next section will add an untaxed firm and analyze the properties of capital market equilibrium.

2.1. A Model of the Firm

The firm's policy is determined by its shareholders who, in line with Fisher's separation theorem, unanimously agree to maximize the initial market value of shares net of the original capital injected. Let t_1 be the point in time t at which the firm's planning problem starts. Shareholders are price takers, look through the corporate veil, and are endowed with perfect foresight of all variables of the model. They can borrow and lend at the going market rate of interest r, r > 0, whose time path they take as exogenously given. The market value of shares is therefore implicitly determined by the following arbitrage condition that requires shareholders to be indifferent between keeping their wealth in the form of bonds or shares:

(1)
$$rM = \Theta D + \dot{m}z + (\dot{z}m - Q) \qquad \text{for } t > t_1$$

The lefthand side of (1) is the return from selling the existing stock of shares at its market value M and investing the funds received in bonds which yield the market rate of interest τ . The righthand side of (1) measures the current return from continued shareholding. D is the gross dividend and $\Theta \equiv 1-\tau$ is one minus the dividend tax rate. It is assumed that $0 < \Theta < 1$ and that Θ is a constant.³ The next term, \dot{mz} , is the capital

³See Howitt and Sinn (1986) for an analysis of anticipated changes of the dividend tax rate in a trapped equity model with debt financing.

gain from existing shares where m is the price of a share and z the number of outstanding shares. The term in round brackets is the value of purchasing options for new shares issued to the existing shareholders. It is the difference between the market value of the new shares, zm, and the flow of funds Q that must be paid to the firm in exchange for the shares. When there are no purchasing options (as in the U.S., for example) it can be assumed that zm = Q as existing shareholders will not vote for a policy of diluting their shares. In general, however, there is no need to assume that zm and Q are linked to one another.

Noting that $\dot{m}z + \dot{z}m = \dot{M}$, (1) can be transformed to $\dot{M} = -\Theta D + Q + rM$ which, upon integration, gives

(2)
$$M(t) = \int_{t}^{\infty} [\Theta D(v) - Q(v)] \exp \int_{t}^{v} - r(u) \mathrm{d}u \, \mathrm{d}v \qquad \text{for } t \geq t_{1}$$

plus some arbitrary constant. The constant is taken to be zero to ensure that the market value of a firm that promises never to issue new shares and never to distribute any dividends is zero. It is assumed for the derivation of (2) that the integral exists which requires that $\lim_{t^* \to \infty} [\Theta D(t^*) - Q(t^*)] \exp \int_t^{t^*} - r(u) du = 0$.

Following Harberger, it is assumed that the firm produces its output only with equity capital K. Moreover, with only small losses in generality, all commodity prices are assumed constant and normalized to unity. Under these assumptions, the firm's revenue, profit, and output can all be described by the function f(K) satisfying the usual properties f' > 0, f'' < 0, $f'(0) = \omega$, $f'(\omega) = 0$. The dividend the firm can pay is

$$D = f(K) - \check{K} + Q,$$

where \dot{K} is the firm's net investment. Let K_0 be the stock of capital available from the firm's past history and K_1 the stock of equity capital reached at t_1 after the initial issue of shares.

Then, the firm's problem can be expressed as

(4)

$$\max \qquad M(t_1) - K_1$$

$$\{\dot{K}, Q, K_1\}$$

$$s.t.$$

$$K_1 \ge K_0 = 0$$

$$D \ge 0,$$

$$Q \ge 0,$$

where K is the state variable and K, Q, and K_1 are the controls. The three constraints implicitly capture the trapped equity assumption that the government participates in corporate distributions but not in what shareholders inject into their corporation. For the time being, the trapped equity assumption is made in the extreme form that it is impossible for the firm to pay cash to its shareholders that is not taxed as dividends; i.e., that $Q \ge 0$ for all $K \ge 0$. This assumption will be relaxed in section 4 to allow the firm to return its original share capital. A similar remark applies to the assumption $K_0 = 0$.

2.2. The Optimality Conditions

The problem of the firm can be solved by using Pontryagin's Maximum Principle. Using (3) and associating a co-state variable q (Tobin's q) with K and Kuhn-Tucker multipliers μ_D and μ_Q with the flow constraints, the current value Hamiltonian of problem (4) can be written as:

$$\mathscr{H}=(\Theta+\mu_{\mathrm{D}})[f(K)-\dot{K}+Q]+q\dot{K}-Q(1-\mu_{\mathrm{D}}).$$

From $\partial \mathcal{H} / \partial \dot{K} = 0$ it follows that

$$(5) q = \Theta + \mu_{\rm D}$$

and from $\partial \mathcal{K}/\partial Q = 0$ that

$$\mu_{\rm D} + \mu_{\rm Q} = \tau \; .$$

Both equations together imply:

$$q = 1 - \mu_{\mathbf{Q}} \quad .$$

Because of (5), the canonical equation $\dot{q} - rq = -\partial \mathcal{H}/\partial K$ can be transformed to

(8)
$$f'(K) + \frac{\dot{q}}{q} = r \; .$$

The Kuhn-Tucker conditions of the problem are

(9)
$$\mu_Q Q = 0, \ \mu_Q \ge 0, \ Q \ge 0,$$

(10)
$$\mu_{\rm D} D = 0, \ \mu_{\rm D} \ge 0, \ D \ge 0$$
.

The firm's starting condition is $\partial M(t_1)/\partial K_1 - 1 = 0$ which, as $\partial M(t)/\partial K(t) \equiv q(t)$ holds by definition, implies that

(11)
$$q(t_1) = 1$$
.

Finally, the transversality condition is

(12)
$$\lim_{t\to\infty} q(t)K(t) \exp \int_{t_1}^t r(v)dv = 0.$$

Notice that, because of (6), (9), and (10), the firm cannot simultaneously issue new shares and pay dividends. Instead, at any point in time after t_1 , it must either be the case that $(Q > 0; D = \mu_Q = 0)$, that $(Q = D = 0; \mu_Q, \mu_D \ge 0)$, or that $(Q = \mu_D = 0; D > 0)$. Together with the initial condition, this implies that the following activity phases are available. The names of these phases anticipate properties yet to be derived.

Phase Ia $(K_1 \ge 0; t = t_1)$

Phase Ia refers to the starting point where the original stock of equity K_1 may be injected. According to (11), this phase is characterized by

$$q(t_1)=1$$

Phase Ib $(Q > 0; D = \mu_Q = 0; t > t_1)$

Phase Ib is a phase of continuing equity injections after the time of foundation. During this phase, $\mu_Q = 0$ and hence (7) implies that

$$q=1, \quad \dot{q}=0.$$

It therefore follows from (8) that

$$(13) r = f'(K) .$$

Phase II $(Q = D = 0; \mu_Q, \mu_D \ge 0; t > t_1)$

If the firm neither issues new shares nor pays out any dividends, then from (8):

(14)
$$\dot{q} = q \left[r - f'(K) \right],$$

and, from (3):

$$\dot{K} = f(K) .$$

Phase III $(D > 0; Q = \mu_D = 0; t > t_1)$ For a firm that pays dividends, (5) and $\mu_D = 0$ indicate that

$$q = \Theta, \dot{q} = 0$$

which, because of (8), implies that

$$(16) f'(K) = r.$$

2.3. The Optimal Growth Path

The optimal growth path of the firm is a combination of the four phases that satisfies the the transversality condition (12) and the Maximum Principle's general requirement that there be no jumps in the state and co-state variables. Assuming for the time being that the market rate of interest is a positive constant⁴ for $t \ge t_1$, the growth path can be uniquely determined in (q, K) space as illustrated in Figure 1.

The position of the vertical line in this diagram characterizes the equity level K_2 implicitly defined by the laissez-faire condition $f'(K_2) = r$. During Phase III, the firm is on this vertical line at $q = \Theta$. It distributes all its profits since $\dot{K} = Q = 0$ and it can stay in this phase forever since the transversality condition (12) is satisfied.

Phase III is the phase on which the new view of corporate taxation concentrates. The shadow price of equity, q, is Θ rather than one, since Θ is the shareholders' net dividend foregone if the firm decides to retain and invest one additional unit of profit. The low value of the shadow price just compensates for the tax on the

⁴In the next section, this assumption will be relaxed and the time path of r will be endogenously determined by the conditions of a market equilibrium.

returns which an additional unit of equity capital will generate and explains why, despite the tax, the firm follows the laissez-faire investment rule f'(K) = r. Let t_2 be the point in time at which Phase III begins.



Figure 1: The optimal growth path under the dividend tax

Clearly, Phase III cannot be a starting phase. The firm first has to raise enough equity capital to get there. A potential candidate for explaining how to reach Phase III is Phase II, for the two differential equations (14) and (15) define a set of possible paths in (q,K) space one of which intersects the vertical at $q = \Theta$. The slope of these paths is given by

(17)
$$dq/dK = \dot{q}/\dot{K} = q [r - f'(K)]/f(K) \qquad (Phase II)$$

As f'' < 0 and K > 0 for K > 0, the slope is negative in the region to the left of the vertical, zero on the vertical, and positive in the region to the right of it. The arrows in Figure 1 indicate the possible movements. Assuming that the production elasticity of K

is bounded away from unity, it is shown in the appendix that the ordinate is an asymptote for all paths:

$$q(K) \to \infty \text{ for } K \to 0$$
 (Phase II).

This property ensures that the path leading to the steady state point (Θ, K_2) intersects the horizontal line of height q = 1 at some strictly positive value of K; call this value K_1^* .

Notice that Phase II cannot be a final phase. If it were, the firm would never again pay dividends. Yet it is clear from (2) that this cannot be an optimum since, at any point in time t^* during this phase and given the then available stock of equity $K(t^*)$, the firm could increase its market value $M(t^*)$ by paying out all future profits as dividends and keeping the stock of equity constant. The existence of such a possibility would violate the Bellman Principle on which the Maximum Principle is built.

Neither can Phase II be an initial phase. Starting with Phase II means starting with an infinite value of q. Given the possibility of injecting equity capital at a price of unity from outside the firm, this cannot be optimal.

Potential candidates for an initial phase are Phases Ia and Ib. It can easily be seen, however, that Phase Ib does not exist. During Phase Ib, q = 1 and the firm issues new shares without distributing any dividends. In the diagram, this means that there is a horizontal movement to the right $(\dot{K} > 0)$ with q being kept constant at the level of unity. Such a horizontal movement clearly implies that condition (13) cannot be met, for this condition and the assumption $\dot{r} = 0$ imply that $\dot{K} = \dot{r}/f'' = 0$. The firm would have to satisfy the laissez faire condition f' = r, if it continued to issue new shares, but it cannot.

Thus, only Phase Ia remains. Like Phase Ib, Phase Ia requires that q = 1. However, since this phase merely refers to the starting point $t = t_1$, the flow condition (13) is not required. The firm issues a sufficient number of shares to reach the Phase-II path in one step when it is founded: $K_1 = K_1^*$. Because of the non-existence of Phase Ib, the subscript "a" is dropped in the remainder of this paper and all references to Phase I are meant to refer to Phase Ia.

With Phase I as the necessary starting phase and Phase III as the only available final one, the question is whether Phase II is needed at all. It might be tempting to believe that the optimal strategy involves issuing enough shares to reach K_2 during Phase I and then to start with Phase III ($t_2 = t_1$). However, such a direct move between the two phases would require a forbidden jump in the co-state variable q from 1 to Θ . Phase II therefore is a necessary link between Phase I and Phase III, and the only continuous transition between the phases is one that satisfies the following pattern. During Phase I, q = 1 and new shares are issued until the desired stock of original capital K_1 is reached. Then the firm drifts along the curved Phase-II path towards the steady state point ($q = \Theta$, $K = K_2$) accumulating a surplus reserve of amount $K_2 - K_1$. Once there ($t = t_2$), it is in Phase III and stays there forever.

It is important to realize that, unlike many other intertemporal models, the steady state is reached in finite time. As Q = D = 0 during Phase II, it follows from (3) that the increase in K per period is positive and bounded away from zero, and, in fact, even the speed of increase is increasing:

$$\dot{K}(t) = f[K(t)] \ge f(K_2) > 0$$
,
 $\ddot{K}(t) = f'[K(t)] \dot{K}(t) > 0$ for all $t_1 \le t \le t_2$ (Phase II).

This clearly implies that $t_2 < \infty$; i.e., that the steady state stock of surplus reserves $K_2 - K_1$ is accumulated in finite time.

The following proposition summarizes these findings. It describes the optimal growth path of a corporation that is subject to dividend taxation.

Proposition 1: When the firm is founded, new shares are issued to generate some equity to start with. The starting stock is smaller than the one at which the marginal product of capital equals the market rate of interest. After the foundation, a phase of internal growth follows during which the firm neither issues new shares nor distributes any profits. This phase terminates in finite time when sufficient surplus reserves have been accumulated to equate the marginal product of capital with the market rate of interest. The firm will then stop growing, issue no shares, and distribute all its profits.

While Proposition 1 refers to the qualitative aspects of the growth path when there is a dividend tax, it does not clarify the role of the dividend tax itself. To understand this role, note that the size of the dividend tax rate neither affects the initial value of q, nor the set of Period-II paths compatible with (14) and (15), nor the steady state value of equity, K_2 . The only thing that is affected is the steady state value of q, $q(t_2) = \Theta$. This value singles out the optimal path during Period II and determines both the length of this period and the size of the original capital K_1 . Obviously, the higher τ , the lower Θ and q, the lower K_1 , and the longer the time span that must elapse before the missing amount of surplus reserves $K_2 - K_1$ is accumulated. When there is no dividend tax, then $q(t_2) = q(t_1) = 1$, and Phase II is not needed to avoid a jump in q. Shareholders inject a sufficient amount of original capital to reach the steady state value K_2 in one step.

Again, this is summarized in a proposition.

Proposition 2: The phase of internal growth is longer and the starting stock of equity smaller the higher the tax rate on dividends. Without the dividend tax, there is no such phase. All equity is then generated through share issues when the firm is founded.

2.4. The Nucleus Theory of the Corporation

While Propositions 1 and 2 are meant to prepare the ground for a discussion of the Harberger problem, they may be interesting in their own right. They show that the policy of maximizing the rate of internal growth and minimizing dividend payments that has been so graphically described by Penrose (1959) and others does not have to be explained by a divergence between manager and shareholder interests. The high burden of dividend taxes can also be an explanation. It is particularly reaffirming in this context to hear what Barlow/Wender (1955, ch. 11) and Penrose (1956, pp. 227-229) say about the growth of foreign affiliates of U.S. corporations. The typical pattern of growth these authors observed was that, when founded, the affiliates were given only a nucleus of equity capital and then had to grow by themselves. Only mature affiliates that had reached their desired size were expected to distribute profits. In the light of the fact that

border crossing profit distributions are frequently subject to international double and triple taxation, this observation is not at all surprising.⁵

A further observation involves the relatively infrequent occurrence of new share issues that has been observed by many authors. For example, in the period from 1980 to 1985, on average 67.8 % of gross investment by U.S. non-financial corporations was internally financed and 31.0 % was debt financed, but only 1.2 % was financed by share issues.⁶ The actual figures may be somewhat different for other countries, but their tendency clearly describes a general empirical phenomenon. In developed economies, corporations are self-perpetuating enterprises that rarely rely on equity injections from the household sector but generate most of their equity capital internally.

2.5. Relationship to the New View

Propositions 1 and 2 complement the new view of corporate taxation. They confirm this view for Phase III and they show that this phase will indeed be reached. In Phase III, profit retentions are a *potential* marginal source of finance and the profit from marginal investment projects is paid out as dividends. As predicted by holders of the new view, the marginal product of capital equals the market rate of interest.

There is less agreement, though, for Phases I and II. As is well-known, the King-Fullerton formula says that, when there is only a dividend tax, the cost of capital is r for a firm that relies exclusively on retained profits, and r/Θ for one that relies exclusively on new issues of shares, to finance its marginal investment projects. By way of contrast, in Phase II, where retained profits are the only marginal source of finance,

⁵Cf. Sinn (1987, pp. 197-200) for short review of tax rules applying to international capital income flows.

⁶Calculated from Survey of Current Business, Volumes 57 (July 1977, p. 24 n.), 61 (1981, special supplement, p. 10), 63 (July 1983, p. 30), 66 (July 1986, p. 33); and Federal Reserve Bulletin, Volumes 55 (November 1969, p. A 71.4), 60 (October 1974, p. A 59.4), 64 (June 1978, p. 433), 65 (December 1979, p. A 44).

the cost of capital exceeds r. Moreover, as shown in the appendix, the cost of capital is not, in general, equal to r/Θ when new share issues are the only marginal source of finance (Phase I). In fact, under mild conditions, it can be proved to be higher than this value. It therefore seems that the King-Fullerton formula underestimates the cost of equity finance for immature firms which are in Phases I or II.

As mentioned in the introduction, the reason for this divergence is the reinvestment of profits generated by marginal investment projects. The King-Fullerton formula is based on the assumption that marginal profits are distributed to shareholders regardless of the marginal source of finance. This assumption is harmless for mature firms whose marginal product equals the market rate of interest. Even if these firms retain their marginal profits, the cost of capital can be calculated as if the profits were distributed since the shareholders are indifferent between dividends and retentions. Retentions generate a present value of future dividends that just equals the value of the dividends foregone. Things are different, though, for young firms that have investment projects yielding a gross rate of return above the market rate of interest. For these firms retentions dominate dividends strictly and so it does make a difference whether the returns from marginal investment projects are retained or distributed. In fact, the reinvestment of profits reduces q, the marginal value of equity, and this reduction is a capital loss which increases the firm's cost of capital beyond the value which simple arbitrage conditions are able to predict.⁷

A useful study in the firm's cost of equity capital that also allows for a change in q is that of Edwards and Keen (1984). These authors calculate cost of capital formulae for situations where adjacent periods are characterized by different combinations of new share issues and retained profits. However, they do not allow for a phase of internal growth, and, except for Phase III, their formulae are not applicable to

⁷This does not mean that the value of equity itself is being reduced. It follows from (1) that, during Phase II where D = Q = 0, $(\dot{m}z + \dot{z}m)/M = \dot{M}/M = r$. Thus, the market value of equity grows at a rate that equals the market rate of interest. The co-state variable q is the slope of a concave curve in (M, K) space that depicts the market value of the firm as a function of its stock of equity capital. During Phase II, there is an upward movement along this curve that comes to a halt where the slope equals Θ .

the present model. Their result (p. 214) that the dividend tax does not affect the cost of capital "whenever the marginal source of funds is the same in two adjacent periods" cannot be confirmed. During Phase II, retained profits are the marginal source of finance in all adjacent "periods", but nevertheless the dividend tax is able to drive a wedge between the marginal product of capital and the market rate of interest. The cited statement is only true in Phase III where retained profits are a potential marginal source of finance and dividends are paid.

The possibility of, and preference for, generating equity capital through profit retentions reduces the need for equity injections when the firm is founded and eliminates this need thereafter. It implies an extended period of internal growth during which the cost of capital is higher than the Edwards-Keen and King-Fullerton formulae predict, but it also implies that the firm will eventually reach a stage of maturity where the cost of capital is lower than Harberger assumed.

3. The Harberger Problem

3.1. Harberger's Own Analysis

Consider now the Harberger problem more closely. Suppose there are two sectors, X and Y, competing for a given aggregate stock of capital \overline{K} :

(18)
$$\overline{K} = K_{\rm r} + K_{\rm r} = {\rm const.}$$

X is the corporate and Y the non-corporate sector. The two sectors produce the same commodity by using their sector-specific production functions $f_{\mathbf{X}}()$ and $f_{\mathbf{Y}}()$. Aggregate output is $f_{\mathbf{X}}(K_{\mathbf{X}}) + f_{\mathbf{Y}}(K_{\mathbf{Y}})$.

Assume that, before t_1 , there was only a non-corporate sector, but at t_1 the corporate sector is "invented". One may think of the corporation as a new form of organizing a firm which increases the efficiency of production and induces the government to impose a dividend tax in order to participate in the rents this form can be expected to generate. Let $f_{\mathbf{x}}(\cdot)$ be the new production function available to the corporate firm. An efficient allocation of capital to the two sectors which maximizes Y, given \overline{K} , is characterized by equality in the marginal products of capital for all points in real time after the corporate firm has been invented $(t > t_1)$:

$$f'_{\mathbf{I}}(K_{\mathbf{I}}) = f'_{\mathbf{Y}}(K_{\mathbf{Y}})$$
 (efficiency).

The question is whether and when this condition will be violated in the presence of a tax on corporate distributions.

Harberger's (1966) model is based on the implicit assumption that capital invested in the corporate sector satisfies the condition

$$\Theta f_{\mathbf{I}}'(K_{\mathbf{I}}) = r$$

and capital invested in the non-corporate sector the condition

$$(19) f'_{v}(K_{v}) = \tau$$

so that, in a capital market equilibrium,

$$\Theta f_{\mathbf{X}}'(K_{\mathbf{X}}) = f_{\mathbf{Y}}'(K_{\mathbf{Y}})$$
 (Harberger).

This means that a constant wedge the size $\tau f'_{\mathbf{X}}$ is being driven between the two marginal products of capital and that there is a permanent welfare loss in terms of a reduced level of output.



Figure 2: The Harberger model

Figure 2 illustrates the Harberger equilibrium. The downward and upward sloping curves are the marginal product curves of the two sectors. The employment of capital in the corporate sector is measured from left to right and in the non-corporate sector from right to left. The distance between the two verticals is the total amount of capital, \bar{K} , that is available. The stock of capital is optimally allocated to the two sectors when $K_{\chi} = DF$ and $K_{\gamma} = FG$, for then aggregate output, the area under the two curves, is maximized. However, the allocation Harberger believed to result from the dividend tax is characterized by $K_{\chi} = DE$ and $K_{\gamma} = EG$, for this allocation implies that the marginal product of capital in the corporate sector exceeds that in the non-coporate sector by an amount sufficient to compensate for the tax discrimination against corporate investment. Obviously, non-corporate output exceeds its optimum level by the amount CBFE, but this is overcompensated by a comparative output loss of size ABFE in the corporate sector. The net output loss of both sectors together is measured by the

Harberger triangle ABC, and this triangle persists for as long as the dividend tax is levied.

The model set up in the last section is not compatible with this result. To see this, only a few steps are necessary.

3.2. Another View of the Harberger Problem

Notice first that the decision problem of the non-corporate firm can be seen as a special case of that model where $\Theta = 1$. The terminology simply has to be changed in an obvious way from a corporate to a non-corporate firm and the constraints $D \ge 0$ and $Q \ge 0$, which, because of (6), (9), and (10), cannot be binding anyway, have to be removed. As revealed by (5) and (8) for the case $\mu_{\rm D} = 0$, the amount of capital employed by the non-corporate sector will always satisfy (19), and because of (18), the equilibrium level of the market rate of interest is given by

(20)
$$r = f_{\mathbf{y}}'(\overline{K} - K_{\mathbf{x}}) .$$

The decision problem of the corporate firm was formulated for the case of an arbitrarily given time path of the market rate of interest r, but the phase diagram of Figure 1 was analyzed assuming a constant rate of interest. Thus the next step required is to generalize the discussion of this diagram to the case where the market rate of interest is endogenously determined by (20).⁸

When r is endogenous, nearly everything that has been said concerning Phase I and Phase III stays valid. In particular, it will still be true that q = 1 in Phase I and $q = \Theta$ in Phase III. Again, Phase Ib cannot exist. On the one hand, Q > 0, D > 0,

⁸Note that the time path of r is endogenous to the equilibrium, but not to the firm's planning problem. The firm is assumed to be a price taker. It is not assumed that it has market power and thus believes that it can affect the time path of the market rate of interest through its own actions.

and (20) imply that $\dot{K}_{\chi} > 0$ and $\dot{r} = -f_{\chi}^{\prime\prime} \cdot \dot{K}_{\chi} > 0$. On the other hand, (13) and $\dot{r} > 0$ indicate that $\dot{K}_{\chi} = \dot{r}/f^{\prime\prime} < 0$, a clear contradiction. The only important addition to the previous analysis is that the steady state stock of capital, call it now K_{χ_2} , that was previously determined by (16), is now implicitly given by

$$f'_{\mathbf{X}}(K_{\mathbf{X}_2}) = f'_{\mathbf{Y}}(\overline{K} - K_{\mathbf{X}_2})$$

There are minor changes with the possible paths during Phase II as defined by (14) and (15). Instead of (17), the slope of a path is now given by

$$dq/K_{\rm X} = q [f_{\rm Y}'(\bar{K} - K_{\rm X}) - f_{\rm X}'(K_{\rm X})]/f_{\rm X}(K_{\rm X}).$$

This equation reveals that the paths are more strongly curved than in the case of a constant r, but clearly none of the qualitative properties described in the previous section change. Thus Figure 1 and Propositions 1 and 2 stay perfectly valid. It is only necessary to keep in mind that the market rate of interest can be identified with the marginal product of capital in the non-corporate sector.

Figure 3 illustrates the intertemporal market equilibrium in a diagram that combines Figures 1 and 2. In the beginning, there is no corporate sector so that the total amount of capital DG is invested by non-corporate firms. Then, at t_1 , the corporate sector defined by the new marginal product curve $f'_{\mathbf{x}}$ and the dividend tax factor Θ come into existence. By issuing shares the corporate sector will immediately withdraw the amount of capital DE from the non-corporate sector and bid up the interest rate from HD to CE. The withdrawal is less than is required by efficiency and implies that there is a Harberger triangle of size ABC. However, the welfare loss is only temporary. With the passage of time, the corporate sector will build up equity capital through profit retentions and claim a growing proportion of the economy's available stock of real assets. This improves the allocation of resources to the two sectors and increases aggregate output. The process comes to a halt when the Harberger triangle has vanished, the



Figure 3: The vanishing Harberger triangle

corporate stock of capital has increased to DF, and the non-corporate stock has fallen to FG. The economy is then in an efficient steady state where its output is maximized given the available stock of capital and where the profits are distributed to the household sector. The following proposition summarizes this conclusion.

Proposition 3: Initially, when the corporate sector is young and reinvests its profits, there is a Harberger triangle. Yet, this triangle gradually vanishes with the passage of time and aggregate output increases. In finite time, the economy reaches a stage of maturity where the corporate sector distributes its earnings and the available stock of capital is being efficiently allocated to the two sectors.

3.3. The Correlation between Dividend Taxes and the Distortions they Cause

There is a final proposition generated by the model which follows directly from the observation that no dividends, and hence no dividend taxes, are paid during the adjustment to the steady state.

Proposition 4: The dividend tax distorts the intersectoral allocation of resources when it is not paid and it is neutral when it is paid.

The payment of dividend taxes signals that the firm is in a stage of maturity where the new view of corporate taxation holds and where the dividend tax no longer affects the investment decisions. The burden of the tax is capitalized in share prices, and there is no way for the corporate sector to reduce this burden by changing its investment policy. By way of contrast, corporations which do not pay dividend taxes signal that they are in need of equity capital and have not yet reached the stage of maturity and efficiency. The point is simply that dividend taxes create distortions *before* they are paid. The threat of dividend taxes that will have to be paid in the future makes shareholders reluctant to inject more than a nucleus of equity capital into their firms. However, when these taxes are actually paid, the process of reinvesting profits must have generated enough equity to compensate for this threat and to eliminate the Harberger triangle. In any given period of time, there is a negative correlation between the size of the tax burden and the magnitude of the Harberger triangle.

4. Extensions and Qualifications

The previous discussion referred only to a single tax and to the case of founding the corporate sector as a whole. It was also based on an extreme version of the trapped equity view. This section comments on possible extensions, generalizations, and qualifications.

4.1. Disturbances with an Existing Corporate Sector

One obvious extension is to think of an initial steady state equilibrium with an existing corporate sector that is disturbed by new and unforeseen inventions which shift this sector's marginal product curve upward. This case can be captured with the formal approach derived above by changing the initial condition of problem (4) from $K_0 = 0$ to $K_0 > 0$. This would not affect the time paths depicted in Figures 1 and 3 but would simply imply that the economy starts at a later stage on these paths. Suppose, the new marginal product curve after the invention is the one illustrated in Figure 3 and the old curve intersected the marginal product curve of the non-corporate sector to the left of point C. In this case, the corporate sector issues new shares at the time the invention occurs and jumps immediately to point C. After this, there is again the finite period of internal growth ending with the stage of maturity, B, where dividends are paid. And once again the dividend tax retards the adjustment process towards an efficient equilibrium, but does not prevent this equilibrium from being eventually reached.

Instead of the initial equilibrium being located to the left of point C in Figure 3, it might also have been to the right of this point. In this case, the corporate sector will not react to the inventions by issuing new shares but will merely stop paying dividends to its shareholders, entering a period of internal growth which eventually leads to the stage of efficiency and maturity.

4.2. Escapes from the Equity Trap

Another extension would be to relax the constraint that dividends are the only way of distributing cash to shareholders. This constraint is certainly not realistic as most countries allow for a tax-exempt return of original capital. If such return were permitted to replace or precede dividend payments, the firm's optimal growth path would be strongly affected. However, the typical provision – one that definitely applies in the United States, for example – is that a return of capital cannot occur before current

profits and all accumulated reserves have been paid out. Formally, the possibility of returning the original capital *after* distributing the reserves and profits implies that the constraint $Q \ge 0$ of problem (4) is removed for $K \le K_1$ and maintained for $K > K_1$ where K_1 is the original capital (see Figures 1 and 3). As this means that a flow constraint is removed for values of the state variable (K) where it is not binding $(q \ge 1)$, and retained where it is (q < 1), the firm's optimal growth path and all of the conclusions based on it would remain unaffected.

Cash payments to shareholders that would undermine both Harberger's results and those derived in this paper are profit financed share repurchases and acquitisions. Share purchases by corporations that are financed out of past and present profits largely avoid the double taxation of dividends and indicate a loophole in the classical and partial imputation systems of capital income taxation. Most countries have effectively closed this loophole by declaring share repurchases illegal. However, in the United States the situation is ambiguous. On the one hand, Section 302 of the Internal Revenue Code prohibits firms to repurchase shares in lieu of dividend payments. On the other hand, share purchases by corporations have recently increased dramatically, constituting a large fraction of corporate cash distributions. A very extensive record of this phenomenon is provided by Shoven (1986).⁹ He showed that, since 1984, the volume of corporate share purchases, predominantly acquisitions, exceeded ordinary dividend payments, yet he also found that the share purchases did not simply replace the dividends but reflected a leverage phenomenon. Quite remarkably, the time path of dividends was unaffected by the rising repurchase volume and debt rather than profits seemed to have been the source of the additional cash that shareholders received.10 Whether and to what extent the recent increase in corporate share purchases can be seen

⁹Cf. also Poterba (1987, p. 471) and Bagwell/Shoven (1988).

¹⁰See Sinn (1987, ch. 6) for a formal analysis of the advantages of profit and debt financed acquisitions.

as a use for profits from marginal investment projects that eliminates Harberger type distortions remains to be seen.

4.3. Other Taxes

Further modifications would involve a richer set of taxes. It would be space consuming, but easy, to introduce personal income taxes on dividends and interest income, personal capital gains taxes, and a corporate tax on retained and distributed profits. This extension could change the nature of the steady state, as there could be a persistent Harberger triangle resulting from a difference between the combined marginal corporate and capital gains tax burden on retained profits on the one hand and the shareholders' marginal personal tax burden on interest income on the other. However, under the classical system of capital income taxation, the basic adjustment pattern described, including the initial share issues, the phase of internal growth, and the shrinking Harberger triangle, would still show up. Moreover, the steady state size of the triangle would continue to be smaller than Harberger believed it to be.11 The driving force behind the results of this paper is a comparatively high tax burden on corporate dividends which exceeds both the burden on personal interest income and that on retained profits. Because of an unmitigated double taxation of distributed profits and a limited taxation of accrued capital gains this force is fully operative in real tax systems of the classical type which are, for example, employed in the United States, Switzerland, and Australia. The present model should have some relevance for these economies although it assumes that the dividend tax is the only tax in the economy.

¹¹A discussion of the steady state properties of the present model in the presence of richer tax structures can be found in Sinn (1987, ch. 6).

5. Conclusion

In one sense, the results of this paper confirm Harberger, in another they contradict him. It is certainly true that the high tax burden on corporate dividends creates efficiency losses as Harberger claimed. This burden is an obstacle to the foundation of firms and prevents capital from being used in the corporate sector although it could usefully be employed there. After a corporation has been founded with a nucleus of external equity capital, there is a phase of internal growth where retentions are the only marginal source of finance. During this phase, the intersectoral distortions are higher than those the cost of capital formulae used by holders of the new view of corporate taxation predict. It is not correct to weight the influence of dividend taxes with the proportion of investment financed by new share issues and it is not true that dividend taxes are neutral when firms generate their equity capital through profit retentions. There is a Harberger triangle when all profits are retained in consecutive periods and no shares are issued.

On the other hand, dividend taxes will not create permanent distortions in the allocation of capital but merely retard the speed with which an efficient allocation is reached. One lesson of the model is that the intersectoral distortions which Harberger claimed to result from the dividend tax are transitory phenomena that may have been important in early stages of the development of the corporate sector, but vanish when the economy matures.

In a mature economy, corporations distribute dividends to their shareholders. Firms that distribute dividends can always generate more equity capital by stopping the distributions. These firms are therefore in the situation which the new view of corporate taxation concentrates on. Their marginal cost of equity capital is the market rate of interest and they follow the same investment rules as their non-corporate competitors.

A stage of full maturity is conceivable for a world with one representative firm, and in some real economies it may have been reached to a high degree of approximation. In general, however, account must be taken of the facts that there are always new entrants into the corporate sector and that new investment opportunities show up for existing firms. These facts imply a continuing incentive to issue new shares. The model developed has immediate implications for this case. One such implication is that the size of the dividend tax rate will have a negative impact on the entrants' starting stocks of equity and the incumbents' share issues after inventions. The tax will therefore adversely affect corporate investment. Perhaps this is the explanation for the empirical result of Poterba and Summers that was cited in the introduction.

If the explanation is correct, it follows that dividend taxation impedes investment not only at the point in time where new shares are issued, but also in the period of internal growth that follows and, in fact, the distortion will be larger, the longer this period. A corollary of this result is that the overall distortion in the economy will be larger, the larger the proportion of investment that is financed with retained profits rather than with newly issued shares. This corollary is contrary to what the weighted average formulations of the cost of capital predict.

Harberger's empirical estimates of the intersectoral distortions created by the existing capital income taxes are strongly affected by these considerations. Unlike Poterba and Summers, Harberger and many of his followers did not focus on statutory tax rates but based their welfare estimates on "effective tax rates" defined as a sector's ratio of total capital income tax liability and total volume of capital income per unit of time. With a classical system of capital income taxation, where the overall tax burden on dividends exceeds that on retained profits, this means that the measured welfare loss will be higher, the higher the proportion of profits paid out as dividends, for the higher this proportion, the higher is the measured value of the effective tax rate.

If the spirit of the model presented in this paper is correct, this method of estimating the welfare loss stands the truth on its head. Given the tax law, a high effective tax rate for the corporate sector signals, among other things, that many corporations are mature and pay dividends; and a low effective tax rate – one that approximates the tax rate of the non-corporate sector – signals that many corporations are in the transitory period of rapid internal growth. A high tax rate therefore signals small, and a low tax rate large, intersectoral distortions or, to put it another way, the true intersectoral distortions are smaller, the larger the distortions that Harberger estimated. The Harberger triangle has not vanished, what has vanished is the idea that the triangle and the visible tax burden take the stage together.

Appendix

By studying the functional form of the Phase-II paths, this appendix

- i) proves that the ordinate is an asymptote for all possible paths and
- ii) derives a sufficient condition for Phase II to start with a capital stock lower than that which follows from Harberger's formula or, equivalently, a sufficient condition for the cost of capital associated with new share issues to be higher than Harberger's formula implies.

The proofs apply to the general model of section 2 where r is endogenous. The constant value of r assumed in section 1 is a special case of this.

The Functional Form of the Phase-II Path

Let $q(K_{\chi})$ be a function that describes the Phase-II path in (q, K_{χ}) space (see Figures 1 and 3) and let β_i denote the production elasticity of capital employed in sector *i*, assuming that this elasticity is bounded away from unity:

(A1)
$$\beta_i(K_i) \equiv f'_i(K_i)K_i/f_i(K_i); \quad i = X, Y; \quad \beta < \overline{\beta} < 1.$$

From (15), the time derivative of q is

$$\dot{q} = q'(K_{\mathbf{I}})\dot{K}_{\mathbf{I}} = q'(K_{\mathbf{I}})f_{\mathbf{I}}(K_{\mathbf{I}})$$

Inserting this into (14) and using (20) to explain τ endogenously, one obtains

(A2)
$$\frac{q'(K_{\mathbf{X}})}{q(K_{\mathbf{X}})} = \frac{f_{\mathbf{Y}}'(K-K_{\mathbf{X}})}{f_{\mathbf{X}}(K_{\mathbf{X}})} - \frac{f_{\mathbf{X}}'(K_{\mathbf{X}})}{f_{\mathbf{X}}(K_{\mathbf{X}})}$$

or, upon logarithmic integration,

(A3)
$$q(K_{\mathbf{X}}) = \frac{\Phi(K_{\mathbf{X}})}{f_{\mathbf{X}}(K_{\mathbf{X}})} \cdot c$$

where c is a strictly positive constant and

(A4)
$$\Phi(K_{\mathbf{X}}) \equiv \exp \int_{0}^{\mathbf{K}_{\mathbf{X}}} \frac{f_{\mathbf{Y}}'(\bar{K}-u)}{f_{\mathbf{X}}(u)} du.$$

Notice that, as $f'_{Y}(\overline{K}-u) < \infty$ for $u < \overline{K}$, the integral in (A4) is finite for all $K_{\chi} < \overline{K}$ if $\int_{0}^{K_{\chi}} 1/f_{\chi}(u) du < \infty$ for $K_{\chi} \ge 0$. As $1/f_{\chi}(u) \to \infty$ for $u \to 0$, this condition is non-trivial. It results from the assumption that β is bounded away from unity as K approaches zero. With $\overline{\beta}$ as the upper bound on β , a positive constant δ exists such that $f_{\chi}(u) > u^{\overline{\beta}}/\delta$ for all u below some arbitrarily small constant ε , $\varepsilon > 0$. It follows that

$$\int_{0}^{\varepsilon} 1/f_{\mathbf{X}}(u) \mathrm{d}u < \int_{0}^{\varepsilon} \delta u^{-\overline{\beta}} \mathrm{d}u = \left[\frac{\delta}{1-\overline{\beta}} u^{1-\overline{\beta}}\right]_{u=0}^{u=\varepsilon} = \frac{\delta}{1-\overline{\beta}} \varepsilon^{1-\overline{\beta}} < \infty$$

which clearly implies that $\int_0^{\mathbf{K}_{\mathbf{X}}} 1/f_{\mathbf{X}}(u) \, \mathrm{d}u < \infty$ for all $K_{\mathbf{X}} \ge 0$.

i) The Limiting Value of q

It is obvious from (A3) and (A4) that $q(K_{\mathbf{X}}) \to \infty$ for $K_{\mathbf{X}} \to 0$ if it is recalled that, by assumption, $f_{\mathbf{X}}(K_{\mathbf{X}}) \to 0$ for $K_{\mathbf{X}} \to 0$ and if one realizes that $\Phi(K_{\mathbf{X}}) \to 1$ for $K_{\mathbf{X}} \to 0$.

ii) The Stock of Original Capital

Let

(A5)
$$q^{\mathsf{H}}(K_{\mathbf{X}}) \equiv f_{\mathbf{X}}'(K_{\mathbf{X}})\Theta/f_{\mathbf{Y}}'(\overline{K}-K_{\mathbf{X}})$$

be the value of q implicit in the Harberger approach and let K^{H} be the size of the corporate stock of capital where $q^{H} = 1$; i.e., the size Harberger believed to result from dividend taxation. The goal is to find sufficient conditions for

(A6)
$$K^{\mathsf{H}} > K_{\mathbf{I}_1}$$

or, equivalently, for

(A7)
$$q(K^{\mathcal{H}}) < q^{\mathcal{H}}(K^{\mathcal{H}}).$$

As K_{χ_2} is defined by $f'_{\chi}(K_{\chi_2}) = f'_{Y}(\overline{K}-K_{\chi_2})$ and as $q(K_{\chi_2}) = \Theta$, it follows from (A5) that $q^{H}(K_{\chi_2}) = q(K_{\chi_2}) = \Theta$. A sufficient condition for (A7) and hence (A6) to hold is therefore

$$\frac{\partial \ln q(K_{\mathbf{X}})}{\partial K_{\mathbf{X}}} > \frac{\partial \ln q^{\mathsf{H}}(K_{\mathbf{X}})}{\partial K_{\mathbf{X}}} \text{ for } K^{\mathsf{H}} \leq K_{\mathbf{X}} < K_{\mathbf{X}_{2}}.$$

Using (A2) and (A5), this inequality can be transformed to

$$\frac{f'_{\mathbf{Y}}}{f_{\mathbf{X}}} - \frac{f'_{\mathbf{X}}}{f_{\mathbf{X}}} > \frac{f''_{\mathbf{X}}}{f'_{\mathbf{X}}} + \frac{f''_{\mathbf{Y}}}{f'_{\mathbf{Y}}}$$

or, after a few manipulations and using (A1), to

(A8)
$$\left(-\frac{f_{\mathbf{X}}^{\prime\prime}}{f_{\mathbf{X}}^{\prime}}K_{\mathbf{X}}-\frac{f_{\mathbf{Y}}^{\prime\prime}}{f_{\mathbf{Y}}^{\prime\prime}}K_{\mathbf{Y}}\frac{K_{\mathbf{X}}}{K_{\mathbf{Y}}}\right)\frac{1}{\beta_{\mathbf{X}}} > \frac{f_{\mathbf{X}}^{\prime}-f_{\mathbf{Y}}^{\prime\prime}}{f_{\mathbf{X}}^{\prime\prime}}.$$

To interpret condition (A8), it is useful to hypothesize that the pure profit or rent which the concave functions $f_{\mathbf{X}}$ and $f_{\mathbf{Y}}$ imply are the returns from a hidden second factor of production. Let $\sigma_i(K_i)$, i = X, Y, denote the Hicksian substitution elasticity between capital and the hidden factor in sector *i* assuming that the production functions are linearly homogenous. It is a standard result that (A8) can then be written as:

(A9)
$$\left(\frac{1-\beta_{\chi}}{\sigma_{\chi}} + \frac{1-\beta_{Y}}{\sigma_{Y}}\frac{K_{\chi}}{K_{Y}}\right)\frac{1}{\beta_{\chi}} > \frac{f_{\chi}' - f_{Y}'}{f_{\chi}'}.$$

Note that, by the definition of $K^{\mathcal{H}}$,

$$\frac{f'_{\mathbf{X}} - f'_{\mathbf{Y}}}{f'_{\mathbf{X}}} \leq \tau \text{ for } K^{\mathsf{H}} \leq K_{\mathbf{X}} < K_{\mathbf{X}_2}.$$

This implies that it is sufficient for (A9) and hence for (A6) to hold if

(A10)
$$\left(\frac{1-\beta_{\chi}}{\sigma_{\chi}} + \frac{1-\beta_{Y}}{\sigma_{Y}}\frac{K_{\chi}}{K_{Y}}\right) > \beta_{\chi}\tau$$

This condition captures the case of a constant rate of interest in the limiting case of a small corporate sector where $K_{\chi}/K_{\gamma} \rightarrow 0$. With a Cobb-Douglas technology ($\sigma_{\chi} = 1$) the condition would then require that the implicit current tax burden on the normal return to equity does not eliminate the pure corporate rent ($\tau\beta_{\chi} < 1-\beta_{\chi}$). With $\sigma_{\chi} < 1$ and/or a "large" corporate sector ($K_{\chi}/K_{\gamma} > 0$), (A10) would be satisfied even under weaker conditions.

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