From the Linear Economy to the Circular Economy: A Basic Model

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This paper sets up a Ramsey model with natural resources to study the optimal recycling of polluting raw materials. Under plausible conditions it is optimal for the economy to go through an initial *linear* phase with no recycling followed by a *circular* phase where a fraction of materials is recycled to alleviate growing natural resource scarcity and environmental degradation. In the presence of a Pigouvian tax on nonrecycled materials a competitive market economy will ensure the optimal degree of recycling.

Keywords: circular economy, linear economy, optimal recycling, Hotelling rule, Pigouvian taxation

JEL classification: Q 53, Q 58, H 21

1. Introduction: The Concept of a Circular Economy

"Reuse, recycle, reduce, rethink!" With this slogan an advisory group of business leaders recently urged the Danish government to move from the current "linear economy" to a "circular economy" (Advisory Board for Circular Economy, 2017). According to this vision the present linear economy is characterized by a "buy-and-throw-away" mentality involving excessive exploitation of natural resources and accumulation of polluting waste products: increasingly scarce raw materials are being extracted from the environment and returned to it as harmful waste as they are put through the "linear" process of production and consumption. By contrast, a circular economy seeks to minimize the use of raw materials per unit of output and to recycle waste products as much as possible in order to reuse them as inputs in production.

The concept of the circular economy is becoming increasingly popular among environmentalists and policymakers and in parts of the business community. The idea has been pushed for some time by think tanks such as the Ellen Macarthur Foundation (2012), and it has featured in the last two Five

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Year Plans of the Chinese government (Zhijun and Nailing, 2007). The European Commission (2015) has recently proposed an EU action plan for the circular economy, and many governments around the world are currently considering policies to promote recycling and more efficient waste treatment.

To economists trained in public economics or environmental economics this hype about the circular economy may seem somewhat exaggerated. For one thing, the idea of promoting recycling is hardly new. For example, in his famous paper on the economics of the coming "spaceship earth," Kenneth Boulding wrote: "Man must find his place in a cyclical ecological system which is capable of continuous reproduction of material form even though it cannot escape having inputs of energy" (Boulding, 1966, pp. 7–8). Boulding also anticipated the concept of the linear economy with his image of the "cowboy economy" where "… the success of the economy is measured by the amount of the throughput from the 'factors of production,' a part of which … is extracted from the reservoirs of raw materials and another part of which is output into the reservoirs of pollution." (Boulding, 1966, p. 8).

For another thing, economists will be skeptical of the idea that the economic system should maximize the amount of output per unit of natural resource input and the degree of recycling. In standard welfare economics and environmental economics the goal is to ensure an efficient use of *all* economic resources, including man-made goods as well as those given to us by nature. After all, a basic tenet of environmental economics is that the optimal level of pollution is generally larger than zero. As William Baumol (1977) pointed out, recycling of the byproducts of production and consumption requires the use of resources that at some point may generate more harm to the environment than the damage prevented through recycling.

Yet the present paper will show that there is a rational core to the proposition that the government should promote the transition from a *linear* economy with little or no recycling to a *circular* economy where a part of the materials used in production is recouped and recycled as inputs. To illustrate this, I will set up a simple model of an economy where production of final goods uses an exhaustible natural resource and (human and physical) capital as inputs and where the use of raw materials generates pollution, which can be mitigated by investing part of the capital stock in a recycling process. If the economy starts out with a good quality of the environment and a sufficiently large reserve stock of the natural resource, it will be optimal for it to go through an initial linear phase with no recycling of materials, but at some point a growing scarcity of natural resources relative to man-made capital and a deteriorating quality of the environment makes it optimal to enter a circular phase with positive recycling. However, in a laissez-faire economy the initial linear phase will involve excessive use of raw materials and the transition to the circular phase will not take place at the appropriate time. Hence government intervention in the form of a Pigou tax on nonrecycled materials is needed to steer the economy to the first-best transition path with the optimal level and timing of recycling.

Earlier writings on recycling such as Smith (1972), Schultze (1974), Lusky (1975, 1976), Hoel (1978), Di Vita (2001, 2007), and Pittel et al. (2010) have had little focus on explaining the transition from the linear to the circular economy and the design of public policy to ensure the optimal timing of this transition. The present paper seeks to fill this gap.

The paper adds to a relatively small environmental economics literature on recycling. An early contribution was made by Smith (1972), who focused on the reuse of household waste. Schultze (1974) illustrated how the recycling of raw materials could ameliorate the exhaustion of nonrenewable resources, and Lusky (1975, 1976) studied the allocation of household time between work in the labor market and recycling activity, showing how the optimal amount of recycling might be secured through a tax on consumption. The more recent papers by Di Vita (2001, 2007) investigate how endogenous technical change driven by R&D may affect the recycling of waste and thereby consumer welfare, and Pittel et al. (2010) set up a Ramsey-type model of exogenous growth with recycling of waste to study how the optimal level of recycling may be implemented through government subsidies. Like the present paper, the article by Andersen (2007) makes the point that the policy problems discussed within the circular economy paradigm can be tackled via the classical Pigouvian policy instruments emphasized in conventional environmental economics.

In contrast to the present paper, the contributions mentioned above did not focus on explaining the transition from a linear to a circular economy. The closest predecessor to the present study is the paper by Hoel (1978), who analyzed the optimal path of economic development and the role of recycling when natural-resource extraction harms the environment. However, Hoel's study was a microeconomic partial-equilibrium analysis, and in his simple model resource extraction and recycling will never take place simultaneously, whereas the present macroeconomic general-equilibrium analysis finds that the two activities can go on at the same time.

Section 2 sets up the model, which is used in section 3 to derive the first-best allocation of resources. Section 4 describes the first-best transition from a linear to a circular economy, and section 5 analyzes the resource allocation and recycling activity generated by a competitive market economy. Section 6 explains how a laissez-faire market economy will fail to attain the optimal volume and timing of recycling and how this failure can be corrected through Pigouvian taxation. The main conclusions are summarized in section 7.

2. The Model

We consider an economy inhabited by a representative family dynasty with an infinite horizon. In each period the family derives utility $u(C)$ from consumption of final goods (C) and utility $v(E)$ from the quality of the environment (E). At time zero the present value of the family's lifetime utility U is

$$U = \int_0^\infty [u(C) + v(E)]e^{-\rho t} dt, \quad u' > 0, \quad u'' < 0, \quad v' > 0, \quad v'' < 0, \tag{1}$$

where $\rho > 0$ is the constant rate of time preference, and the variables C and E are understood to be functions of time t. The total output of final goods (Y) may be used for consumption or for investment (I):

$$Y = C + I. \tag{2}$$

The output of final goods is given by the linearly homogeneous production function

$$Y = F\left(K^Y, M\right), \quad F_K > 0, \quad F_{KK} < 0, \quad F_M > 0, \quad F_{MM} < 0, \tag{3}$$

where the subscripts indicate first and second partial derivatives. The variable K^Y is the stock of capital used in final-goods production, and M is the input of a flow of raw materials. A part of these materials may be recycled by investing a capital stock K^R in the recycling process. The flow of recycled materials is given by the following recycling technology:

$$R = g(K^R/M)M, \quad g(0) = 0, \quad g' > 0, \quad g'' < 0,$$
$$\lim_{K^R/M \to \infty} g(K^R/M) = 1. \tag{4}$$

According to the last assumption in (4) a complete recycling of all materials ($g = 1$) would require an infinitely high capital intensity of the recycling process and is therefore infeasible due to the Second Law of Thermodynamics discussed by Georgescu-Roegen (1971). The assumption $g(0) = 0$ reflects that no recycling is possible if no capital is invested in recycling equipment.

Raw materials may be extracted at zero cost from a stock of an exhaustible natural resource. When there is recycling, the flow of new materials extracted from the ground each period is $M - R > 0$. Abstracting from new discoveries, the reserve stock of the natural resource (S) therefore evolves as

$$\dot{S} = -(M - R), \tag{5}$$

where a dot above a variable indicates its derivative with respect to time. The total stock of man-made capital (K) is

$$K = K^Y + K^R. \tag{6}$$

We may think of K as a composite of physical and human capital where optimizing behavior ensures that investment in the two forms of capital yields the same marginal return. Ignoring depreciation, the change in the capital stock over time is

$$\dot{K} = I. \tag{7}$$

The throughput of raw materials in the production process generates polluting waste products, so the quality of the environment deteriorates by an amount γ for each unit of raw material that is not recycled. The ability of the environment to assimilate waste and regenerate itself is proportional to the existing stock of environmental goods (proxied by E), with a proportionality factor δ. Hence the change in environmental quality over time is

$$\dot{E} = \delta E - \gamma(M - R), \quad \rho > \delta > 0, \quad \gamma > 0. \tag{8}$$

The assumption $\rho > \delta$ ensures that the shadow value of environmental quality is finite (cf. equation (20) below).

3. The First-Best Allocation

A utilitarian social planner will maximize the lifetime utility function (1) subject to the constraints implied by (2) through (8), given the predetermined initial values of K, S, and E. The current-value Hamiltonian for this optimal control problem can be written as

$$\begin{aligned} H = u(C) + v(E) + \mu \overbrace{[F(K - K^R, M) - C]}^{\dot{K}} \\ + \lambda \overbrace{[g(K^R/M) - 1]M}^{\dot{S}} + \eta \overbrace{\{\delta E - \gamma[1 - g(K^R/M)]M\}}^{\dot{E}} \end{aligned} \tag{9}$$

where μ, λ, and η are the current shadow values of the state variables K, S, and E, respectively, and the control variables are C, K^R, and M. The first-order conditions for the solution to the social planning problem are found to be

$$u'(C) = \mu, \tag{10}$$

$$mF_M = \frac{\lambda + \gamma\eta}{\mu}, \quad m \equiv \frac{1}{1 - (1 - \varepsilon)g}, \quad \varepsilon \equiv \frac{dg/g}{d(K^R/M)/(K^R/M)}, \tag{11}$$

$$K^R = 0 \text{ if } g'(0)\left(\frac{\lambda + \gamma\eta}{\mu}\right) \le F_K, \tag{12a}$$

$$K^R > 0 \text{ and } g'(K^R/M)\left(\frac{\lambda + \gamma\eta}{\mu}\right) = F_K \text{ if } g'(0)\left(\frac{\lambda + \gamma\eta}{\mu}\right) > F_K, \tag{12b}$$

$$\dot{\mu} = (\rho - F_K)\mu, \tag{13}$$

$$\dot{\lambda} = \rho\lambda, \tag{14}$$

$$\dot{\eta} = (\rho - \delta)\eta - v'(E). \tag{15}$$

Equation (10) states that the marginal utility of consumption must equal the marginal welfare gain from investment. The fraction $(\lambda + \gamma\eta)/\mu$ appearing in (11) and (12) is the marginal social cost of using an additional unit of nonrecycled raw material in production. It is measured in units of the final good (since we are dividing by the marginal utility of consumption, μ) and consists of the marginal cost of depleting the natural-resource stock, captured by the shadow price λ/μ, plus the marginal welfare cost $\gamma\eta/\mu$ of the damage to the environment when an extra unit of nonrecycled materials is put through the production process. The variable m in (11) is a *recycling multiplier* reflecting that a unit of materials can be used more than once when there is recycling. Each time an extra unit of materials enters the production process, a fraction $(1-\varepsilon)g$ of it can be used again, so an initial unit increase of materials input results in a total increase of $m \equiv 1/[1-(1-\varepsilon)g]$ units.[1] The presence of the dampening elasticity ε in the expression for m reflects that adding an extra unit of materials to the recycling process while keeping the recycling equipment K^R constant reduces the effectiveness of the process, thereby reducing the fraction of materials that can be recycled. Note that diminishing returns in the recycling process imply that the elasticity ε defined in (11) is smaller than 1.[2]

With these observations in mind, we see that (11) is a condition for optimal use of materials, stating that the marginal productivity of materials should equal the marginal social cost of their use, taking accout of the degree of recycling. The optimal degree of recycling is determined by (12a) and (12b), where the term $g'(0)(\lambda + \gamma\eta)/\mu$ is the marginal social gain from investing a unit of capital in recycling, starting from a level of zero investment. This gain reflects the alleviation of natural-resource scarcity and the improvement of environmental quality resulting from initiating recycling. The right-hand side of (12a) and (12b) is the marginal social opportunity cost of reallocating capital from final-goods production to recycling, given by the marginal productivity of capital in final-goods production. Thus (12a) says that if the marginal social gain from recycling is smaller than its marginal opportunity cost, society should not invest in recycling. But if $g'(0)(\lambda + \gamma\eta)/\mu > F_K$, so

1 To verify this, note that $m = 1 + (1-\varepsilon)g + [(1-\varepsilon)g]^2 + [(1-\varepsilon)g]^3 + \cdots = 1/[1-(1-\varepsilon)g]$.

2 The recycling process specified in (4) can be thought of as resulting from a linearly homogeneous *recycling function* $R = R(K^R, M) = g(K^R/M)M$ where $g(K^R/M) \equiv R(K^R/M, 1)$. With diminishing returns to each of the inputs in the recycling function $R(K^R, M)$, the function $g(K^R/M)$ will also display diminishing returns to the capital intensity K^R/M.

that some amount of recycling is worthwhile, (12b) says that investment in recycling should be carried to the point where its marginal social benefit equals its marginal social opportunity cost.

We can boil down the conditions for a first-best allocation into a *wealth accumulation rule* determining how much wealth society should transfer from the present to the future and a *portfolio composition rule* indicating how society should allocate its wealth between man-made capital and natural capital. The wealth accumulation rule in the present model is the familiar Keynes–Ramsey rule for an optimal intertemporal allocation of consumption that is implied by (10) and (13):

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}(F_K - \rho), \quad \sigma \equiv -\frac{u''C}{u'} > 0. \tag{16}$$

The portfolio composition rule can be found by differentiating (11) with respect to time and inserting (11) plus (13) through (15) into the resulting expression to obtain

$$F_K = \frac{\dot{F}_M}{F_M} + \left(\frac{\gamma\eta}{\lambda + \gamma\eta}\right)\left(\delta + \frac{v'(E)}{\eta}\right) + \frac{\dot{m}}{m}. \tag{17}$$

The left-hand side of (17) is the marginal social rate of return on investment in man-made capital, given by its marginal productivity. In optimum this must equal the marginal social rate of return on investment in natural capital appearing on the right-hand side of (17). The investment in natural capital takes the form of postponing the extraction of an extra unit of materials from "today" until "tomorrow." A part of the gain from doing so consists in the rise of the marginal productivity of materials as they become scarcer over time. This is captured by the first term on the right-hand side of (17). The second term reflects that postponing extraction implies a lower current use of materials, which generates an environmental gain, partly because the lower current emission of waste products increases the future assimilative capacity of the environment (captured by the parameter δ), and partly because the postponement of emissions directly benefits consumers by delaying the damage to the environment (reflected in the term v'/η). We see that the environmental gain carries a heavier weight the greater the importance of improving environmental quality relative to the importance of alleviating natural-resource scarcity, i.e., the larger the fraction $\gamma\eta/(\lambda + \gamma\eta)$. Finally, there is a gain from postponement of extraction to the extent that the *materials multiplier* m increases over time so that materials can be used more effectively in the future. This is captured by the third term on the right-hand side of (17). From the definition of m stated in (11) it follows that if the elasticity ε is roughly constant, we have $\frac{\dot{m}}{m} \approx \frac{(1-\varepsilon)\dot{g}}{1-(1-\varepsilon)g}$, so that (17) may be written as

$$F_K = \frac{\dot{F_M}}{F_M} + \left(\frac{\gamma\eta}{\lambda+\gamma\eta}\right)\left(\delta + \frac{v'(E)}{\eta}\right) + \frac{(1-\varepsilon)\dot{g}}{1-(1-\varepsilon)g}. \tag{18}$$

Recalling that $g < 1$ and $\varepsilon < 1$ because of diminishing returns to recycling, we see from (18) that an increase over time in the recycling rate g increases the marginal gain from postponing the extraction and use of materials, which is intuitive.

4. From the Linear to the Circular Economy

If the economy starts out at an early stage of economic development, it is likely that an optimal development path will involve an initial *linear* stage with no recycling and a deteriorating environment followed by a *circular* stage with positive recycling that reduces the pressure on the environment and slows down the depletion of the natural-resource stock.

To see this, note that (14) and (15) imply

$$\lambda(t) = \lambda(0)e^{\rho t}, \tag{19}$$

$$\eta(t) = \int_t^{\infty} v'(E(z))e^{-(\rho-\delta)(z-t)}dz. \tag{20}$$

According to (19) the shadow value of an extra unit of the natural resource rises steadily over time at the rate ρ as the resource gets scarcer. Equation (20) states that the shadow value of a unit improvement in environmental quality equals the present value of the future marginal utilities of environmental quality.[3]

Now suppose the economy starts out at an early stage of economic development where the reserve stock of the natural resource is large, the quality of the environment is good, and the stock of man-made capital is relatively low. With abundant natural resources, a well-preserved environment, and a relatively low level of material consumption due to a low capital stock, the marginal social cost $(\lambda+\gamma\eta)/\mu$ of using a unit of nonrecycled raw material will be low, since λ and η will be small whereas μ (the marginal utility of consumption) will be large. At the same time the marginal productivity of capital in final-goods production will be high due to its scarcity. In these circumstances the marginal social gain $g'(0)(\lambda+\gamma\eta)/\mu$ from investing in recycling will most likely be lower than the marginal opportunity cost F_K of doing so. According to (12a)

3 Note that since $\rho > \delta$ by assumption, the integral in (20) is finite. The presence of the parameter δ in the effective discount rate $\rho-\delta$ reflects that an improvement in current environmental quality increases the future ability of the environment to absorb waste, thereby increasing the future quality of the environment.

the economy should therefore start out in a linear phase with no recycling. During this phase, where $g = 0$, the optimality conditions (11) and (18) simplify to

$$F_M = \frac{\lambda + \gamma\eta}{\mu}, \tag{21}$$

$$F_K = \frac{\dot{F_M}}{F_M} + \left(\frac{\gamma\eta}{\lambda + \gamma\eta}\right)\left(\delta + \frac{v'(E)}{\eta}\right), \tag{22}$$

and with $R = 0$ it follows from (8) that the quality of the environment will evolve as

$$\dot{E} = \delta E - \gamma M. \tag{23}$$

When the marginal social cost of materials use is low, the optimality condition (21) will encourage a large input of materials in final-goods production. In the absence of recycling it is therefore likely that the pollution from materials use (γM) will exceed the absorption capacity of the environment (δE), causing the environment to deteriorate. Since the marginal utility of environmental quality increases as the quality goes down, it follows from (20) that the fall in environmental quality will drive up its shadow value η over time. According to (19) the shadow value λ of natural resource reserves will likewise increase systematically with time. Moreover, as long as man-made capital is relatively scarce, its marginal product is likely to exceed the rate of time preference, inducing positive savings and capital accumulation that will cause consumption to rise (cf. (16)) and drive down the marginal utility of consumption μ over time. At the same time the accumulation of capital will gradually reduce its marginal productivity.

Thus the linear economy is likely to be characterized by falling values of μ and F_K and rising values of η and λ as capital and pollution accumulate and the natural-resource stock diminishes. With the passing of time the economy will therefore reach a point where $g'(0)\left(\frac{\lambda+\gamma\eta}{\mu}\right) = F_K$. Beyond this point it becomes optimal to move from the linear phase to a circular phase with a positive level of recycling determined by the arbitrage condition (12b), which ensures identical marginal social returns to investment in recycling and investment in final-goods production. The transition from the linear economy with $R = 0$ to the circular economy with $R > 0$ alleviates the pressure on the environment as the evolution of environmental quality becomes governed by (8) rather than (23).

5. Resource Allocation in the Market Economy

Let us now compare the resource allocation generated by competitive markets with the socially optimal allocation described above. Consider a representative competitive mining firm owning a natural-resource stock S from which it extracts a flow of new raw materials N per period. Extraction is costless, and raw materials can be sold at the real market price p. In each period the mining firm can therefore pay out the following (time-varying) net dividend D^M to its owners:

$$D^M = pN. \tag{24}$$

The market value V_t^M of the mining firm at time t is the present value of its future dividend payouts, which is

$$V_t^M = \int_t^{\infty} D_z^M e^{-\int_t^z r_q dq} dz, \tag{25}$$

where r is the real market interest rate. The mining firm draws up a plan for the future levels of extraction that will maximize its market value (25) at time t subject to the stock-flow constraint $\dot{S} = -N$ and the predetermined initial reserve stock S_t. The first-order conditions for the solution to this problem yield the classical Hotelling rule stating that the equilibrium natural-resource price rises at the rate of interest:

$$r = \frac{\dot{p}}{p}. \tag{26}$$

The mining firm sells the extracted raw materials to the representative competitive firm in the final-goods industry, and the price of materials adjusts to ensure that supply equals demand, so that

$$N = M - R. \tag{27}$$

The final-goods firm uses the production technology (3) and the recycling technology (4) (when recycling is profitable). The government may choose to levy a unit tax at the (time-varying) rate τ on materials that are not recycled. Using the final good as numeraire, the real dividend D^Y paid out by the final-goods firm after deduction for investment expenditure may therefore be written as

$$\begin{aligned} D^Y &= Y - (p+\tau)(M-R) - I \\ &= F(K-K^R, M) - (p+\tau)[1-g(K^R/M)]M - I. \end{aligned} \tag{28}$$

By analogy to (25), the market value V^Y of the final-goods firm is

$$V_t^Y = \int_t^{\infty} D_z^Y e^{-\int_t^z r_q dq} dz. \tag{29}$$

Given (28) and its initial total stock of capital, the final-goods firm chooses K^R, M, and I with the purpose of maximizing (29) subject to the stock-flow constraint $\dot{K} = I$. The first-order conditions for the solution to this problem imply that

$$F_K = r, \tag{30}$$

$$mF_M = p + \tau, \tag{31}$$

$$K^R = 0 \text{ if } g'(0)(p+\tau) \leq F_K, \tag{32a}$$

$$g'(K^R/M)(p+\tau) = F_K \text{ if } g'(0)(p+\tau) > F_K. \tag{32b}$$

Equation (30) is the standard condition for profit maximization, that the marginal productivity of capital must equal the real rate of interest. Equation (31) says that materials are used until their marginal productivity equals their tax-inclusive price, allowing for the multiplier effect of recycling captured by the variable m. According to (32a), no capital is invested in recycling unless the resulting saving on materials expenses exceeds the marginal revenue from investing capital in final-goods production. In the early stage of development where natural resources are abundant and man-made capital is scarce, the materials price p will be low and the marginal productivity of capital in final-goods production will be high, so (32a) suggests that the market economy will go through an initial linear phase with no recycling. However, (26) implies that the materials price will rise over time, and as capital accumulates its marginal productivity will fall. At some point recycling therefore becomes profitable, and the market economy will enter the circular phase where the profit-maximizing level of recycling is determined by the arbitrage condition (32b), which requires identical marginal returns to investment in recycling and investment in final-goods production.

The household finances its consumption by the net dividends received from firms and by a government lump-sum transfer B financed by the revenue from the tax on nonrecycled materials. Hence

$$C = D^M + D^Y + B, \quad B = \tau(M - R). \tag{33}$$

Note that D^M and D^Y are dividend payouts *minus* any new capital that households inject in firms, so (33) allows for financial savings. The total household wealth V is

$$V \equiv V^M + V^Y. \tag{34}$$

From the expressions for V^M and V^Y in (25) and (29) it follows that total wealth evolves as

$$\dot{V} \equiv \dot{V^M} + \dot{V^Y} = r(V^M + V^Y) - D^M - D^Y = rV - (D^M + D^Y). \tag{35}$$

Combining (33) and (35), we obtain the dynamic household budget constraint:

$$\dot{V} = rV + B - C. \tag{36}$$

The household maximizes the present value of its lifetime utility (1) subject to the budget constraint (36) and the initial stock of wealth, taking the government transfer B as given. The first-order conditions for the solution to this problem yield the standard Keynes–Ramsey rule,

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}(r - \rho), \quad \sigma \equiv -\frac{u''C}{u'} > 0. \tag{37}$$

When the condition for profit maximization $r = F_K$ is inserted, (37) takes the same form as the wealth accumulation rule (16) for the planned economy. The market economy will therefore accumulate wealth at the optimal rate provided the marginal product of capital $F_K(K^Y, M)$ is at its first-best level at each point in time. For this to be the case, resource allocation in the market economy must also obey the portfolio composition rule (17). The next section shows how this may be achieved.

6. Securing the Optimal Transition from the Linear to the Circular Economy

Differentiating (31) with respect to time and inserting (26), (30), and (31) into the resulting equation, we obtain the following expression characterizing the portfolio composition in the market economy:

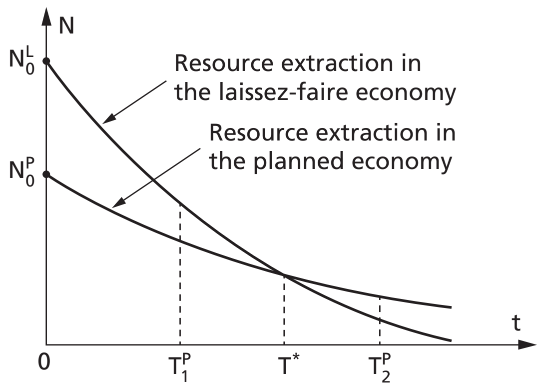
$$F_K = \frac{\dot{F}_M}{F_M} + \frac{\dot{m}}{m} + \frac{r\tau - \dot{\tau}}{P}, \quad P \equiv p + \tau. \tag{38}$$

Comparing (17) with (38), we see that, in a laissez-faire economy where $\tau = \dot{\tau} = 0$, the marginal private gain from postponing resource extraction given by the right-hand side of (38) will tend to be lower than the marginal social rate of return, which includes the environmental gain from slower extraction. In the initial linear phase of the laissez-faire economy, natural-resource extraction will therefore tend to be too rapid relative to the first-best pace of extraction. Intuitively one would also expect the transition to the circular phase to occur too late in the laissez-faire economy. However, this cannot be taken for granted, since the more intensive use of raw materials in the linear laissez-faire economy also means that the scarcity of natural resources increases faster over time.

The situation is illustrated in figure 1, where the flatter curve starting at the initial extraction level N_0^P shows the time path of materials extraction in the planned economy, and the steeper curve starting at the higher extraction level N_0^L depicts the evolution of extraction in the laissez-faire economy. Since the total area under each curve must add up to the same initial reserve stock S_0, the curve for the laissez-faire economy must cut through the curve for the planned economy from above at some point in time, denoted by T^* in figure 1. Now suppose it is optimal for the planned economy to move from the linear to the circular stage at time T_1^P. At that time, where the recycling multiplier *m* is still 1 but just about to become larger than 1, it follows from (11) and (12) that

$$\overbrace{g'(0)F_M(K,M)}^{\substack{\text{Marginal return to investment in recycling} \\ \text{(starting from zero recycling)}}} = \overbrace{F_K(K,M)}^{\substack{\text{Marginal return to investment} \\ \text{in final-goods production}}} . \quad (39)$$

Figure 1



In the laissez-faire economy, where $\tau = 0$, (31) and (32) likewise imply that the transition to the circular economy will take place at the time when the condition (39) is met. However, at time T_1^P the laissez-faire economy is seen to involve a larger materials input and is therefore likely to have a higher materials intensity M/K than the planned economy, implying a lower marginal productivity of materials and a higher marginal productivity of capital. In the laissez-faire economy the left-hand side of (39) will then be smaller than the right-hand side at time T_1^P, so the transition to the circular phase will not take place until some later time when the materials intensity has fallen sufficiently to satisfy the equality in (39). In this example the laissez-faire economy will thus move too slowly to the circular phase.

But suppose the initial marginal return to investment in recycling, $g'(0)$, is very low, so that it is not optimal for the planned economy to become circular until time T_2^P in figure 1. At that time the laissez-faire economy has a lower materials use and therefore most likely a lower materials intensity than the planned economy, implying (by simple reversal of the reasoning above) that it must have moved from the linear to the circular phase at some earlier time. Without imposing further restrictions on the model, we therefore cannot say whether the transition from the linear to the circular phase in the laissez-faire economy happens too early or too late.

What we *can* say is that the transition will take place at the "wrong" time and that the levels of materials use and recycling at any given point in time will be distorted compared to the first-best levels. These market failures may be corrected by imposing a Pigouvian tax on nonrecycled materials at a rate equal to the present value of the marginal environmental cost of materials use. Specifically, this Pigou tax must be levied at the following rate, where η and λ are the shadow values of the environment and of the natural resource stock prevailing along the economy's first-best time path, and where mec_z is the marginal external cost of using a unit of nonrecycled materials in some future period z, measured as a fraction of its tax-inclusive price P_z:

$$\tau_t = \int_t^{\infty} mec_z P_z e^{-\int_t^z r_q dq} dz, \quad mec_z \equiv \left(\frac{\gamma\eta}{\lambda + \gamma\eta} \right) \left(\delta + \frac{v'(E)}{\eta} \right). \tag{40}$$

To see that this tax rate does indeed guarantee optimality, note that (40) implies

$$\dot{\tau} = r\tau - mec \cdot P = r\tau - \left(\frac{\gamma\eta}{\lambda + \gamma\eta} \right) \left(\delta + \frac{v'(E)}{\eta} \right) P. \tag{41}$$

When (41) is inserted in (38), the resulting portfolio composition rule for the market economy becomes identical to the corresponding portfolio composition rule (17) for the planned economy, both in the linear phase with $m = 1$ and $\dot{m} = 0$ and in the circular phase with $m > 1$ and $\dot{m} \neq 0$. At any point in time the values of K and M in the market economy will then be at their first-best levels, and profit-maximizing behavior will therefore ensure that the transition from the linear to the circular economy takes place at the right time determined by (39).

In his influential study of the Green paradox, Sinn (2008) pointed out that an environmental tax on the use of a polluting exhaustible raw material may actually backfire if the present value of the tax rate increases over time, since resource owners will then have an incentive to accelerate the extraction of the resource, thereby accelerating the accumulation of pollution in the environment. The optimal Pigouvian tax determined by (40) is not vulnerable to such a Green paradox, since we see from (41) that the tax rate will grow at a rate below the rate of interest, so that its present value will fall over time.

On the other hand, from (40) and (41) we cannot exclude the possibility that the Pigou tax should start out from a high level and be gradually lowered as the relative price of raw material increases over time. As Sinn (op. cit.) pointed out, there may be serious political-economy obstacles to such a time profile of environmental taxation.

7. Conclusions

Our simple Ramsey model with natural resources that can be recycled has generated the following insights.

First, the proponents of the circular-economy paradigm are right in claiming that the economy should at some point move from a linear phase with no recycling to a circular phase where a part of the polluting materials used in production is recycled. The rationale for moving to the circular economy is a growing scarcity of natural resources relative to man-made capital combined with a deterioration of environmental quality as a result of their use.

Second, as natural resources become scarcer, the transition to a circular economy will occur even in a laissez-faire economy, but it will happen at the wrong time, and the volume of recycling will be distorted due to lacking internalization of the environmental cost of materials use.

Third, this market failure can be eliminated through a Pigouvian tax on nonrecycled materials that reflects their marginal environmental costs. When such a tax is levied, there is no need for further intervention, as profit-maximizing behavior will then secure the appropriate level and timing of recycling.

Thus the analysis suggests that the subsidy schemes and other forms of regulation (like mandatory sorting of waste) that have been implemented in many countries with the aim of promoting recycling may be poor substitutes for environmental taxes designed and calibrated according to time-honored Pigouvian principles. It seems that the case for other forms of regulation must rest mainly on political-economy barriers to Pigouvian taxation and/or a lack of information or administrative capacity to implement Pigou taxes at the correct level.

Our simple model could be extended in numerous ways. A fruitful topic for future research might be to include pollution from waste generated in the process of consumption and the possibility of sorting and recycling of household waste. In such a setting the optimal policy is likely to include a tax on nonrecycled household waste in addition to a tax on nonrecycled materials used by firms.

8. Appendix

8.1. Derivation of Equation (17)

Rearrangement of (11) yields

$$\mu m F_M = \lambda + \gamma \eta. \tag{42}$$

Differentiating both sides of (42) with respect to time, we get

$$\mu(\dot{m}F_M + m\dot{F}_M) + mF_M\dot{\mu} = \dot{\lambda} + \gamma\dot{\eta}$$
$$\Leftrightarrow \quad \mu m F_M\left(\frac{\dot{m}}{m} + \frac{\dot{F}_M}{F_M} + \frac{\dot{\mu}}{\mu}\right) = \dot{\lambda} + \gamma\dot{\eta}. \tag{43}$$

Dividing through by $\mu m F_M$ in (43) and inserting (42), we find

$$\frac{\dot{m}}{m} + \frac{\dot{F}_M}{F_M} + \frac{\dot{\mu}}{\mu} = \frac{\dot{\lambda} + \gamma\dot{\eta}}{\lambda + \gamma\eta}. \tag{44}$$

Using the first-order conditions (13), (14), and (15) to eliminate $\dot{\mu}/\mu$, $\dot{\lambda}$, and $\dot{\eta}$ from (44), we obtain

$$\frac{\dot{m}}{m} + \frac{\dot{F}_M}{F_M} + \rho - F_K = \frac{\rho(\lambda + \gamma\eta) - \gamma[\eta\delta + v'(E)]}{\lambda + \gamma\eta} \quad \Leftrightarrow$$
$$\frac{\dot{m}}{m} + \frac{\dot{F}_M}{F_M} - F_K = -\left(\frac{\gamma\eta}{\lambda + \gamma\eta}\right)\left(\delta + \frac{v'(E)}{\eta}\right). \tag{45}$$

A simple rearrangement of (45) now yields (17).

8.2. Derivation of Equation (38)

Differentiation of both sides of the first-order condition (31) with respect to time gives

$$\dot{m}F_M + m\dot{F}_M = \dot{p} + \dot{\tau} \quad \Leftrightarrow \quad mF_M\left(\frac{\dot{m}}{m} + \frac{\dot{F}_M}{F_M}\right) = \dot{p} + \dot{\tau}. \tag{46}$$

According to (31) we have $mF_M = p + \tau$, which may be inserted in (46) to give

$$\frac{\dot{m}}{m} + \frac{\dot{F}_M}{F_M} = \frac{\dot{p} + \dot{\tau}}{p + \tau}. \tag{47}$$

From the Hotelling rule (26) we know that $\dot{p} = rp$, and according to (30) value maximization by the final-goods firm implies $r = F_K$. Using these results, we can rewrite (47) as

$$\frac{\dot{m}}{m} + \frac{\dot{F}_M}{F_M} - F_K = \frac{rp + \dot{\tau}}{p + \tau} - r\left(\frac{p + \tau}{p + \tau}\right) \quad \Leftrightarrow \quad F_K = \frac{\dot{m}}{m} + \frac{\dot{F}_M}{F_M} + \frac{r\tau - \dot{\tau}}{p + \tau}, \tag{48}$$

which is identical to (38).

References

Advisory Board for Circular Economy (2017), Anbefalinger til Regeringen, Copenhagen, http://mfvm.dk/fileadmin/user_upload/MFVM/Miljoe/Cirkulaer_oekonomi/Advisory_Board_for_cirkulaer_oekonomi_Rapport.pdf (Access Date: 2017-07-19).

Baumol, W. (1977), On Recycling as a Moot Environmental Issue, Journal of Environmental Economics and Management 4, 83–87.

Boulding, K. (1966), The Economics of the Coming Spaceship Earth, in: Jarrett, H. (Ed.), Environmental Quality in a Growing Economy, Resources for the Future/Johns Hopkins University Press, Baltimore, MD, 3–14.

Di Vita, G. (2001), Technological Change, Growth and Waste Recycling, Energy Economics 23, 549–567.

Di Vita, G. (2007), Exhaustible Resources and Secondary Materials: A Macroeconomic Analysis, Ecological Economics 63, 138–148.

Ellen Macarthur Foundation (2012), Towards the Circular Economy, Vol. 1: Economic and Business Rationale for a Circular Economy, Cowes: Ellen Macarthur Foundation.

European Commission (2015), Closing the Loop – An EU Action Plan for the Circular Economy, COM (2015) 614 Final, https://ec.europa.eu/transparency/regdoc/rep/1/2015/EN/1-2015-614-EN-F1-1.pdf (Access Date: 2017-07-19).

Georgescu-Roegen, N. (1971), The Entropy Law and the Economic Process, Harvard University Press, Cambridge, MA.

Hoel, M. (1978), Resource Extraction and Recycling with Environmental Costs, Journal of Environmental Economics and Management 5, 220–235.

Lusky, R. (1975), Optimal Taxation Policies for Conservation and Recycling, Journal of Economic Theory 11, 315–328.

Lusky, R. (1976), A Model of Recycling and Pollution Control, Canadian Journal of Economics 9, 91–101.

Pittel, K., Amigues, J.-P., and Kuhn, T. (2010), Recycling under a Material Balance Constraint, Resource and Energy Economics 32, 379–394.

Schultze, W. (1973), The Optimal Use of Non-Renewable Resources: The Theory of Extraction, Journal of Environmental Economics and Management 1, 53–73.

Sinn, H.-W. (2008), Public Policies Against Global Warming: A Supply Side Approach, International Tax and Public Finance 15, 360–394.

Smith, V. (1972), Dynamics of Waste Accumulation: Disposal Versus Recycling, Quarterly Journal of Economics 86, 600–616.

Zhijun, F., and Nailing, Y. (2007), Putting a Circular Economy into Practice in China, Sustainability Science 2, 95–101.