NBER WORKING PAPER SERIES

A THEORY OF THE WELFARE STATE

Hans-Werner Sinn

Working Paper No. 4856

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 1994

This paper has been prepared for the SJE conference on "The Future of the Welfare State" in Aarhus, Denmark, September 1994. It is part of the CEPR project "Monitoring European Integration." The author gratefully acknowledges research assistance by Marcel Thum. This paper is part of NBER's research program in Public Economics. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #4856
September 1994

A THEORY OF THE WELFARE STATE

ABSTRACT

The welfare state can be seen as an insurance device that makes lifetime careers safer, increases risk taking and suffers from moral hazard effects. Adopting this view, the paper studies the trade-off between average income and inequality, evaluating redistributive equilibria from an allocative point of view. It identifies the properties of an optimal welfare state and shows that constant returns to risk taking are likely to imply a redistribution paradox where more redistribution results in more inequality. In general, optimal taxation will either imply that the redistribution paradox is present or that the economy operates at a point of its efficiency frontier where more inequality implies a lower average income.

Hans-Werner Sinn
Department of Economics
University of Munich
Schackstrasse 4
80539 München
GERMANY
and NBER

1. Redistribution and Insurance

While this may be the time to turn the welfare state around, it is also the time to warn against throwing the baby out with the bathwater. Economists have learned so much about the Laffer curve, Leviathan, and a myriade of disincentive effects brought about by government intervention that they have lost sight of the allocative advantages of the welfare state.[1]

From an allocative point of view, the main advantage of the welfare state is the insurance or risk reducing function of redistributive taxation. To finance commonly accessible public goods and public transfers, governments take more taxes from the rich than from the poor, thus reducing the variance of real lifetime incomes. To the extent that the variance of lifetime incomes is not predictable when people are born, this activity can be regarded as welfare increasing insurance. Every insurance contract involves a redistribution of resources from the lucky to the unlucky, and most redistributive measures of the state can be interpreted as insurance if the time span between judging and taking these measures is sufficiently long. Redistributive taxation and insurance are two sides of the same coin.

It is true, in principle, that the insurance function of the government budget could possibly have been privately provided. However, it is difficult to imagine endowing private agencies with the extensive monitoring and enforcement rights enjoyed by tax authorities, and in the absence of such rights, moral hazard and adverse selection problems render a broad based private solution impossible. Also, of course, if the need for fiscal taxation is taken as given, then the marginal cost of making the existing tax system redistributive may well be lower than the cost of introducing additional private insurance. The historical growth of the welfare state can, in part, be seen as a response to the private insurance system's inability to offer the cheaper solution.

While the production of safety is an important function of the welfare state, the Domar-Musgrave effect of increased risk taking may be even more important. Protected by the welfare state people engage in risky and profitable activities which they otherwise would not have dared to undertake. Risky occupations might not be chosen without the protection of the

[1]In fact, none of the favourable allocative effects of the welfare state discussed in this paper have been mentioned in the illuminating and important book "Turning Sweden Around" by Lindbeck, Molander, Persson and others (1994). The authors do not even include the redistribution of income in their list of "Basic responsibilities of the state"; cf. pp.14-16.

welfare state, and it would be difficult to find entrepreneurs to supervise risky investment if the debtor's prison were all the society provided in the case of failure. It is perhaps the most important function of the social welfare net that it makes people jump over the dangerous chasms which otherwise would have put a halt to their economic endeavors.

It may, in fact, make them too eager to jump. Protected by the welfare state, people may neglect to take the necessary care, may take too much risk, and end up in a worse situation than without such protection. This is the fear that an overwhelming majority of policy advisors seems to have.

The effect on risk taking has important repercussions for the observable degree of inequality in the economy, for, if a given set of people will chose more risk ex ante, they will typically be more unequal ex post. Risk averse societies may exhibit relatively little inequality, and the more redistribution there is, the larger the pre-tax inequality tolerated may be.

The paper offers a simple model that makes it possible to analyse the interaction between redistributive taxation, risk taking, and inequality and provides unambiguous welfare evaluations of the allocations achieved. As suggested by Harsanyi (1953, 1955), Rawls (1971), and others, the social welfare function for evaluating the income distribution is identical with a representative individual's utility function for risk evaluations. However, in the model, people really are behind the veil of ignorance when they make their decisions and evaluate the resulting income distribution. Their amount of risk taking ex ante determines their degree of inequality ex post.

The paper's main focus is on the policy trade-off between income equality and per-capita income. This is not on the trade-off between equity and efficiency, because equity is an aspect of efficiency. Will redistributive taxation induce too much or too little risk taking? How does it compare with ideal insurance? Will the pie shrink when it is more evenly distributed? Will more redistribution result in less inequality? What are the properties of an optimal redistributive tax system? These are among the questions addressed in this paper.

While little is known about the issue, there are noteworthy exceptions from the general lack of interest in the insurance function of redistributive taxation. The exceptions include the literature on risk taking and taxation, in the context of asset choice, savings or occupational decisions (see, e.g., Ahsan 1974, 1976, Allingham 1972, Atkinson and Stiglitz 1980, Bamberg and Richter 1984, Domar and Musgrave 1944, Kanbur 1979, and Stiglitz 1969), as well as the welfare theoretic literature extending the theory of optimal taxation to the case of income risks (Diamond, Helms and Mirrlees 1980, Eaton and Rosen 1980, Varian 1980, and Sinn 1981). This paper owes an intellectual debt to all of these approaches. Above all, however, it gained from Friedman's (1953) "Choice, Chance and the Personal Distribution of Income" and Buchanan and Tullock's (1962) "Calculus of Consent", chapter 13. The paper can be seen as an attempt to formalize, apply and extend their approaches.

2. The Model

A very simple model that is able to incorporate the issues discussed is the following. There is a large number of identical individuals, each facing the same choice problem under uncertainty. With stochastically independent income risks and identical choices, each person's probability distribution of income converts to the economy's frequency distribution of realized incomes. If, say, a single person's probability of having a lifetime income of between $ 500,000 and $ 510,000 is 1 %, then the law of large numbers will ensure that 1% of the population will have an income in this range. Risk and expected income ex ante will turn out as inequality and average income ex post.

To reduce the dimensionality of risk, a broad-based definition of income including market income, non-market income, public goods and public transfers is used. The risk occurs in the form of a random income loss $L \geq 0$ whose magnitude depends on the random state of nature θ and the cost of self-insurance e in terms of foregone market and non-market resources. One may think, for example, of investment in physical and human capital limiting the risk of not reaching one's income goals. Let m and n be the maximum values of market and non-market income attainable if the individual makes no effort and the loss nevertheless happens to be zero, p be the value of transfers (monetary transfers and public goods) received, and T be the individual's tax liability which also depends on θ and e. Then the individual's (post-tax) income is

$$Y = m + n - L(e,\theta) - e - T(e,\theta) + p. \tag{1}$$

An increase in effort e reduces the size of the income loss for all states of the world, albeit with diminishing marginal returns. It is assumed that

$$L(e,\theta) = \lambda(e)\cdot\theta,\ \theta \geq 0, \lambda > 0, \qquad (2)$$
$$\lambda' < 0, \lambda'' \geq 0,\ \lambda'(0) = -\infty,$$

where λ is a twice continously differentiable function reflecting the efficacy of self-insurance.

There is a linear tax on market income. Let α be the fraction of self-insurance efforts consisting of foregone market income and $1-\alpha$ the fraction consisting of foregone non-market income. Then

$$T(e,\theta) = \tau\left[m - L(e,\theta) - \alpha e\right] \qquad (3)$$

where τ is the tax rate. Note that, despite the linearity of the tax, the tax system is redistributive because the public transfer p is independent of the state of nature.[2] Lucky individuals are net payers and unlucky net recipients of public funds.

To balance the government budget, the public transfer is chosen so as to make it equal to the average tax liability:[3]

$$p = E\left[T(e,\theta)\right]. \qquad (4)$$

The income distribution in the economy described is specified once the government has chosen τ and α, and the individuals have chosen e. It is convenient to describe this distribution in terms of its mean μ (the average income) and its standard deviation σ. It follows from (1)-(4) that

$$\mu = m + n - \lambda(e)E(\theta) - e \qquad (5)$$

and

[2]The formal structure of the redistribution mechanism is similar to the progressive linear tax used by Ahsan (1974, 1976) for a portfolio selection problem with fiscal taxation.

[3]Alternatively, it could have been assumed that $p = \sum_{j=1}^{x} T\left(e_j,\theta_j\right)/x$ where x is the number of individuals in the economy. Because of the assumption of stochastic independence of the θ_j, $j = 1, \ldots, x$, the transfer specified this way converges stochastically to $E\left[T(e,\theta)\right]$ as x goes to infinity.

$$\sigma = (1-\tau)\lambda(e)R(\theta) \tag{6}$$

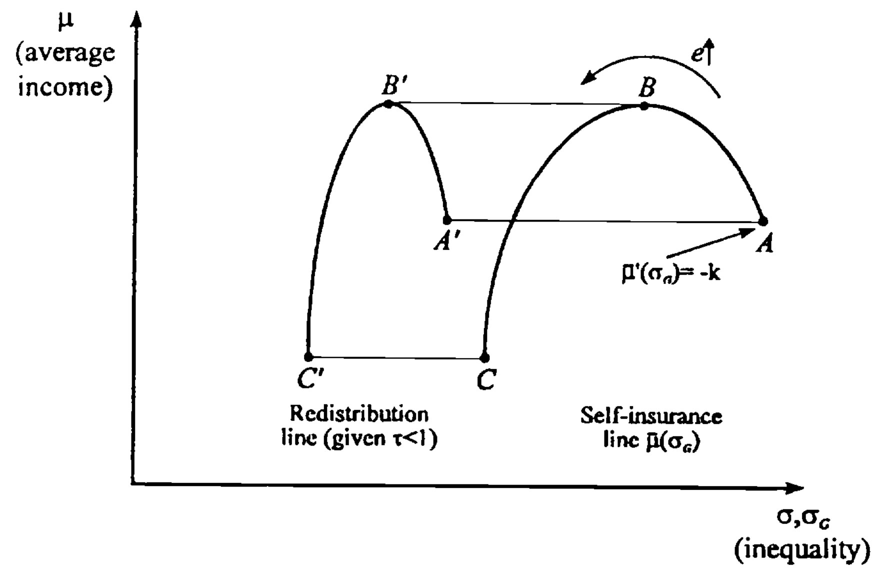
where R (.) is the standard deviation operator.[4] Equations (5) and (6) show that, with any given amount of self-insurance effort e, redistributive taxation will not affect the average income, μ, but will reduce the deviation from the average, σ. Seen from an ex-ante perspective this is the insurance aspect of redistributive taxation. The important question of how redistributive taxation will in turn affect the amount of effort chosen will be postponed to the next sections.

Figure 1 depicts the combinations of μ and σ attainable with an appropriate choice of e and for two alternative values of the tax rate: $\tau = 0$ and $\tau > 0$.

The opportunity set of (μ, σ) combinations attainable with $\tau = 0$ will be called the "self-insurance line" and the set attainable with a given $\tau > 0$ will be called the "redistribution line". Geometrically, the redistribution line can be constructed by shifting all points on the self-insurance line horizontally to the left where the percentage reduction of the distance from the ordinate equals the tax rate. The movements of A, B, and C towards A', B', and C' are examples of this shift. It is unclear at this stage which amount of self-insurance effort and which pair of points on the two lines the individual choses. However, whatever his choice, all attainable post-tax income distributions that satisfy the government's budget constraint (4) are represented by points on the redistribution line.

[4]Throughout the paper E and R are used as expectation and standard deviation operators while μ and σ are the mean and standard deviation of post-redistribution income. Recall that $R(X) = \left[E(X^2)-E^2(X)\right]^{1/2}$ and note that $E(a+bX) = a+bE(X)$ and $R(a+bX) = |b|R(X)$.

Figure 1: *The Sets of Feasible Pre-Tax and Post-tax Distributions of Income*



Let σ_G be the pre-tax standard deviation chosen:

$$\sigma_G \equiv \sigma|_{\tau=0} = \lambda(e)R(\theta). \tag{7}$$

Since $\lambda'(e) < 0$ implies that σ_G is a monotonically declining function of e, it is possible to treat σ_G as the choice variable of the individual. Accordingly (5) and (6) can be written as

$$\mu = \bar{\mu}(\sigma_G) \tag{8}$$

and

$$\sigma = (1-\tau)\cdot\sigma_G \tag{9}$$

where

$$\begin{aligned}\bar{\mu}(\sigma_G) &\equiv m+n-E(L)-e \\ &= m+n-\sigma_G k-\lambda^{-1}[\sigma_G / R(\theta)]\end{aligned} \tag{10}$$

is the function defining the self-insurance line with

$$\sigma_G k = E[L(e,\theta)] = \lambda(e)E(\theta) \tag{11}$$

and

$$k \equiv \frac{E(\theta)}{R(\theta)} = \text{const.} > 0. \tag{12}$$

It is easy to derive a boundary condition for the slope of the self-insurance line ,[5]

[5]Equation (13) follows from (5), (7), and the assumption $\lambda'(0) = -\infty$.

$$\bar{\mu}'(\sigma_G) = -k \text{ when } e = 0, \tag{13}$$

and to show that the line has a maximum where $\lambda'(e)E(\theta) = -1$ and is concave throughout:[6]

$$\bar{\mu}''(\sigma_G)\left\{\begin{smallmatrix}<\\=\end{smallmatrix}\right\}0 \text{ when } \lambda''(e)\left\{\begin{smallmatrix}>\\=\end{smallmatrix}\right\}0. \tag{14}$$

To close the model, the representative agent's preference structure has to be specified. It is assumed that the agent is a globally and locally risk averse expected utility maximizer. Since the set of distributions implied by (1), (2), and (3) forms a linear class, any given von Neumann-Morgenstern function can be exactly represented in terms of (μ,σ) preferences without any loss of generality.[7] As shown by Meyer (1987) and Sinn (1983, 1989), there exists a well-behaved utility function $U(\mu,\sigma)$ if the von Neumann-Morgenstern function is well-behaved. Its properties can best be summarized by the properties of the function

$$i(\mu,\sigma) \equiv \left.\frac{d\mu}{d\sigma}\right|_U = -\frac{U_\sigma}{U_\mu} \tag{15}$$

which indicates the indifference-curve slope - the marginal risk or inequality aversion - at a particular combination of μ and σ:

(a) $i(\mu,0) = 0$ (enter ordinate perpendicularly)

(b) $i(\mu,\sigma) > 0$ for $\sigma > 0$ (upward bending)

(c) $\left.\frac{di}{d\sigma}\right|_U > 0$ (strictly convex)

(d) $i_\sigma > 0$ (slope increases with σ, given μ)[8]

---

[6]It follows from (10) that $\bar{\mu}''(\sigma_G) = \lambda''(e)/\left[\lambda'^3(e)R^2(\theta)\right]$. Since $\lambda'' \geq 0$ and $\lambda' < 0$ the sign of this expression is zero or negative.

[7]To prove that the attainable distributions belong to the same linear class, it is necessary to show that the standardized distribution $Z = [Y - E(Y)]/R(Y)$ is independent of the model's choice variables e, τ, and α. Inserting (2) and (3) into (1) gives

$$Z = \frac{m+n-\lambda\theta - e - \tau[m-\lambda\theta-\alpha e] + p - \{m+n-\lambda E(\theta) - e - \tau[m-\lambda E(\theta) - \alpha e] + p\}}{(1-\tau)\lambda R(\theta)}$$

or, after a few simplifications,

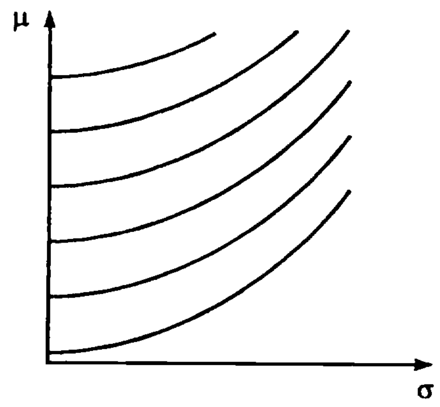
$$z = \frac{-\theta + E(\theta)}{R(\theta)}, \text{ q.e.d.}$$

[8]Condition (d) drives some of the results of this paper. It has been proved under the condition that absolute risk aversion is decreasing, is constant, or does not increase faster than with the "fastest" quadratic utility function

(e) $i_\mu \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ for $\begin{Bmatrix} \text{increasing} \\ \text{constant} \\ \text{decreasing} \end{Bmatrix}$ absolute risk aversion (slope change with μ, given σ).

Figure 2 illustrates an example of the indifference-curve system for the case of constant absolute risk aversion.

Figure 2: *Evaluating Income Distributions*



While the preference map of Figure 2 makes it possible to evaluate probability distributions, it allows an equally appropriate evaluation of the realized income distributions. Since people have identical risk preferences and since the probability distribution chosen translates into an identical frequency distribution of realized incomes, an unambiguous social welfare function is available.

3. Laissez Faire and the Social Optimum

Imposing the "indifference map" of Figure 2 on the "feasibility map" of Figure 1 gives two kinds of optima, illustrated by points T and Q' in Figure 3. Point T is the laissez faire optimum without redistributive taxation and Q' is the optimum with redistribution at a given tax rate $\tau > 0$. Let T' and Q be the counterparts of these two points on the redistribution line and the self-insurance line, respectively.[9] Formally, the two solutions follow from the problem

$$\max_{\sigma_G} U(\mu,\sigma) \qquad \text{s. t. } \mu = \bar{\mu}(\sigma_G),\ \sigma = (1-\tau)\sigma_G \tag{16}$$

compatible with strictly positive marginal utility in the relevant range. See Sinn (1989). It is assumed that this condition will hold.

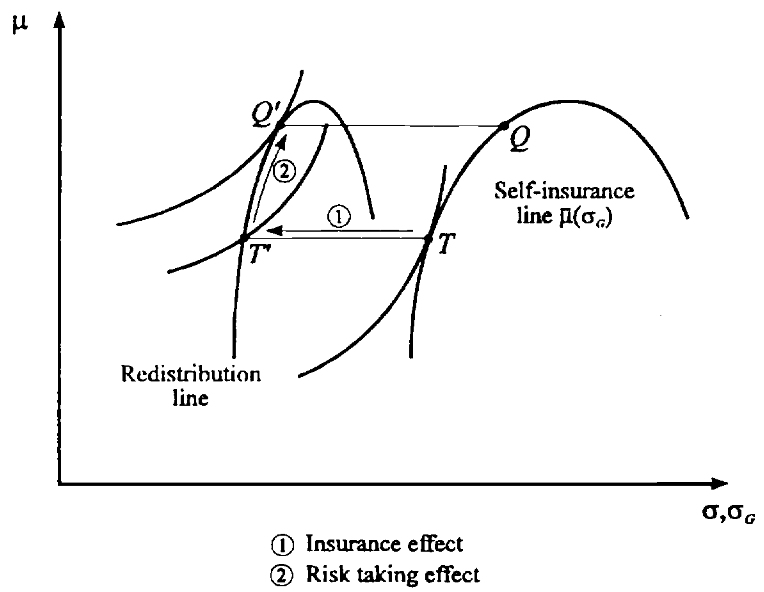
[9]Throughout the paper, points labelled with a prime are located on the redistribution line horizontally left of the respective points without a prime which are located on the self-insurance line. Points labelled by the same letter indicate the same self-insurance effort.

which implies the first order condition

$$i\left[\bar{\mu}(\sigma_G),(1-\tau)\sigma_G\right]=\frac{\bar{\mu}'(\sigma_G)}{1-\tau}. \tag{17}$$

The left-hand side of (17) is the indifference curve slope and the right-hand side is the slope of the redistribution line. In general, (17) refers to a point like Q'; however, in the limiting case where $\tau = 0$ it also captures the laissez faire solution T.

Figure 3: *The Socially Optimal Degree of Risk Taking, given the Tax Rate*



The solution illustrated in Figure 3 is a constrained Pareto optimum, defining the optimal level of self-insurance efforts given the tax rate. It will not necessarily be reached by private actions, since the redistribution line may not coincide with the opportunity set as perceived by the individual. It would, however, be attained in an ideal insurance market where individual actions can be monitored by the company and a fair premium is announced for each self-insurance strategy the individual may chose. It would also be attained if a strict equivalence principle of taxation could be met. The government would have to be able to monitor the individual self-insurance activities and announce a separate value of the public transfer for every feasible action, obviously an unrealistic requirement.

Having made these reservations two lessons can be learned from Figure 3.

Proposition 1: *Under laissez faire, or with ideal insurance, the society operates at a point in its opportunity set where an increase in inequality would increase the average income.*

Proposition 2: *Redistributive taxation has the potential for creating two kinds of welfare gain. It can increase welfare by increasing the equality of incomes, and it can increase it even more when more risk is taken and some equality is sacrificed for a higher level of average income. The socially optimal level of pre-tax inequality is an increasing function of the tax rate.*

While Proposition 1 is obvious, Proposition 2 needs a

Proof: Assume that $0 < \tau < 1$, let $r(\cdot)$, $i(\cdot)$, and $s(\cdot)$ denote the slopes of the redistribution line, the indifference curve, and the self-insurance line at the respective points (in Figure 3) named in the brackets. By the definition of T, $s(T)=i(T)$, and, because of (8) and (9), $r(T') = r(T)/(1-\tau) > i(T)$. Property (d) of the indifference curve system ensures that $i(T) > i(T')$. Thus $r(T') > i(T')$. Together with the convexity of the indifference curves and the concavity of $\bar{\mu}$, this implies $\sigma(Q') > \sigma(T')$ and $\sigma_G(Q) > \sigma_G(T)$.[10] While this proves that taxation increases risk taking and pre-tax inequality in the large, the marginal effect of τ on the optimal level of $\sigma_G, \sigma_G(Q)$, follows from implicitly differentiating (17):

$$\frac{\mathrm{d}\sigma_G(Q)}{\mathrm{d}\tau} = \frac{i + i_\sigma \cdot \sigma_G(Q)(1-\tau)}{(1-\tau)\{i_\mu \cdot \bar{\mu}[\sigma_G(Q)] + i_\sigma(1-\tau)\} - \bar{\mu}''[\sigma_G(Q)]} > 0 \qquad (18)$$

The denominator of this expression is strictly positive if the second-order condition of problem (17) is satisfied. This is the case since the indifference curves are strictly concave and the redistribution function is convex. The numerator is strictly positive since all items occurring there are strictly positive. [Cf. property (d) of the indifference curve system.] Q.e.d.

Proposition 1 is the model's confirmation of the frequently expressed belief that the pie can grow when a more unequal distribution of its slices is tolerated. Risk aversion (or inequality aversion) requires a compromise between the goals of maximizing the size of the pie and minimizing the degree of inequality. It makes it wise to operate at a point of the efficiency

[10]The notation should be self-explanatory. For example $\sigma(T')$ is the post-tax standard deviation at point T' which is the counterpart of $\sigma_G(T)$, the pre-tax standard deviation. Note that $\sigma(T') = (1-\tau)\sigma_G(T)$.

frontier where a little more tolerance with regard to the latter makes it possible to come a bit closer to the former.

Proposition 2 confirms the discussion of the introduction to this paper. Given that the government offers public insurance, the need for self-insurance is reduced. Redistributive taxation increases the marginal post-tax return to risk taking (the slope of the redistribution line as compared to that of the self-insurance line) and lowers the marginal compensation for risk taking that the agent requires (the indifference curve slope). This makes it socially optimal to tolerate more risk and inequality in exchange for a higher level of average income. Under the protection of the welfare state more can be dared.[11]

The risk taking effect of the welfare state may have wide reaching implications. In a broader context, risk can be seen as a factor of production, a necessary input for the economy without which a high level of productivity could not be achieved.[12] The factor "risk" is probably no less important than "waiting", the factor economists have familiarized themselves with under the name of capital. If the real rate of interest is a measure of the importance of waiting and if the unexplained remainder of the "return to capital" is in fact the reward for risk taking, then risk taking should be considered at least as responsible for economic prosperity as capital investment. The enhancement of risk taking may be the most important economic function the welfare state can perform.

4. Redistributive Taxation and the Optimality of Individual Choice

While the previous section demonstrated the potential for gains from redistributive taxation, this section addresses the more interesting question of whether the exploitation of this potential through individual choice can be expected. The crucial assumption of this section is that the government transfer p is not tailored to the individual decision. The individual agent takes this transfer as exogenous to its own decisions, notwithstanding the fact that it will endogenously be determined in equilibrium through the government budget constraint, equation (4).

---

[11] Surprisingly, the benefits from increased risk taking have been largely ignored in the private insurance literature. Often the insurance-induced increase of risk taking is confused with moral hazard resulting from a lack of observability of individual actions.

[12] See Pigou (1932, Appendix I, pp.771-781), Sinn (1986), or Konrad (1992).

The individual opportunity set of decision alternatives is given by equation (1). Taking expectations, noting that $\bar{\mu}(\sigma_G) = m + n - E(L) - e$ from (10), and using (3) yields

$$\mu = \bar{\mu}(\sigma_G) - \tau\{m - E[L(e,\theta)] - \alpha e\} + p. \qquad (19)$$

After a few algebraic manipulations making use of (11), equation (19) can also be written as

$$\mu = \bar{\mu}(\sigma_G)(1 - \alpha\tau) - \tau(1 - \alpha)(m - k\sigma_G) + \alpha\tau n + p. \qquad (20)$$

The standard deviation as perceived by the individual follows from (1), (3), and (7):

$$\sigma = (1 - \tau)\sigma_G. \qquad (21)$$

Since p was also non-stochastic in the social planning problem, this is the same as equation (9). Equations (20) and (21) imply an opportunity locus in (μ, σ) space that will be called the "individual opportunity line".

The agent's optimization problem is

$$\max_{\sigma_G} \; U(\mu, \sigma) \qquad \text{s. t. (20) and (21).} \qquad (22)$$

Using (15), the first order condition of this problem can be written as[13]

$$i(\mu, \sigma) = \bar{\mu}'(\sigma_G)\frac{1 - \alpha\tau}{1 - \tau} + \frac{\tau}{1 - \tau}(1 - \alpha)k. \qquad (23)$$

The left-hand side of equation (23) is the indifference curve slope, and the right-hand side is the slope of the individual opportunity line.

A redistributive equilibrium is defined as a situation where the agent has chosen σ_G so as to maximize his utility and the government has chosen the public transfer so as to satisfy its budget constraint (4). In equilibrium, therefore, (23) has to hold on the redistribution line (cf. Figures 1 and 3) which means that the indifference curve slope $i(\mu, \sigma)$ refers to a point where $\mu = \bar{\mu}(\sigma_G)$ and $\sigma = (1 - \tau)\sigma_G$.

---

[13]The second-order condition is satisfied since the indifference curves are convex and (20) and (21) define a concave curve in (μ, σ) space representing the individual opportunity set as perceived by the agent.

A comparison with (17) reveals that the equilibrium satisfying (23) is not in general identical with the constrained Pareto optimum characterized by the pair (Q, Q') in Figure 3. The next three sections analyze the differences.[14]

4.1 Deductible Efforts

In the case $\alpha = 1$, the cost of self-insurance occurs exclusively in the form of foregone market resources and will therefore enjoy full tax deductibility. One may think in particular of pecuniary investment outlays or business expenses that are fully tax deductible. In an intertemporal context, a cash flow tax would be an exact example for the case $\alpha = 1$ because it allows an immediate write-off of investment expenses.[15] A capital income tax with annual economic depreciation allowances would instead be equivalent to $0 < \alpha < 1$, because the present value of depreciation allowances falls short of the investment.

The implications of (23) for the case $\alpha = 1$ are summarized in

Proposition 3: *When self-insurance efforts are fully tax-deductible (as with investment under a cash flow tax) redistributive taxation is welfare increasing. In addition to the direct gain from insurance there is a gain from increased risk taking. However, risk taking and the resulting increase in inequality are less than what would be socially optimal.*

Proof: If $\alpha = 1$, condition (23) becomes

$$i\left[\bar{\mu}(\sigma_G),(1-\tau)\sigma_G\right] = \bar{\mu}'(\sigma_G). \tag{24}$$

Assume that $\tau > 0$ and let $i(\cdot)$ and $s(\cdot)$ denote the slopes of the indifference curve and the self-insurance line at the respective points (from Figure 4) named in the brackets. Using this notation, condition (24) defines a point V' on the redistribution line and its counterpart V horizontally to the right on the self-insurance line such that $i(V') = s(V)$. From (17) it is known that $i(Q') = s(Q)/(1-\tau) > s(Q)$. On the other hand, property (d) of the indifference curve system and the definition of T imply that $i(T') < i(T) = s(T)$. Continuity implies that a solution

[14] The sections also prove the existence of equilibrium. Stability is analyzed in the appendix.
[15] The variables of the model will then have to be interpreted in terms of present values.

exists between T' and Q' on the redistribution line; i.e., $\sigma(T') < \sigma(V') < \sigma(Q')$ and $\sigma_G(T) < \sigma_G(V) < \sigma_G(Q)$, q.e.d.

The intuition for the suboptimality of individual risk taking can best be gained by inspecting (19). Suppose the individual had chosen the socially optimal level of σ_G and considers a small variation by changing his self-insurance effort. This variation will, in general, change his expected tax liability, $\tau\{m - E[L] - \alpha e\}$. If the public transfer p is changed accordingly so as to satisfy the government budget constraint (4), then the variation in σ_G implies no change in the expected net payment to the government, and, by assumption, expected utility stays constant. However, if p stays constant despite the change in the expected tax liability, expected utility will change. The individual will have an incentive to deviate from the social optimum in the direction where the expected tax liability declines and where he can expect to become a net recipient of public funds. Assuming an endogenous change in p would require collective rationality. With individual rationality, p has to be taken as exogenous, because the agent knows that his taxes will contribute only a negligible fraction to the government budget and will therefore not be able to affect the volume of public transfers returned.

For the case $\alpha = 1$, this argument implies that the representative agent takes less risk and choses a lower degree of inequality than is socially optimal, optimality being judged by his own preferences. The expected tax base is $\{m - E(L) - e\}$. Since it differs from the expected income $\bar{\mu}(\sigma_G)$ only by the non-market component of income, n, which is a constant, the expected tax liability can be reduced by lowering income and enjoying the advantage of lower risk.

Figure 4: *Less than Optimal Inequality with Full Deductibility of Self-Insurance Efforts (Cash Flow Tax)*

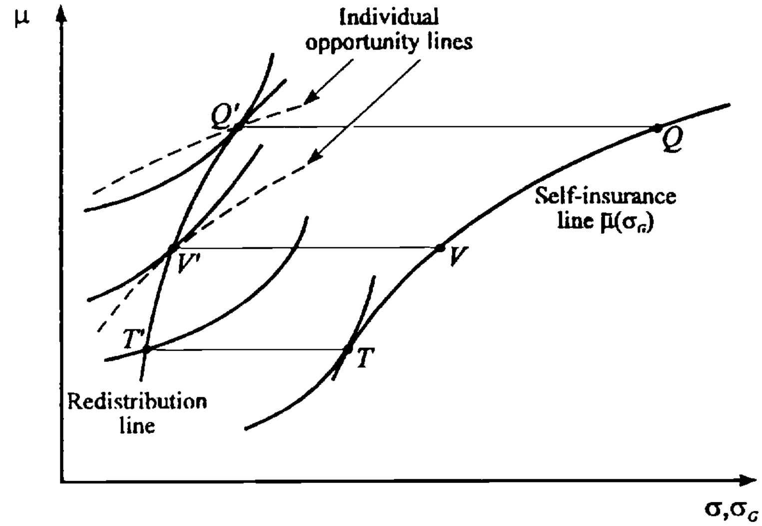
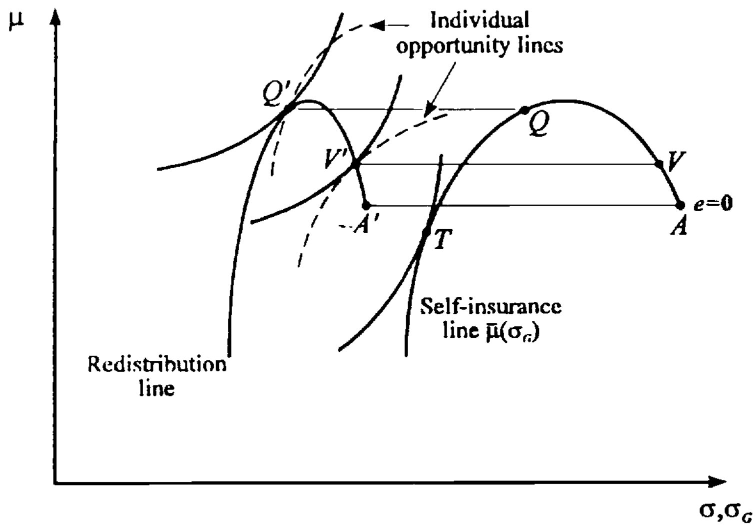


Figure 4 illustrates this reasoning. The broken line through Q' is the individual opportunity line, given the level of public transfers p that would be paid if the agents chose the socially optimal level of self-insurance effort. The individual believes that he will be able to reach a higher indifference curve by moving to the left of Q'; i.e., by reducing σ_G. In fact, however, if everyone does so, the transfer will have to be reduced and the realized point in (μ, σ) space is pushed down, back to the redistribution line. The equilibrium is at a point such as V'. Here an indifference curve is tangent to an individual opportunity line, and the point of tangency is also on the redistribution line. The individual does not want to change his behavior, and the government budget is balanced.

4.2 *Non-deductible Efforts*

Consider now the other extreme case $\alpha = 0$. Here, the opportunity cost of effort occurs exclusively in the form of non-market income foregone, and non-market income is untaxed. The case can be interpreted in terms of the familiar labor-leisure distortion if leisure is, in fact, an activity producing non-market income and if the tax is imposed on labor income alone. The tax system discourages the self-insurance effort because this effort cannot be deducted from a tax base.

Figure 5: *Excessive Inequality without Deductibility of Self-Insurance (Labor Income Tax)*



Inspecting (19) shows that the expected tax base now reduces to $\{m - E[L]\}$. Since m is a constant, the base is smaller the greater $E[L]$ and hence the larger the amount of risk taking as measured by σ_G [cf. equation (11)]. Thus the intuitive argument raised above suggests that the individual will want to deviate to the right from the social optimum Q' in Figure 5 in order to become a net recipient of public funds. There is an individual opportunity line cutting through the redistribution line at point Q' from below such that a higher indifference curve seems to be attainable by increasing σ and σ_G. Again, however, if everyone behaves that way, the public transfer p will have to be reduced, and the individual's position will be pushed downward, back to the redistribution line. The equilibrium V' where an indifference curve is tangent to the individual opportunity line, and where the point of the tangency is, in addition, located on the redistribution line, will be to the right of Q', possibly even to the right of the maximum as shown in the figure. This intuitive result is confirmed by

Proposition 4: *When self-insurance efforts are not tax-deductible (as with a labor income tax) there will be some self-insurance effort but not enough: risk taking overshoots the social optimum, and too much inequality will result.*

Proof: In the case $\alpha = 0$, condition (23) becomes

$$\bar{\mu}'(\sigma_G) - i[\bar{\mu}(\sigma_G),(1-\tau)\sigma_G](1-\tau) = -\tau k. \tag{25}$$

Assume $0 < \tau < 1$ and let $r(\cdot)$ and $i(\cdot)$ denote the slopes of the self-insurance line and the indifference curve at the respective points (from Figure 5) named in the brackets. Let A be the end point of the self-insurance line where $e = 0$ and recall from (13) that $r(A) = -k$, k being a strictly positive parameter characterizing the distribution of θ (the state of the world). Recall furthermore from (17) that the social optimum is defined by $r(Q) - i(Q')(1-\tau) = 0$. Equation (25) defines a point V' on the redistribution line and its counterpart V horizontally to the right on the self-insurance line such that $r(V) - i(V')(1-\tau) = -\tau k$. Since $i \geq 0$, this implies $r(V) > r(A)$ which, because of the concavity of the self-insurance line, defines a point to the left of A. Moreover the concavity of the self-insurance line and the strict convexity of the indifference curves imply that $r(V) - i(V')(1-\tau) < 0$ can only hold true to the right of the the social optimum. Thus $\sigma(Q') < \sigma(V') < \sigma(A')$ and $\sigma_G(Q) < \sigma_G(V) < \sigma_G(A)$, q.e.d.

4.3 The General Case

Since $\alpha = 0$ implies too much and $\alpha = 1$ too little inequality relative to the social optimum, there should be an intermediate value of α where the right amount of inequality can be generated. Equating the right-hand sides of (17) and (23) gives

$$\bar{\alpha}(\sigma_G) = \frac{k}{\bar{\mu}'(\sigma_G) + k} \tag{26}$$

where $\bar{\alpha}(\sigma_G)$ is a function that indicates the level of α that equates the slope of the individual opportunity line with the slope of the redistribution line at a given level of σ_G. Let $\sigma_G(Q)$ be the socially optimal level of σ_G. Then setting $\alpha = \bar{\alpha}[\sigma_G(Q)]$ will ensure that the equilibrium coincides with the social optimum. Uniqueness of (26) and continuity of (23) imply that higher levels of α will induce too little, and lower levels too much, risk taking and inequality.

Note that the optimal level of α depends on the size of the tax rate because the optimal amount of risk taking does so. From Proposition 2 and equation (18) it is known that $\sigma_G(Q)$ is a strictly increasing function of τ. Since $\bar{\mu}'' \leq 0$, the optimal level of α increases with τ where $\bar{\mu}'' < 0$ and stays constant where $\bar{\mu}'' = 0$.

It is known from property (a) of the indifference system that $i(\mu,\sigma)=0$ when $\sigma = 0$. In the limit, where $\tau \to 1$ and the redistribution line is compressed to a narrowing range near the ordinate in the (μ,σ) diagram, this property and equation (17) imply that the socially optimal amount of risk taking, $\sigma_G(Q)$, converges to that value of σ_G where $\bar{\mu}$ has its maximum and $\bar{\mu}'=0$. The optimal level of α will then converge towards unity such that, with any given $\alpha < 1$, there will be too much risk taking.

In fact, when τ goes to unity, effort e approaches zero and $\sigma_G(V)$ approaches $\sigma_G(A)$, the maximum feasible value of σ_G. To see this, rewrite (23) in the form

$$i[\bar{\mu}(\sigma_G),\sigma_G(1-\tau)](1-\tau)=(1-\alpha)\left[\bar{\mu}'(\sigma_G)\frac{1-\alpha\tau}{1-\alpha}+\tau k\right]. \tag{27}$$

Clearly, $\tau \to 1$ implies that $\bar{\mu}'(\sigma_G) \to -k$, the condition characterizing point A. Conversely, if $\tau < 1$, an equilibrium at point A is impossible. For one thing, the left hand side of (27) is now strictly positive since $i > 0$ and $(1-\tau) > 0$. For another, the right-hand side of (27) would be negative if $\bar{\mu}' = -k$ and $\tau < 1$. This becomes immediately obvious by differentiating the right-hand side of equation (27) with regard to τ. As the derivative is positive (namely $+k$), $\tau < 1$ implies a value less than zero.

These findings can be summarized as follows.

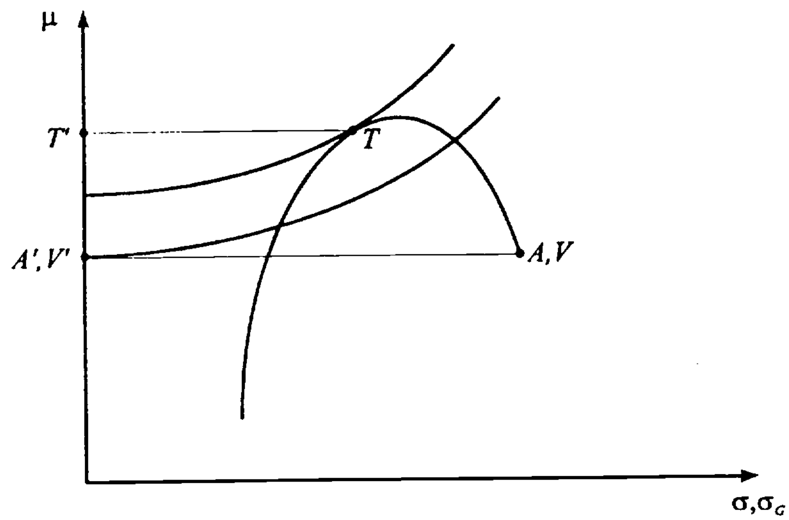
Proposition 5: *There is a critical value for the deductible proportion of self-insurance efforts greater than zero and smaller than one which generates an equilibrium with the optimal amount of risk taking and inequality. Higher values imply too little risk taking and inequality, lower values too much. The critical value is an increasing function of the tax rate and approaches unity as the tax rate does so.*

Proposition 6: *While there is always some self-insurance effort if the tax rate is less than one, this effort will go to zero when the tax rate approaches one while the deductible proportion of self-insurance efforts is a constant strictly less than one. In the limiting case, the society will operate beyond the maximum of the self-insurance line where a higher average income could be reached by a reduction in pre-tax inequality.*

Proposition 6 confirms the scepticism of those who doubt that redistribution is an efficiency enhancing or even legitimate part of government activity. Since it will be impossible in practice to make all self-insurance efforts tax deductible ($\alpha = 1$), it is unavoidable that an ongoing growth of the welfare state will eventually push the economy to the wrong side of its risk-return opportunity space and eliminate all self-insurance efforts. When the government absorbs all risks, excessive risk taking is the obvious consequence.

The disincentive effects of the welfare state may indeed be so strong that society on the whole loses from the existence of this state. Figure 6 demonstrates such a possibility.

Figure 6: *The Welfare Loss from an Overdrawn Welfare State* ($\tau \rightarrow 1$)



On the other hand, Proposition 5 ensures that with moderate tax rates and a suitably chosen value of α between zero and one, the socially optimal risk allocation can, in principle, be reproduced with decentralized decision making. Redistributive taxation would then indeed have the beneficial insurance and risk taking effects described in section 3.

Of course it is difficult to draw direct policy conclusions on the size of α from an abstract model like this one. However, to be on the safe side it would be better to chose a high value of α rather than a low one. Truly detrimental effects can only occur when α is too small. When it is too high, the welfare gain from redistributional taxation will not be maximal, but at least there will be some gain. The insurance effect will in this case be fully present, and part of the potential welfare gain from risk taking can also be exploited.

For practical tax systems this means that a move from capital income taxes towards cash flow taxes on capital is advisable as well as all measures which the optimal tax literature recommends for minimizing the labor-leisure distortion. In particular, the investment in human capital which may be the most important self-insurance activity in a market economy should be made fully tax deductible.

5. The Redistribution Paradox

How redistributive taxation will affect the equality of incomes is an old economic question. With any pre-tax income distribution the variance of post-tax incomes is clearly reduced by redistributive taxation. However, people may react by taking more risks so that the pre-tax inequality rises. How strong is this countervailing effect? Is it possible that it offsets the primary effect?

Section 4 showed, among other things, that the introduction of a linear redistribution system will increase the equilibrium pre-tax inequality. Before the impact of a tax rate change on the post-tax distribution can be considered, the marginal analogue of that result has to be proved.

Proposition 7: *A marginal increase in the tax rate will increase the equilibrium inequality of pre-tax incomes.*

Proof: Implicit differentiation of (23) yields

$$\frac{d\sigma_G}{d\tau} = \frac{i_\sigma \sigma_G + [\beta/(1-\tau)]}{\gamma - \delta} > 0 \tag{28}$$

where

$$\beta \equiv i - \alpha\bar{\mu}'(\sigma_G) + (1-\alpha)k, \tag{29}$$

$$\gamma \equiv i_\mu \cdot \bar{\mu}'(\sigma_G) + i_\sigma \cdot (1-\tau), \tag{30}$$

$$\delta \equiv \bar{\mu}''(\sigma_G)(1-\alpha\tau)/(1-\tau). \tag{31}$$

Here, the indifference curve slope i and its derivatives i_μ and i_σ are functions of μ and σ, where $\mu = \bar{\mu}(\sigma_G)$ and $\sigma = (1-\tau)(\sigma_G)$.

To sign (28) consider first the numerator. It is clearly positive. For one thing, property (d) of the indifference curves ensures that $i_\sigma \sigma_G > 0$. For another, if $\alpha\bar{\mu}' - (1-\alpha)k$ is substracted from both sides of equation (23), it follows after a few algebraic manipulations that

$$\beta = (\bar{\mu}' + k)(1-\alpha)/(1-\tau). \tag{32}$$

Since it is known from Proposition 6 and the preceding discussion that $\bar{\mu}' + k$ is positive and will only in the limiting case $\tau \to 1$ approach zero, it follows that

$$\beta \begin{Bmatrix} > \\ = \end{Bmatrix} 0 \text{ for } \alpha \begin{Bmatrix} < \\ = \end{Bmatrix} 1 \text{ and } \tau < 1, \tag{33}$$

a result that will also be needed below.

Consider the demoninator next. The terms γ and δ measure the marginal changes of the slope of the indifference curve and the individual opportunity line, respectively, brought about by a rightward movement *along the redistribution line* (and along neither a given indifference curve nor a given individual opportunity line). It is shown in the appendix that $\gamma - \delta > 0$ is a stability condition for the equilibrium and that the existence of a stable equilibrium is ensured. The correspondence principle therefore implies that $\sigma_G / d\tau > 0$. Q.e.d.

Consider now post-tax incomes. Since $\sigma = (1-\tau)\sigma_G$ [from (9) and (21)] is the standard deviation of the income distribution net of taxes and public transfers, it holds that

$$\frac{d\sigma}{d\tau} = (1-\tau)\frac{d\sigma_G}{d\tau} - \sigma_G. \tag{34}$$

Using (28), (29), and (30) this expression can be transformed to

$$\frac{d\sigma}{d\tau} = \frac{\beta - \sigma_G \cdot i_\mu \cdot \bar{\mu}'(\sigma_G) + \bar{\mu}''(\sigma_G)\dfrac{1-\alpha\tau}{1-\tau}}{\gamma - \delta}. \tag{35}$$

The sign of (35) is ambiguous. Since $\gamma - \delta > 0$, it equals the sign of the numerator.

Note first that $d\sigma / d\tau < 0$ if $\bar{\mu}''$ is sufficiently strongly negative. A negative sign for μ'' indicates a curved self-insurance line and decreasing returns to risk taking. With a strongly negative value of $\bar{\mu}''$, the scope for individual reactions to a tax increase is small, and obviously the direct effect of a tax increase dominates.

A more interesting possibility is the one where $\bar{\mu}'$ is a positive constant in the relevant range such that $\bar{\mu}''=0$. In this case, equation (35) simplifies to

$$\frac{d\sigma}{d\tau}=\frac{\beta-\sigma_G\cdot i_\mu\cdot\bar{\mu}'}{\gamma-\delta}\text{ for }\bar{\mu}'=\text{const.}\tag{36}$$

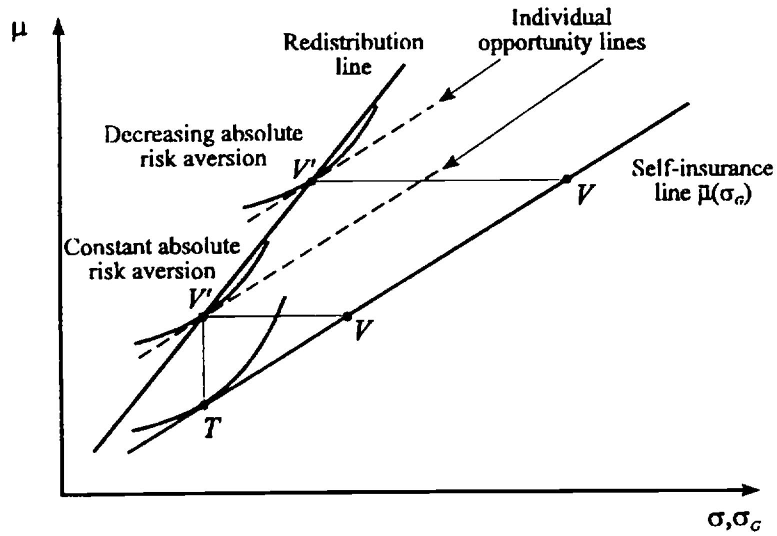
Recalling property (*e*) of the indifference curve system and (33) this expression can easily be interpreted.

Proposition 8: *Suppose there are constant returns to risk taking in the relevant range. Then, with decreasing absolute risk aversion* ($i_\mu<0$), *an expansion of the redistribution system will imply an equilibrium with more post-tax inequality. The same will be true with constant absolute risk aversion* ($i_\mu=0$) *provided that less than 100 % of self-insurance efforts are tax deductible. With constant absolute risk aversion and full deductibility of self-insurance efforts the equilibrium post-tax inequality will not be affected by the tax rate.*[16]

Proposition 8 describes a redistribution paradox because it specifies conditions under which the primary effect on equality of increased taxes will be overcompensated by the secondary effect of increased risk taking. This gives a deeper meaning to the statement made in the introduction that the risk taking effect of redistributive taxation may be more important than the insurance effect. In the cases considered, people transform more than 100 % of the increase in equality through redistributive taxation into income increases. Redistributive taxation does not improve the distribution of the pie's slices, but it makes the pie bigger.

[16]The proposition is related to a result that in another context had been derived by Atkinson and Stiglitz (1980, p.119). These authors studied redistributive taxation in the context of the standard two asset portfolio problem [where the (μ,σ) trade-off is automatically constant] and found that taxation increases "private risk taking" if the wealth elasticity of demand for the risky asset is positive.

Figure 7: *More Inequality through Redistributive Taxation*



An intuitive explanation of Proposition 8 can be given with the aid of Figure 7. This figure incorporates the cases of constant and decreasing absolute risk aversion and assumes that α equals unity (full deductibility of effort). The self-insurance line is linear in the relevant range, and so is the redistribution line. The equilibrium is characterized by a point on the redistribution line which is also a point of tangency between an indifference curve and the individual opportunity line. Depending on the level of government transfers, the latter can have a continuum of alternative positions. For the case at hand ($\alpha = 1$), it is known from (24) that the individual opportunity line has the same slope as the self-insurance line. The possible positions of the individual opportunity line can therefore be constructed by parallel shifts of the self-insurance line to the left. When absolute risk aversion is constant, the indifference curve slope stays constant when μ increases, given σ ($s_\mu = 0$). The equilibrium point V' on the redistribution line will therefore be vertically above the laissez-faire point T, while the point characterizing the pre-tax distribution shifts from T to V on the self-insurance line. The advantage of the protection that the redistribution scheme offers is entirely translated into a higher average income.

On the basis of this neutrality result, it is easy to see under which conditions the equilibrium point V' will be to the right of the laissez-faire point T. A first and obvious possibility is the case where, vertically above T, the indifference-curve slope is lower than at T.

This case prevails under decreasing absolute risk aversion. For any given level of post-tax inequality, pre-tax inequality and average income rise with an introduction of the redistribution scheme. The rise in average income lowers the required marginal compensation, $i(\mu,\sigma)$, for risk taking. The actual marginal compensation perceived by the individual, $\bar{\mu}'$, is constant, on the other hand. Hence, an equilibrium with a higher level of post-tax inequality will result. Figure 5 illustrates this with the upper of the two solution points labelled V'.

The second reason (not demonstrated in the figure) for an equilibrium with a higher inequality in post-tax incomes is incomplete deductibility of self-insurance efforts ($\alpha < 1$). The incomplete deductibility means that the decision maker perceives an additional incentive to reduce his effort and to move along the self-insurance line towards higher values of pre-tax inequality. In Figure 7, the individual opportunity line would have a higher slope than the self-insurance line and so the solution point V' would be to the right of T even in the case where absolute risk aversion is constant ($s_\mu = 0$).[17]

The conditions under which the redistribution paradox emerges are not implausible. From an empirical point of view, there can be little doubt that decreasing absolute risk aversion and less than full deductibility of self-insurance efforts are realistic assumptions. So the assumption of constant returns to risk taking is crucial. With the specifications of this model this assumption is only a limiting case. However, other model specifications may rather give the impression that constant returns to scale are an intermediate case in the spectrum of possibilities. For example, when there are decreasing returns to self-insurance while, at the same time, it is possible for an agent to add up independent income risks, then it is entirely unclear whether there will be increasing or deceasing returns to risk taking, since adding up independent income risks in itself implies increasing returns to risk taking. Increasing returns to risk taking would strengthen the mechanism underlying the redistribution paradox.

6. The Optimal Welfare State

Up till now it has been assumed that the government is a fairly passive agent satisfying itself with adjusting the public transfer so as to balance the government budget. What if the

---

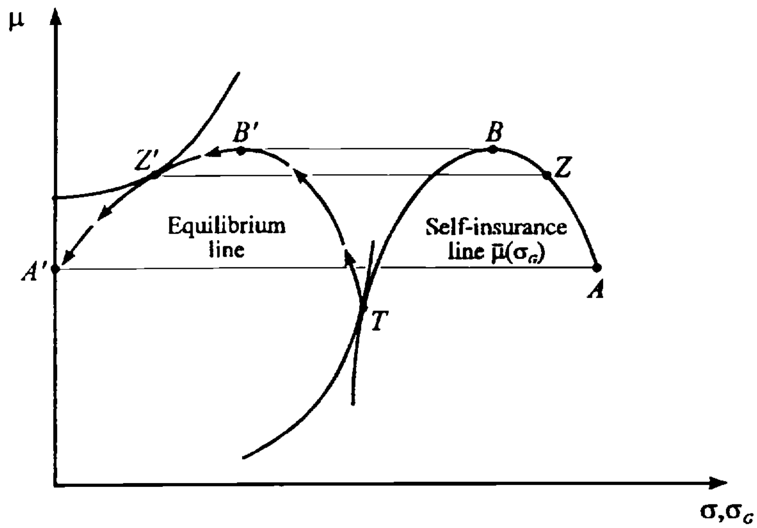
[17]This effect is operative even when $\bar{\mu}' = 0$. Cf. the discussion of the next section, in particular equation (40).

government choses the tax rate so as to maximize the representative individual's expected utility? What are the characteristics of the optimal welfare state?

To make the problem interesting it has to be assumed that $\alpha < 1$ so that at least some moral hazard effect is present. With $\alpha = 1$ the model would predict an optimal tax rate of one, since successive tax increases would always generate welfare increasing insurance and risk taking effects. Assuming that at least part of the agent's effort results in a loss of non-market income is common to the optimal tax literature.

The problem of optimal taxation is illustrated in Figure 8. For every tax rate τ, there is an equilibrium as described by equation (23). Starting from the laissez-faire point T, an increase in the tax rate will therefore induce a movement to the right along the self-insurance line (Proposition 7). In addition, the tax increase will move the redistribution line (cf. Figure 1) to the left. The net effect on the equilibrium combinations of μ and σ attainable through successive tax rate changes is illustrated by the arrowed curve in Figure 8 which will be called the "equilibrium line". It is known from Proposition 6 that the equilibrium line ends at point A' on the ordinate when the tax rate approaches one. (A' is the counterpart of A on the self-insurance line which is characterized by an absence of self-insurance effort.) The optimal tax rate is determined by a point like Z' where an indifference curve is tangent to the equilibrium line. Z' and its counterpart Z on the self-insurance line coincide with points like V' and V in Figure 5 if that figure is drawn for the optimal tax rate. The magnitude of the tax rate equals the distance $Z'Z$ relative to the distance between Z and the ordinate.

Figure 8: *One Version of the Optimal Tax Problem*



Let $\bar{\sigma}_G(\tau)$ be a function that summarizes the relationship between the equilibrium amount of pre-tax inequality and the tax rate as calculated with (28). Then the problem of optimal taxation can be stated as follows:[18]

$$\max_{\sigma_G} \; U(\mu,\sigma)$$
$$\text{s.t. } \mu = \bar{\mu}(\sigma_G),\ \sigma = (1-\tau)\sigma_G,\ \sigma_G = \bar{\sigma}_G(\tau). \tag{37}$$

Let $(\mathrm{d}U/\mathrm{d}\tau)/U_\mu$ denote the tax-induced welfare change in terms of certainty equivalents or what Atkinson (1970) called "equally distributed equivalent incomes". Differentiation of $U(\mu,\sigma)$ yields:

$$\frac{\mathrm{d}U/\mathrm{d}\tau}{U_\mu} = i\cdot\sigma_G + \bar{\sigma}_G'(\tau)\left[\bar{\mu}'(\sigma_G) - i\cdot(1-\tau)\right] \tag{38}$$

where $i = i(\mu,\sigma)$ is the indifference curve slope as defined in (15). A change in the tax rate generally alters μ and σ. The right-hand side of equation (38) evaluates these alterations. The term $i\cdot\sigma_G$ is the direct gain from redistribution, given individual behavior; i.e., the insurance effect. The term $\bar{\sigma}_G'(\tau)[\bar{\mu}'-i(1-\tau)]$ is the welfare change resulting from the increase in risk taking: it consists of a change in per capita income, $\bar{\sigma}_G'\cdot\bar{\mu}'$, and a change in post-tax inequality evaluated at the individual's "price of risk" (the indifference curve slope), $\bar{\sigma}_G'\cdot i\cdot(1-\tau)$.

[18]This formulation incorporates the government budget constraint through the assumption $\mu = \bar{\mu}(\sigma_G)$.

From (17) it is known that, if risk taking is at the socially optimal level given the tax rate, then $\bar{\mu}' - i \cdot (1-\tau) = 0$. As this includes the laissez-faire situation where $\tau = 0$, the first bit of redistributive taxation must increase welfare through the direct gain from redistribution; i.e., $(dU/d\tau)/U_\mu = i \cdot \sigma_G > 0$ at $\tau = 0$. At $\tau = 1$, according to Proposition 6, effort is zero so that $\bar{\mu}' = -k < 0$. Since, in addition, $i = i(\mu, \sigma) = 0$, from property (a) of the indifference curve system, the marginal increase in welfare approaches $(dU/d\tau)/U_\mu = -\bar{\sigma}'_G(\tau)k < 0$ as $\tau \to 1$. This implies that there is an interior solution for the optimal tax rate such as the one illustrated in Figure 8.

In the optimum, it is necessary that $(d\mu/d\tau)/U_\mu = 0$, which means that the welfare gain from the insurance effect is outweighed by a welfare loss resulting from excessive risk taking:

$$i \cdot \sigma_G = -\bar{\sigma}'_G(\tau)\left[\bar{\mu}'(\sigma_G) - i \cdot (1-\tau)\right] > 0. \tag{39}$$

Since $i \cdot \sigma_G > 0$ and $\bar{\sigma}'_G > 0$, it is necessary for (39) to be true that $\bar{\mu}'/(1-\tau) < i$. A comparison with (17) shows that this condition implies an equilibrium point on the redistribution line to the right of the constrained social optimum Q'. The result can be summarized as follows.

Proposition 9: *When self-insurance efforts are not fully tax-deductible, there is an interior solution for the socially optimal tax rate. In the optimum, risk taking and inequality overshoot the constrained social optimum, given a tax rate at the level of the optimal rate.*

The overshooting of risk taking may be substantial. In the case considered in Figure 8, it even implies moving to a point to the right of the maximum of the self-insurance line, where the marginal return to risk taking is negative.

Figure 9: *Optimal Taxation and the Redistribution Paradox*

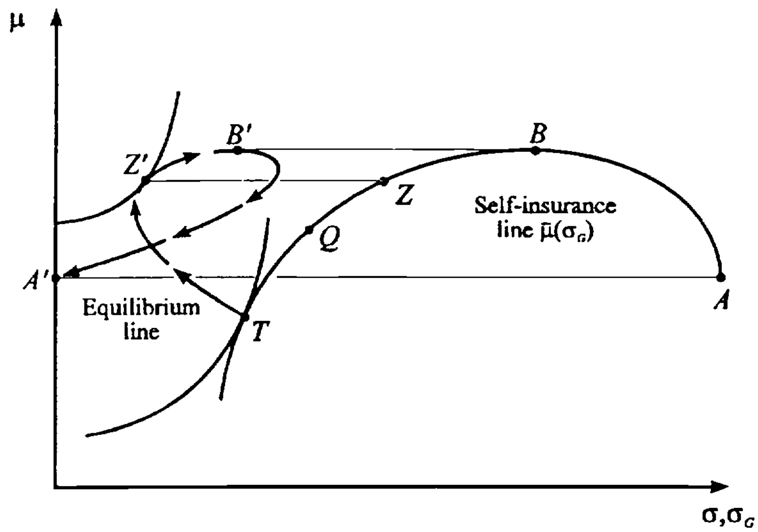


Figure 8 does not, however, depict the only possible case. An alternative possibility is illustrated in Figure 9. Here the equilibrium line performs a loop, and the optimal size of the redistributive system is found before the maxima of the self-insurance line and the equilibrium line are reached. The solution is now located in the range of positive marginal returns to risk taking (albeit still in the range where the marginal return to risk taking is unable to compensate for the resulting marginal increase in inequality).

Since σ_G is a monotonically increasing function of τ, and μ is a concave function of σ_G, a necessary and sufficient condition for a loop in the equilibrium line is that, at the maxima of the two curves, a redistribution paradox is present; i.e., it is necessary that, in the neighborhood of the point where $\bar{\mu}' = 0$, post-tax inequality rises with an increase in the tax rate.

To check whether and under what conditions this can be the case, insert (29),(30), and (31) into (35). If $\bar{\mu}' = 0$, this expression becomes

$$\frac{d\sigma}{d\tau} = \frac{i + (1-\alpha)k + \bar{\mu}'' \cdot \frac{1-\alpha\tau}{1-\tau}}{i_\sigma(1-\tau) - \bar{\mu}'' \cdot \frac{1-\alpha\tau}{1-\tau}}. \tag{40}$$

Equation (40) shows that the curvature of the self-insurance line, $|\bar{\mu}''|$, is essential for the existence of a loop. If the self-insurance line is sufficiently curved, then $d\sigma / d\tau < 0$ and there will be no loop. If it is sufficiently flat, there will be one. General continuity arguments imply that $d\sigma / d\tau$ will be strictly positive in the neighborhood of the maximum of $\bar{\mu}(\sigma_G)$ if $|\bar{\mu}''|$ stays sufficiently small in that neighborhood.

The interesting aspect of the solution illustrated in Figure 9 is that the redistribution paradox is present when the size of the welfare state has been optimized. A marginal increase in the tax rate increases average income, but this advantage is outweighed by an increase in post-tax inequality.

The nature of the two kinds of solution becomes apparent when equation (34) is inserted into (39). The resulting version of the optimality condition,

$$\frac{d\sigma_G}{d\tau}\bar{\mu}'(\sigma_G) = i \cdot \frac{d\sigma}{d\tau} \tag{41}$$

shows that $\bar{\mu}'$ and $d\sigma / d\tau$ will have the same sign. In the case depicted in Figure (9), the common sign is indeed positive; in the case depicted in Figure 8 it is negative. The following proposition emphasizes the interesting aspects of this result.

Proposition 10: *With an optimal size of the redistributive tax system, one of the two following conditions will hold. Either the economy operates at a point on its self-insurance line where, given the tax rate, more inequality results in a smaller average income. Or more redistribution causes more inequality in post-tax incomes and a higher average income.*

Although it contradicts popular views, Proposition 10 is a very natural and straightforward implication of a preference for equality when - as in the present model - the inequality of pre-tax incomes is an increasing function of the tax rate. Obviously, in the optimum, a marginal tax change must not induce adverse movements of average income and post-tax inequality for, if it did, a tax reform could be designed that increases welfare. Instead a marginal tax change must either decrease post-tax inequality and average income or have precisely the reverse effect. In the former case, a fall in average income coincides with an increase in pre-tax inequality; thus, given the redistribution scheme, the economy's technology implies a positive relationship between the size of the pie and the equality in the distribution of its slices. In the latter case, more redistribution increases the pie, but makes its distribution more unequal.

7. Concluding Remarks

In this paper an attempt has been made to contribute to the understanding of why the welfare state emerged and which laws and regularities govern its performance. The model developed for that purpose is admittedly abstract, leaving out many aspects that economic models of distribution normally contain. However, it concentrates on the uncertainty created for the individual by the inequality of market incomes. Arguably, insuring against this uncertainty, and thus stimulating private risk taking, constitute the main functions of the welfare state.

At first sight, it may seem that income uncertainty is too unimportant to justify the insurance interpretation of redistributive taxation. Indeed, if the model is interpreted as applying to a short period, say a year, there does not appear to be all that much uncertainty. As uncertainty is resolved only gradually with the passage of time, most of the observable inequality by the end of the year would have been predictable from the inequality already known at the beginning of the year. Things are very different, though, when the model is seen as applying to lifetime inequality or perhaps even to inequality evolving over the life span of dynasties. In the absense of information about both their unborn children's innate abilities and the opportunities the economy will offer them, parents-to-be may well be interested in the broad-based "career insurance contract" that the welfare state provides. They should, in principle, be able to design this contract in a way that ensures an efficient comromise between its insurance, risk taking and moral hazard effects. It is true that parents will be able to predict some of the next gerneration's inequality. After all, the parents-to-be may well know how many children they will have and what they will bequeath to them. Yet, it is also true that major aspects of their children's lifetime careers will remain totally obscure. It is this that may induce parents to protect their children with a redistribution contract even in cases where this contract may not be perfectly fair in the actuarial sense.

Part of the difficulty with the insurance interpretation of the welfare state is a time consistency problem. After the children have been born, have grown up, and have found their positions in the market economy, nearly all of the uncertainty will have been resolved and, naturally, they will then segregate into groups who like the welfare state and others who dislike it.The latter may, indeed, even try to opt out of the social contract which they, or their parents, had welcomed from an ex ante point of view. Obviously, a binding redistribution commitment is necessary to make the insurance contract possible and to enjoy the protection it offers.

When the welfare state was firmly embedded into a closed nation state, such a commitment was not a problem. The rich who had to pay could not leave the country, and the poor from other countries who could qualify as recipients of public funds could not immigrate. Things will be different in the new Europe where the nation states are facing open borders that allow free migration of goods, factors, people, and tax bases. Opening the borders, however beneficial this will be for the allocation of resources, has the disadvantages of loosening the commitment to fulfill the obligations from the redistribution contract and of admitting people at a stage where it is known that they are needy.

Suppose the model developed above is extended to a set of identical welfare states between which costless migration is possible at any time, and consider a competitive situation where the single states choose their taxes and transfer levels independently of one another. Under these circumstances, the optimal size of the welfare state as derived in section 6 cannot be maintained and, in fact, the welfare state as such will collapse.

Each single state will have an incentive to cut its taxes and reduce its public expenditure, so that people from the lower end of the income scale will emigrate and people from the upper end will immigrate. Those who emigrate will not be hurt, because they can find the same conditions elsewhere. Those who stay will gain, because they pay less or receive more from the government. And those who immigrate will gain because they pay less than they would have had to pay in other countries. From the single country's isolated perspective, the policy of "turning the welfare state around" seems an unambiguous welfare improvement. The only problem is that, if all countries behave that way they will end up in a situation without redistribution. The welfare gains from the insurance and risk taking effects, on which this paper elaborated, cannot survive in systems competition.

Created as a protection against outside attacks and internal disparities, the European nation state has since developed into the modern welfare state, which takes care of individuals and shelters them against the inequalities inherent to a market economy. The rapid economic and political integration of Europe has gradually begun to erode the foundations of the nation state. Protection against the neighbors will no longer be necessary, and protection against internal risks may become less and less feasible, as economic integration proceeds. The nation state will lose its insurance function. Only time can tell whether a replacement can be found.

Appendix: Stability of Equilibrium

Consider equation (23). Let

$$i(\sigma_G) \equiv i[\bar{\mu}(\sigma_G),(1-\tau)\sigma_G] \tag{A1}$$

denote the value of the left hand side of equation (23) on the redistribution line and let

$$q(\sigma_G) \equiv \bar{\mu}'(\sigma_G)\frac{1-\alpha\tau}{1-\tau}+\frac{\tau}{1-\tau}(1-\alpha)k \tag{A2}$$

denote the value of the right-hand side. The functions $i(\sigma_G)$ and $q(\sigma_G)$ give slopes of the indifference curves and the individual opportunity lines, respectively, along the redistribution line; i.e. for the set of potential equilibria satisfying the government budget constraint. By the definitions of γ and δ in equations (30) and (31),

$$i'(\sigma_G) = \gamma \equiv i_\mu \cdot \bar{\mu}'(\sigma_G) + i_\sigma(1-\tau), \tag{A3}$$

$$q'(\sigma_G) = \delta \equiv \bar{\mu}''(\sigma_G)(1-\alpha\tau)/(1-\tau). \tag{A4}$$

Since the second-order condition of problem (23) is satisfied (concavity of individual opportunity line given p and convexity of indifference curves), the individual will increase σ_G when $i(\sigma_G) < q(\sigma_G)$ and reduce σ_G, when $i(\sigma_G) > q(\sigma_G)$. The government, in turn, will adjust the transfer p so as to balance it budget. Obviously, local stability of the mutual adjustment process requires that, in some neighborhood of the equilibrium level of $\sigma_G, \sigma_G(V)$,

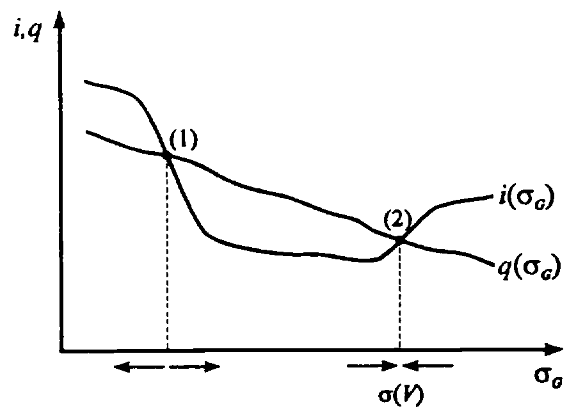
$$i(\sigma_G) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} q(\sigma_G) \Leftrightarrow \sigma_G \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \sigma_G(V) \tag{A5}$$

which is equivalent to saying that, when $\sigma_G = \sigma_G(V)$,

$$\gamma - \delta > 0. \tag{A7}$$

Figure A1 illustrates this. Points (1) and (2) represent equilibria, but only (2) is stable, because there the curve $i(\sigma_G)$ cuts the curve $q(\sigma_G)$ from below.

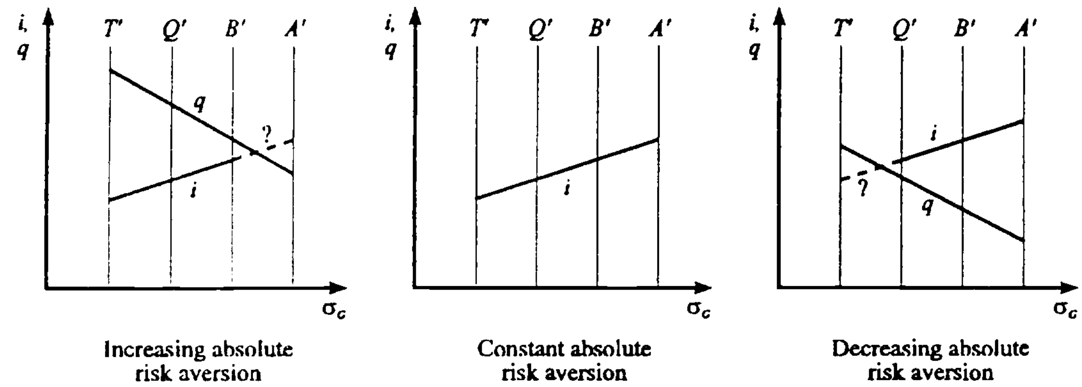
Figure A1: *Stable and Unstable Equilibria*



Knowing the properties of a stable equilibrium raises the question of whether the equilibria analyzed in section 4 are stable. To answer this question, note first that $q'(\sigma_G) \leq 0$ since $\bar{\mu}'' \leq 0$. Thus q is a declining function of σ, though not necessarily a monotonically declining function: there may be linear segments in the self-insurance line.

Consider next the function $i(\sigma_G)$. It is not obvious whether this function is "well-behaved" since μ and σ change along the redistribution line and both variables affect the indifference curve slope. Figure A2 illustrates in a schematic way how the slope depends on the shape of the von-Neumann-Morgenstern function and the segment of the redistribution line in which σ_G may lie. The segment $T'Q'$ is the range between the laissez-faire point T' and the social optimum Q'. The segment $Q'B'$ is the range between the social optimum and the maximum. And the segment $B'A'$ is the range between the maximum and the no-effort point A'. (Cf. the figures in the main text and recall properties (a)-(e) of the indifference curve system as listed in section 2).

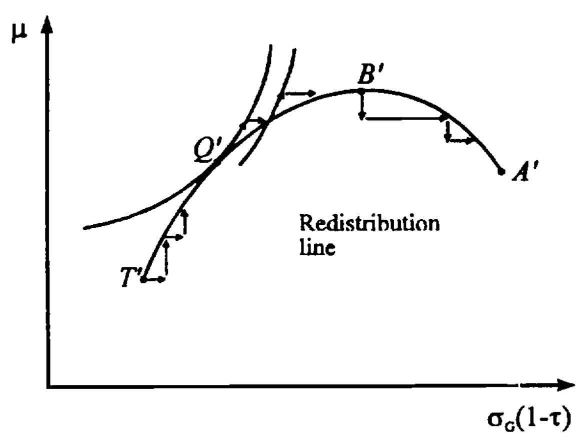
Figure A2: *Possible Shapes of the Function* $i(\sigma_G)$
(stylized as linear curves)



In the case of constant absolute risk aversion, $i'(\sigma_G) > 0$ holds true throughout, since the changes in μ brought about by a rightward movement along the redistribution line have no influence on the indifference curve slope. In the cases of increasing and decreasing absolute risk aversion the slope of the curve *i* is less obvious.

Note first, regardless of the preference structure, $i(\sigma_G)$ is positively sloped in the range between Q' and B'. The reason is that an indifference curve is tangent to the redistribution line at point Q'. Movements to the right of Q' up to B' can be decomposed into movements along an indifference curve plus horizontal movements to the right. Properties (c) and (d) of the indifference curves ensure that both components imply increases in the indifference curve slope. Figure A3 illustrates the argument.

Figure A3: *Changes of Indifference Curve Slope along the Redistribution Line*



A similar argument can be applied to the range $T' Q'$ when absolute risk aversion is increasing. A movement along the redistribution line can be decomposed into rightward and upward movements as illustrated in Figure A3. Because of properties (e) and (d) of the indifference curve system such movements will increase the indifference curve slope.

Finally, with decreasing absoute risk aversion, a rightward movement along the segment $B'A'$ will increase the indifference curve slope since properties (d) and (e) of the indifference curve system ensure that both a decline in μ, given σ, and an increase in σ, given μ, will bring about such an increase in the slope.

As illustrated by the interrogation marks in Figure A5, ambiguities remain with the segment $B'A'$ under increasing absolute risk aversion and with the segment $T'Q'$ under decreasing absolute risk aversion. Equilibria found here may be unstable, because the curve $q(\sigma_G)$ may be cutting the curve $i(\sigma_G)$ in these segments from below, even though q itself is downward sloping.

It can be shown, however, that if equilibria occur in the ambiguous segments then they will always include at least one stable equilibrium.

Proof: By the definition of the function $\bar{\alpha}(\sigma_G)$ from (26) it holds that

$$q[\sigma_G(Q)] = i[\sigma_G(Q)] \tag{A7}$$

if $\alpha = \bar{\alpha}[\sigma_G(Q)]$, where $\sigma_G(Q)$ is the socially optimal amount of risk taking. Differentiating (A2) with regard to α at $\sigma_G(Q)$ gives

$$\left.\frac{dq}{d\alpha}\right|_{\sigma_G=\sigma_G(Q)} = -\frac{\tau}{1-\tau}\{\bar{\mu}'[\sigma_G(Q)]+k\} < 0. \tag{A8}$$

From Proposition 5 it is known that

$$\sigma_G(V) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \sigma_G(Q) \Leftrightarrow \alpha \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \bar{\alpha}[\sigma_G(Q)]. \tag{A9}$$

Suppose an equilibrium occurs in the range $B'A'$ under increasing absolute risk aversion. Then $\sigma_G(V) > \sigma_G(Q)$ and $\alpha < \bar{\alpha}[\sigma_G(Q)]$. Because of (A8) and (A9) it follows that $q[\sigma_G(Q)] > i[\sigma_G(Q)]$. Figure A2 makes it clear that this excludes the possibility that q will cut

i only once from below in the range $B'A'$. (For this to happen it would be necessary that $q[\sigma_G(Q)] < i[\sigma_G(Q)]$.)

Suppose alternatively that an equilibrium occurs in the range $T'Q'$ under decreasing absolute risk aversion. Then $\sigma_G(V) < \sigma_G(Q)$ and $\alpha > \bar{\alpha}[\sigma_G(Q)]$. Because of (A8) and (A9) it follows that $q[\sigma_G(Q)] < i[\sigma_G(Q)]$, and again it becomes clear from Figure A2 that it is impossible for q to cut i only once from below in the range $T'Q'$, q.e.d.

Remark: For the special case of constant relative risk aversion a proof is available (Sinn 1985) that i is upward sloping even in the range $T'Q'$. So the equilibrium is always stable when absolute or relative risk aversion is constant.

References

Ahsan, S. M. (1974): "Progression and Risk-Taking," *Oxford Economic Papers* 26, 318-328.

---, (1976): "Taxation in a Two-Period Temporal Model of Consumption and Portfolio Allocation," *Journal of Public Economics* 5, 337-352.

Allingham, M. G. (1972): "Risk Taking and Taxation," *Zeitschrift für Nationalökonomie* 32, 203-224.

Atkinson, A. B. (1970): "On the Measurement of Inequality," *Journal of Economic Theory* 2, 244-263.

Atkinson, A. B., and J. Stiglitz (1980): *Lectures on Public Economics,* New York: McGraw-Hill.

Bamberg, G., and W. F. Richter (1984): "The Effects of Progressive Taxation on Risk-Taking," *Zeitschrift für Nationalökonomie* 44, 93-102.

Buchanan, J. M., and G. Tullock (1962): *The Calculus of Consent,* Ann Arbor: University of Michigan Press.

Diamond, P. H., L. J. Helms, and J. A. Mirrlees (1980): "Optimal Taxation in a Stochastic Economy," *Journal of Public Economics* 14, 1-29.

Domar, E., and R. A. Musgrave (1944): "Proportional Income Taxation and Risk-Taking," *Quarterly Journal of Economics* 58, 388-422.

Eaton, J., and H. S. Rosen (1980): "Taxation, Human Capital, and Uncertainty," *American Economic Review* 70, 705-715.

Friedman, M. (1953): "Choice, Chance, and the Personal Distribution of Income," *Journal of Political Economy* 61, 277-290.

Harsanyi, J. C. (1953): "Cardinal Utility in Welfare Economics and the Theory of Risk-Taking," *Journal of Political Economy* 61, 434-435.

---, (1955): "Cardinal Welfare, Individualistic Ethics and Interpersonal Comparisons of Utility," *Journal of Political Economy* 63, 309-321.

Kanbur, R. (1979): "Of Risk Taking and the Personal Distribution of Income," *Journal of Political Economy* 87, 769-797.

Konrad, K. (1992): *Risikoproduktivität,* Berlin, Heidelberg, New York: Springer.

Lindbeck, A., P. Molander, T. Persson, A. Sandmo, B. Swenborg and N. Thygesen (1994): *Turning Sweden Around,* Cambridge, Mass., and London, England: The MIT Press.

Meyer, J. (1987): "Two-Moment Decision Models and Expected Utility Maximization," *American Economic Review* 77, 421-430.

Pigou, A. C. (1932): *The Economics of Welfare*, London: McMillan.

Rawls, J. (1971): *A Theory of Justice,* Cambridge, Mass.: Harvard University Press.

Sinn, H.-W. (1981): "Die Grenzen des Versicherungsstaates. Theoretische Bemerkungen zum Thema Einkommensumverteilung, Versicherung und Wohlfahrt", in: H. Göppl and R. Henn, eds., *Geld, Banken und Versicherungen,* Königstein: Athenäum, 907-928. Reprinted in: G. Rolf, P. B. Spahn, and G. Wagner, eds., *Sozialvertrag und Sicherung - Zur ökonomischen Theorie staatlicher Versicherungs- und Umverteilungssysteme,* Frankfurt and New York: Campus, 1988, 65-84.

---, (1983): *Economic Decisions under Uncertainty,* Amsterdam, New York and Oxford: North Holland.

---, (1986):"Risiko als Produktionsfaktor," *Jahrbücher für Nationalökonomie und Statistik* 201, 557-571.

---, (1985): "Redistributive Taxation, Risk Taking and Welfare," University of Munich: *Münchener Wirtschaftswissenschaftliche Beiträge,* Discussion paper No. 85-13.

---, (1989): "Two-Moment Decision Models and Expected Utility Maximization: Comment", *American Economic Review* 79, 601-602.

---, (1990): "Expected Utility, μ,σ Preferences, and Linear Distribution Classes: A Further Result," *Journal of Risk and Uncertainty* 3, 277-281.

Stiglitz, J. E. (1969): "The Effects of Income, Wealth, and Capital Gains Taxation on Risk Taking," *Quarterly Journal of Economics* 83, 263-283.

Varian, H. R. (1980): "Redistributive Taxation as Social Insurance," *Journal of Public Economics* 14, 49-68.