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TAXATION, GROWTH, AND RESOURCE EXTRACTION

A General Equilibrium Approach

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This paper deals with taxation in a framework which is a synthesis between the neoclassical growth model, augmented by a (separable) sector of resource-extracting firms, and the Fisherian intertemporal general equilibrium model: market forces bring about the neoclassical optimal growth path under laissez faire, but taxation may result in welfare-reducing distortions. The taxes studied comprise ad valorem, capital-income, and capital-gains taxes, where the tax revenue is assumed to be redistributed in a lump-sum fashion. Particular attention is paid to the second-best problem of whether capital-income taxation should be supplemented by capital-gains taxation.

Editorial note

The article is a slightly revised translation of a paper presented at the 1979 annual conference of the German economic association 'Verein für Socialpolitik'. The German paper, without the appendix, was published in the conference review [Sinn (1980)]. An English discussion paper was circulated in Spring 1979. The aim of this translation is to make the results of the article accessible to a wider English-speaking audience, particularly, since there seems to be a renewed interest, among North American economists, in the intertemporal effects of taxation in the Fisherian perfect-foresight general equilibrium world. Compare, e.g., the paper by Chamley (1981); the approach chosen in that paper is surprisingly similar to, though less explicit than, that presented in this article.

1. Introduction

The present paper is a study in the intertemporal theory of taxation. The

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analysis is carried out in a very simple general equilibrium model with capital accumulation and resource extraction. It is predominantly an exercise in positive economics, but also offers an evaluation of taxes from a welfare point of view.

It might be claimed that a study in the intertemporal aspects of taxation is unnecessary. A frequent argument is that the well-established static theory of taxation is general enough to deal with the time aspect of allocation by simply distinguishing commodities by the time of their appearance. But this argument is a doubtful one. How, for example, could a capital-gains tax, which is one of the taxes studied in this paper, be explained by merely reinterpreting the static theory of taxation?

The economy we depict satisfies the usual assumptions for a competitive general equilibrium with well-established property rights, perfect information, and rational behavior of all private agents. Present and future prices are determined such that the quantities planned by the economy's agents are compatible and feasible under the given set of constraints. This is consistent either with a situation where there is a complete set of future markets or with a situation in which agents are endowed with perfect foresight.[1]

There are two goods and two factors. The goods are a non-replenishable natural resource and a 'normal' commodity. Whereas the normal commodity is used for consumption and investment, the resource is only consumed. The normal commodity, net of sales taxes, also serves as the numéraire in the economy. The two factors of production are labor and capital, both of which are used to produce the normal commodity.

The economy has four types of agents, resource firms, firms producing the normal commodity, households, and the government. Firms maximize their market values and households maximize their utilities. All private agents are intertemporal optimizers with an infinite time horizon.

Government is introduced in a very rudimentary form. It levies various taxes and redistributes them evenly to all households in the form of lump-sum transfers. The taxes comprise separate ad valorem and unit taxes on both commodities, a general income tax on both interest and wage income[2] and, as already mentioned, a capital-gains tax on the natural resource.[3] The latter will receive our particular attention, since from the view point of the ability-to-pay principle it might be regarded as a necessary supplement to an income tax.

[1]If we cannot rely on perfect foresight and if future markets are incomplete, then we can still hope that turnpike properties ensure a reasonably good approximation to the general equilibrium result at least for the near future.

[2]Since labor income is assumed to be exogenous, in the present model the allocative effects of the general income tax only stem from the capital-income tax.

[3]We study a tax on realized and unrealized capital gains. For the question of whether such a tax can be approximated by a suitably constructed tax on realized capital gains alone, see Helliwell (1969) and Green and Sheshinski (1978).

There are various studies in the dynamic theory of taxation which are related to our problem. We can separate them into a group of studies concerned with the intertemporal effects of taxation on capital formation, and a second group dealing with resource taxation.

The first group includes a large number of studies. Some of these can best be understood as reinterpretations of the static theory of taxation in an intertemporal context.[4] As is usual in static theory, they assume that the economy works perfectly in the absence of taxes, but that taxes introduce various kinds of welfare-reducing distortions. No attempt is made in these studies to relate the results of the analysis to those of neoclassical growth theory. Other investigations, also belonging to the first group, are carried out in the framework of neoclassical growth theory, but these do not assume that the laissez faire solution is optimal in any meaningful sense.[5]

The present paper provides a synthesis of the two kinds of approaches. It incorporates a decentralized version of the neoclassical growth model and assumes that individual preferences and social welfare are compatible.[6] Together with other assumptions this ensures that our model brings about the neoclassical optimal growth path automatically in the absence of taxes.

A first step in the direction of synthesis has already been made by Schenone (1975), who was able to demonstrate that an income tax will distort the optimal growth path. Schenone, however, does not explicitly depict a decentralized economy, as is done here. Instead he assumes that the market behaves as if it were steered by a central planner who maximizes welfare under some constraints about marginal conditions.

The second group of studies, which concern the taxation of natural resources, is rather small. The contributions most directly related to the present paper are those of Gray (1914), Hotelling (1931), and Burness (1976).[7] Each of these authors examines how the imposition of various taxes affects the supply behavior of a single resource-owning firm. This paper extends their approach to the general equilibrium case and introduces the capital-gains tax.

[4]See Feldstein and Tsiang (1968), Wright (1969), Feldstein (1978), and Boskin (1978). Cf. I. and H.W. Fisher (1942) and Kaldor (1957).

[5]See Krzyzaniak (1966), Sato (1967), Diamond (1970), Feldstein (1974a, b) and Atkinson and Sandmo (1980).

[6]This assumption is necessary if the evaluation of taxes we attempt is to be free from the merit-good problem.

[7]Other contributions include those of McRae (1974), Simmons (1977), and Kemp and Long (1979). All these authors attempt to determine an optimal time schedule of taxes (or tariffs) in the sense that the economy is pushed to some desired path. They do not analyze the economy's reaction to the imposition of existing or suggested tax instruments. Furthermore, McRae is only concerned with the case of common property resources which is irrelevant to the present study. Some structural elements of the model presented here were however chosen in the light of the excellent paper by Kemp and Long. Descriptive studies of resource taxation are those of Agrian (1969) and Page (1977a, b). Cf. also footnote 35.

2. Individual optimization of market agents

2.1. Resource firms

There are n_R identical resource-owning firms, each of which has initially the stock $q(0)=q_0$ of the non-renewable resource. The single resource firm behaves as a quantity setter. Its task is to determine the optimal time path of its resource supply[8] $\{g^s\}$, given the time paths of the resource price $\{p\}$ and the momentary rate of interest $\{r\}$, which are assumed to be differentiable. The firm faces the self-evident constraint that its accumulated sales cannot exceed its total resource stock. In addition, it faces the constraint that its rate of extraction cannot exceed some maximum rate $\bar{g}$, which reflects physical constraints, and cannot become negative since the resource is assumed not to be storable once extracted.[9]

The resource firm has to pay an ad valorem tax at the rate ε and a unit tax at the rate μ, denominated in terms of the numéraire,[10] on its resource extraction g^s, where $0 \leqq \varepsilon < 1$, $0 \leqq \mu < 1$, $\varepsilon + \mu/p < 1$. In addition, realized and unrealized capital gains of its resource stock q are taxed at the rate Ω, $0 \leqq \Omega < 1$. Thus, since the price of the unextracted resource is $p(1-\varepsilon)-\mu$, the capital-gains tax volume is[11] $\Omega \dot{p}\,(1-\varepsilon)q$.[12] Finally, income from investment in the capital market is taxed at the rate τ, $0 \leqq \tau < 1$.

The aim of a single firm is to maximize the present value of all sales minus taxes, i.e., the present value of the net dividend flow which accrues to its shareholders. So it faces the following dynamic optimization problem:

$$\max_{\{g^s\}} J_R(0) = \int_0^\infty \exp\left(-\int_0^t r(s)(1-\tau)\,\mathrm{d}s\right)\{g^s(t)\,[p(t)(1-\varepsilon)-\mu] - \Omega\dot{p}(t)(1-\varepsilon)q(t)\}\,\mathrm{d}t, \tag{1}$$

$$\text{s.t.}\quad q(0)=q_0,\quad \dot{q}=-g^s,\quad 0 \leqq g^s \leqq \bar{g},\quad q \geqq 0.$$

The Hamiltonian for this problem is

[8] $\{X\}$ denotes the time path of the variable X from point in time zero to infinity.

[9] It will be seen below that neither constraint is binding in a market equilibrium provided that $\bar{g}$ is large enough. The constraints are introduced simply to guarantee a solution to the resource firm's optimization problem even outside the market equilibrium.

[10] Since in reality unit taxes are denominated in terms of money, this assumption implicitly means that the money price level of current production stays constant over time.

[11] $\dot{X} \equiv \partial X/\partial t$, where t is a time index.

[12] With this formulation it is assumed that government calculates the price of the unextracted resource using the market price of the extracted resource. If government can directly monitor the market value of the firm, then the capital-gains tax volume is $\Omega\dot{\lambda}_R q$, where $\lambda_R = \partial J_R/\partial q$ with J_R defined in (1) below. Because of (2), both methods coincide in market equilibrium.

$$H_{\mathrm{R}}=\exp\left(-\int_0^t r(s)(1-\tau)\,\mathrm{d}s\right)\{g^s[p(1-\varepsilon)-\mu]-\Omega\dot{p}(1-\varepsilon)q+\lambda_{\mathrm{R}}(-g^s)\}.$$

After differentiation with respect to the control variable g^s we obtain the condition

$$p(1-\varepsilon)-\mu\{\gtreqless\}\,\lambda_{\mathrm{R}}\Rightarrow\left\{\begin{array}{c} g^s=\bar{g} \\ 0\leqq g^s\leqq\bar{g} \\ 0=g^s \end{array}\right\}. \tag{2}$$

Furthermore, from

$$\frac{\partial}{\partial t}\left\{\exp\left(-\int_0^t r(s)(1-\tau)\,\mathrm{d}s\right)\lambda_{\mathrm{R}}\right\}=-\partial H/\partial q,$$

we have

$$\dot{\lambda}_{\mathrm{R}}-\lambda_{\mathrm{R}}r(1-\tau)=\Omega\dot{p}(1-\varepsilon). \tag{3}$$

Together with the transversality condition

$$\lim_{t\to\infty}\exp\left(-\int_0^t r(s)(1-\tau)\,\mathrm{d}s\right)\lambda_r(t)q(t)=0 \tag{4}$$

and the initial condition $q(0)=q_0$, eqs. (2) and (3) are sufficient conditions for the policy to be optimal.

If $\{p\}$ and $\{r\}$ are arbitrarily given, generally the equality sign in (2) will not hold. Thus a 'bang-bang' policy would be pursued where partly $g^s=0$ and partly $g^s=\bar{g}$. But for an interior solution the time paths $\{p\}$ and $\{r\}$ have to be chosen so that[13]

$$\hat{p}[(1-\varepsilon)(1-\Omega)/(1-\varepsilon-\mu/p)]=r(1-\tau). \tag{5}$$

Note that in this case the time path $\{g^s\}$ is not yet uniquely determined, for if the equality sign in (2) holds, it holds for all g^s in the interval $0\leqq g^s\leqq\bar{g}$. Thus (5) is the condition for the firm being, within limits, indifferent between alternative resource extraction paths.[14] What this path eventually will look like depends on the demand side of the market which will be analyzed below.

2.2. Normal-commodity firms

In the normal-commodity sector there are n_{N} identical firms, which

[13] $\hat{X}\equiv\dot{X}/X$.
[14] Cf. footnote 20.

produce the normal commodity from capital, k, and labor, l, according to the linear homogeneous production function[15]

$$
\begin{aligned}
&f(k,l)-\delta k, \qquad \delta>0,\\
&f_i>0,\quad f_{ii}<0,\quad \lim_{i\to 0} f_i=\infty,\quad \lim_{i\to\infty} f_i=0, \qquad i=1,2, \\
&f(0,l)=f(k,0)=0.
\end{aligned}
\tag{6}
$$

The single firm has to determine its optimal time paths of commodity supply, $\{c^s\}$, and labor demand, $\{l^d\}$, given its initial capital stock, $k(0)=k_0$, and given the continuous time paths for the momentary rate of interest, $\{r\}$, and the wage rate, $\{w\}$. Implicitly, with the choice of c^s and l^d, the firm determines also the optimal path of its capital stock, $\{k\}$, for at each point in time that part of net production which is not sold, $f(k,l^d)-\delta k-c^s$, is all used to increase its capital stock. The firm faces the constraints $l^d\geqq 0$ and $\underline{c}\leqq c^s\leqq f(k,l^d)$, where $\underline{c}$ is a technically determined, absolutely large, value in the range $-\infty<\underline{c}<0$.[16]

In its optimization calculus the firm has to take into account the fact that the government levies an income tax[17] at the rate τ and a consumption sales tax at the rate Θ; $0\leqq\tau<1$, $\Theta\geqq 0$. Since the normal good, net of the sales tax, is the numéraire in the economy the sales tax is both an ad valorem and a unit tax.

The aim of an individual firm is to maximize its market value, i.e., the present value of dividends net of all taxes. If there were no income taxes, dividends as planned by the firm would be given by the difference between planned sales, net of the sales tax, c^s, and planned wage costs, wl^d, but taking all taxes into account they are only[18] $c^s-wl^d-\tau[f(k,l^d)-\delta k-wl^d]$. So the single producer faces the following control problem:

[15]The natural resource is treated as a consumption good rather than a factor of production. This specification, although unrealistic, has the advantage of facilitating a comparison between the intertemporal allocation in the normal-commodity sector and the well-known neoclassical optimal growth path. The reader who is primarily interested in the distortions created through capital-income taxation may reduce the present model to a one-sector growth model by simply setting the initial resource stock equal to zero.

[16]It will turn out that c cannot be a binding constraint in market equilibrium.

[17]It is assumed that economic income is the tax base. A relaxation of this assumption is studied in Sinn (1981b).

[18]Note that the personal income tax of the firm's owners (shareholders) has to be included, since their evaluation of the dividend flow determines the market value of the firm. It is assumed that each unit of capital income is taxed a single time at the rate τ. There is no additional corporate income tax. Note also that the inclusion of debt financing would not lead to changes in the firm's policy. If A denotes the outstanding debt and $\dot{A}$ its time change, then the flow of dividends, net of taxes, increases by $\dot{A}-r(1-\tau)A$. Taking $\dot{A}$ as an additional control variable and A as an additional state variable in the firm's problem, it can easily be shown that the marginal conditions (12) and (13) are unaffected.

$$\max_{\{c^s, l^d\}} J_N(0) = \int_0^\infty \exp\left(-\int_0^t r(s)(1-\tau)\,ds\right)\{c^s(t) - w(t)l^d(t)$$

$$-\tau(f[k(t), l^d(t)] - \delta k(t) - wl^d(t))\}\,dt, \tag{7}$$

$$\text{s.t.}\quad k(0) = k_0, \quad \dot{k} = f(k, l^d) - \delta k - c^s, \quad \underline{c} \leqq c^s \leqq f(k, l^d), \quad l^d \geqq 0, \quad k \geqq 0.$$

The Hamiltonian for this problem is

$$H_N = \exp\left(-\int_0^t r(s)(1-\tau)\,ds\right)\{c^s - wl^d(1-\tau)$$

$$-\tau[f(k, l^d) - \delta k] + \lambda_N[f(k, l^d) - \delta k - c^s]\}.$$

After differentiation with respect to the control variables we obtain

$$1\{\gtreqless\}\lambda_N \Rightarrow \left\{\begin{array}{c} c^s = f(k, l^d) \\ \underline{c} \leqq c^s \leqq f(k, l^d) \\ \underline{c} = c^s \end{array}\right\}, \tag{8}$$

and

$$f_2[(\lambda_N - \tau)/(1-\tau)] = w. \tag{9}$$

Eq. (9) does not allow for a corner solution, since the properties of the production function ensure that the employment level can be chosen such that (9) is satisfied if $w > 0$ and $\lambda_N > \tau$, conditions which, as shown in a footnote,[19] can reasonable be assumed for a market equilibrium.

As a further piece of information we obtain from

$$\frac{\partial}{\partial t}\left\{\exp\left(-\int_0^t r(s)(1-\tau)\,ds\right)\lambda_N\right\} = -\partial H_N/\partial k:$$

$$-\hat{\lambda}_N + r(1-\tau) = (f_1 - \delta)(1 - \tau/\lambda_N). \tag{10}$$

Together with the transversality condition,

[19]Assume that (i) $w > 0$ is a necessary condition for a strictly positive labor supply on the part of households and (ii) that, as is done explicitly in the next section, households want to consume at least a strictly positive subsistence minimum flow of the normal commodity. Then, since $f(k, 0) = 0$ and $c^s \leqq f(k, l^d)$, it is obvious that households are unable to satisfy their goal (ii) if $w \leqq 0$. If $\lambda_N \leqq \tau$, but $w > 0$, the Hamiltonian is a decreasing function of l^d, such that $l^d = 0$ is optimal. Once again this means that (ii) is not satisfied. So a market equilibrium with $w \leqq 0$ and/or $\lambda_N \leqq \tau$ would not exist.

$$\lim_{t\to\infty} \exp\left(-\int_0^t r(s)(1-\tau)\,\mathrm{d}s\right)\lambda_{\mathrm{N}}(t)k(t)=0, \tag{11}$$

and the starting condition $k(0)=k_0$, the differential eqs. (8)–(10) yield sufficient conditions for an optimal growth path from the viewpoint of a single firm.

If interior solutions are to prevail, then according to (8) and (10) the rate of interest has to be chosen such that, independently of the tax rates,

$$r=(f_1-\delta), \tag{12}$$

and according to (8) and (9) the wage rate has to be

$$w=f_2. \tag{13}$$

The nature of the optimal supply path for the normal commodity is similar to that for the resource: If the equality sign in (8) holds, then it holds for all c^{s} in the interval $\underline{c}\leqq c^{\mathrm{s}}\leqq f(k,l^{\mathrm{d}})$. So, at a given point in time, the firm is indifferent with respect to (limited) variations in its sales and it is up to the demand side of the market to determine exactly the level of trade.[20] However, given w and k, the level of labor demand is uniquely determined by (13).

2.3. Households

There are n_{H} identical households. They have equal shares in all firms and they offer each the time-independent flow of labor $l^{\mathrm{s}}=\bar{l}^{\mathrm{s}}$ provided that $w>0$ as is assumed.[21] Their personal disposable income consists of dividends from firms, net wages, and lump-sum transfers from the government.[22] Wealth at time $t, x(t)$, is the discounted value of these flows. Given the (continuous) price paths $\{r\}$, $\{p\}$, and $\{w\}$, given the constant price $1+\Theta$ of the consumption commodity, and given the plans of all firms, the single household calculates its wealth for point in time zero in the following way:

$$\begin{aligned} x(0)=x_0=&(n_{\mathrm{R}}/n_{\mathrm{H}})\,J_{\mathrm{R}}(0)+(n_{\mathrm{N}}/n_{\mathrm{H}})\,J_{\mathrm{N}}(0)\\ &+\int_0^\infty \exp\left(-\int_0^t r(s)(1-\tau)\,\mathrm{d}s\right)\{w(t)\bar{l}^{\mathrm{s}}(1-\tau)+T(t)/n_{\mathrm{H}}\}\,\mathrm{d}t. \end{aligned} \tag{14}$$

[20] This is related to the well-known problem of the horizontal investment demand curve. See Knight (1949), Haavelmo (1960, chs. 25, 28, 29), and Arrow and Kurz (1970, pp. 74ff). Note that despite the indeterminateness of market supply at any point in time, the paths $\{r\}$ and $\{w\}$ uniquely determine the path $\{k\}$. Hence there is one, and only one, path of market demand compatible with the firm's conditions for an interior optimum. The indeterminateness of supply at a particular point in time is one of the puzzles of continuous-time analysis.

[21] Cf. footnote 19.

[22] We could add the single household's net claims against other households. However, in a market equilibrium with symmetrically placed households, these net claims are zero.

Here T is the volume of transfers which, because of the government budget constraint, equals the total tax revenue,

$$T = n_{\mathrm{R}}[g^{\mathrm{s}}(p\,\varepsilon + \mu) + \Omega\dot{p}(1-\varepsilon)q] + n_{\mathrm{N}}\{c^{\mathrm{s}}\Theta + \tau[f(k, l^{\mathrm{d}}) - \delta k]\}.$$

With this formulation it is assumed that the single household calculates the tax volume via its information about the plans of the firms it owns, disregarding however any influence of its personal decisions on the aggregate tax volume and its refund out of this volume.

Given its wealth $x(0)$, the price paths $\{p\}$ and $\{r\}$, and the income tax rate τ, the single household chooses demand paths for the normal commodity, $\{c^{\mathrm{d}}\}$, and the resource, $\{g^{\mathrm{d}}\}$, which maximize its utility.

A single household's utility is represented by the present value of the 'felicities' from consuming the normal commodity and from consuming the natural resource. Felicity is discounted at the subjective rate of discount ρ, $\rho > 0$. It is assumed that households want at least to consume the subsistence minimum flow m, $m > 0$, of the normal commodity. The felicity function for consumption of the normal commodity is $U(c^{\mathrm{d}} - m)$ and has the properties $U' > 0$.

$$\lim_{c^{\mathrm{d}} \to m} U'(c^{\mathrm{d}} - m) = \infty, \quad \lim_{c^{\mathrm{d}} \to \infty} U'(c^{\mathrm{d}} - m) = 0, \qquad 0 < \underline{\eta}_U < \eta_U(c^{\mathrm{d}} - m) < \bar{\eta}_U < \infty,$$

where

$$\eta_U(c^{\mathrm{d}} - m) \equiv -[U''(c^{\mathrm{d}} - m)/U'(c^{\mathrm{d}} - m)]\,(c^{\mathrm{d}} - m). \tag{15}$$

Similarly, the felicity function for the resource is $V(g^{\mathrm{d}})$ with $V' > 0$,

$$\lim_{g^{\mathrm{d}} \to 0} V'(g^{\mathrm{d}}) = \infty, \quad \lim_{g^{\mathrm{d}} \to \infty} V'(g^{\mathrm{d}}) = 0, \qquad 0 < \underline{\eta}_V < \eta_V(g^{\mathrm{d}}) < \bar{\eta}_V < \infty,$$

where

$$\eta_V(g^{\mathrm{d}}) \equiv -[V''(g^{\mathrm{d}})/V'(g^{\mathrm{d}})]\,g^{\mathrm{d}}. \tag{16}$$

The terms η_U and η_V are elasticities of marginal felicity. Note that, although it is assumed that these elasticities are bounded, they are not required to be constant.[23]

[23]The special case of constant elasticities would imply the felicity functions

$$F(.) = \begin{Bmatrix} a + b(1-\eta_F)(.)^{1-\eta_F}, & \eta_F \neq 1 \\ a + b\ln(.), & \eta_F = 1 \end{Bmatrix}, \; -\infty < a + \infty, \quad 0 < b < \infty, \quad F = U, V.$$

So the household faces the following decision problem:[24]

$$\max_{\{c^{\mathrm{d}},g^{\mathrm{d}}\}} \int_0^\infty \mathrm{e}^{-\rho t}[U(c^{\mathrm{d}}-m)+V(g^{\mathrm{d}})]\,\mathrm{d}t, \tag{17}$$

$$\text{s.t.}\quad x(0)=x_0,\quad \dot{x}=r(1-\tau)x-c^{\mathrm{d}}(1+\Theta)-g^{\mathrm{d}}p,$$

$$c^{\mathrm{d}}\geqq m,\quad g^{\mathrm{d}}\geqq 0,\quad x\geqq 0.$$

The corresponding Hamiltonian is

$$H_{\mathrm{H}}=\mathrm{e}^{-\rho t}\{U(c^{\mathrm{d}}-m)+V(g^{\mathrm{d}})+\lambda_{\mathrm{H}}[r(1-\tau)x-c^{\mathrm{d}}(1+\Theta)-g^{\mathrm{d}}p]\}.$$

After differentation with respect to the control variables we obtain

$$U'(c^{\mathrm{d}}-m)=\lambda_H(1+\Theta),\qquad m\leqq c^{\mathrm{d}}<\infty, \tag{18}$$

and

$$V'(g^{\mathrm{d}})=\lambda_{\mathrm{H}}p,\qquad 0\leqq g^{\mathrm{d}}<\infty. \tag{19}$$

We consider here only the case of an interior maximum, for the properties of the felicity functions ensure that (18) and (19) can always be satisfied for strictly positive and finite values of λ_{H} and p.[25]

As a further condition for a maximum we obtain from $\partial(\mathrm{e}^{-\rho t}\lambda_{\mathrm{H}})/\partial t=-\partial H_{\mathrm{H}}/\partial x$

$$\hat{\lambda}_{\mathrm{H}}=\rho-r(1-\tau). \tag{20}$$

Together with starting condition, $x(0)=x_0$, and the transversality condition,

$$\lim_{t\to\infty}\mathrm{e}^{-\rho t}\lambda_{\mathrm{H}}(t)x(t)=0, \tag{21}$$

conditions (18)–(20) uniquely determine the optimal multiperiod plan of a single household.

Using the elasticities of marginal felicity defined in (15) and (16), we can explicitly calculate the equations of motion for c^{d} and g^{d}. From (18) and (19),

$$\dot{c}^{\mathrm{d}}=-[(c^{\mathrm{d}}-m)/\eta_U(c^{\mathrm{d}}-m)]\,[\rho-r(1-\tau)] \tag{22}$$

[24]With the assumption of an infinite planning horizon it is implicitly assumed that a household does not only plan its own consumption path but also that of its heirs. Bellman's principle of optimality ensures that (17) can be reformulated as a lifecycle optimization problem with a bequest motive without changing the results we derive.

[25]The case $p\leqq 0$ is not admissible since then $H_{\mathrm{H}}=g^{\mathrm{d}}=\infty$ would be chosen.

is obtained and, from (19) and (20),

$$\dot{g}^{\mathrm{d}} = -[g^{\mathrm{d}}/\eta_V(g^{\mathrm{d}})]\,[\rho - r(1-\tau) + \hat{p}]. \tag{23}$$

3. Market equilibrium

Having derived the conditions for the microeconomic optima of the single market agents, we now have to study their implications for a general equilibrium at the macro level. Such an equilibrium prevails if the price paths $\{r\}$, $\{p\}$, and $\{w\}$ are such that at each point in time the quantities supplied in all four markets (capital, labor, normal commodity, resource) equal the quantities demanded. It can be shown that this is the case if

$$\bar{L}^{\mathrm{s}} = L^{\mathrm{d}}(\equiv \bar{L}), \qquad C^{\mathrm{s}} = C^{\mathrm{d}}(\equiv C), \qquad G^{\mathrm{s}} = G^{\mathrm{d}}(\equiv G), \tag{24}$$

where

$$\bar{L}^{\mathrm{s}} \equiv n_{\mathrm{H}}\,\bar{l}^{\mathrm{s}}, \qquad L^{\mathrm{d}} \equiv n_{\mathrm{N}}\,l^{\mathrm{d}},$$

$$C^{\mathrm{s}} \equiv n_{\mathrm{N}}\,c^{\mathrm{s}}, \qquad C^{\mathrm{d}} \equiv n_{\mathrm{H}}\,c^{\mathrm{d}},$$

$$G^{\mathrm{s}} \equiv n_{\mathrm{R}}\,g^{\mathrm{s}}, \qquad G^{\mathrm{d}} \equiv n_{\mathrm{H}}\,g^{\mathrm{d}}.$$ [26]

The equilibrium condition in the labor market is easy to discover: we only have to choose w such that (9) is satisfied for $f_2 = f_2(K, \bar{L})$ where, by the assumption of linear homogeneity, $f_2(K, \bar{L}) = f_2(K/n_{\mathrm{N}}, \bar{L}/n_{\mathrm{N}})$. Given the employment level $\bar{L}$ we now have to examine the conditions for equilibria in the markets for the normal commodity and the resource.

3.1. The market for the normal commodity

The properties of an intertemporal market equilibrium for trade in the normal commodity are studied in a (C, K) diagram (fig. 1) which is well known from conventional growth models.

The solution space in this diagram has two boundaries. The upper one is given by $f(K, \bar{L}) = n_{\mathrm{N}}\; f(K/n_{\mathrm{N}}, \bar{L}/n_{\mathrm{N}})$ due to our assumption that firms cannot sell more than their gross production. The lower one is given by $M \equiv m\,n_{\mathrm{H}}$ which is the aggregate subsistence minimum. From the decision problem of the firm there is another lower boundary at $\underline{C} \equiv n_{\mathrm{N}}\,\underline{c}$, but since $\underline{c} \leqq 0$, and

[26]While the output of each industry is well defined, the output of the individual producers, resource and normal, is indeterminate. Since all firms of a given kind are identical, it is assumed that the quantities supplied are evenly distributed. This assumption is typically used in general equilibrium models under constant returns to scale and is not really necessary for the subsequent analysis.

$m>0$ this boundary cannot be binding for the market solution and is therefore not shown in fig. 1.

Consider first the possibilities for interior solutions in the firm's decision problem. Since $\dot{c}^{\mathrm{d}}/(c^{\mathrm{d}}-m)=\dot{C}/(C-M)$ the conditions (12) and (22) can be combined to obtain the following conditions for an equilibrium in the market for the normal commodity:

$$\dot{C}=(C-M)[(f_1-\delta)(1-\tau)-\rho]/\eta_U,$$

$$f_1=f_1(K,\bar{L}), \qquad \eta_U=\eta_U((C-M)/n_{\mathrm{H}})). \tag{25}$$

Together with the macroeconomic equation of motion

$$\dot{K}=f-\delta K-C, \qquad f=f(K,\bar{L}), \tag{26}$$

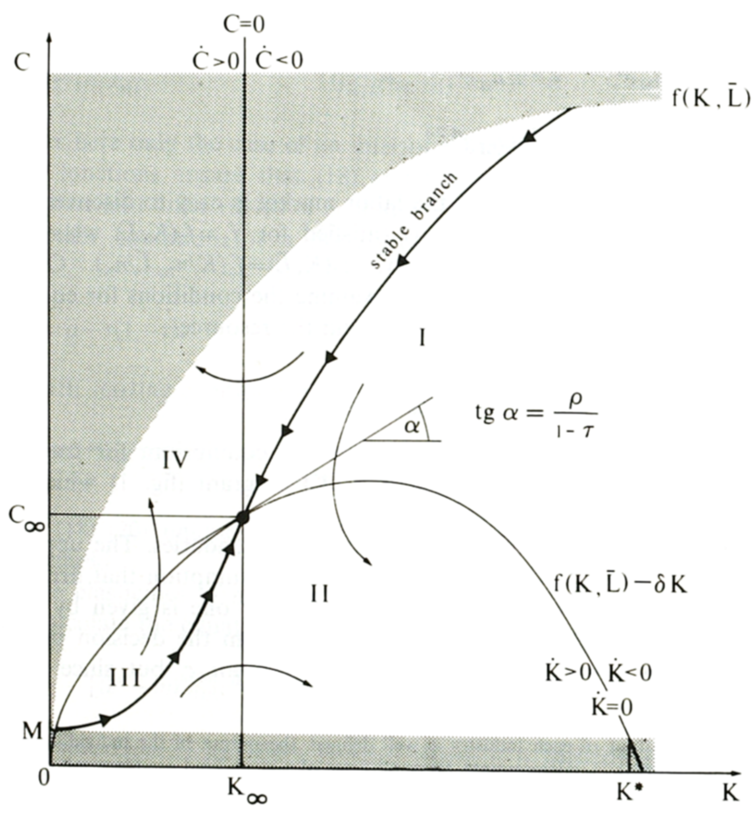

Fig. 1. The intertemporal equilibrium in the normal-commodity market.

derived from its microeconomic counterpart (7), this equation describes a continuum of possible paths in (C, K) space. Note, for later use as well, that the slope of any possible path is given by the equation

$$\mathrm{d}C/\mathrm{d}K = (C-M)[(f_1-\delta)(1-\tau)-\rho]/[(f-\delta K-C)\eta_U], \tag{27}$$

which combines the information from (25) and (26).

If we set $\dot{C}=\dot{K}=0$ in (26) and (27), we obtain the correspondingly labelled curves of fig.1. At the point of intersection there is a steady-state position with the consumption of the normal commodity C_∞ and the capital stock K_∞. The steady-state position is characterized by

$$f_1-\delta=\frac{\rho}{1-\tau}, \tag{28}$$

and is thus (recall that $\rho>0$) left of the maximum of the $\dot{K}=0$ curve. Furthermore it is below the maximum consumption schedule, i.e., below the gross-output curve $f(K,\bar{L})$, since $\delta>0$. It is assumed that it is also high enough to allow for permanent consumption above the subsistence minimum, M.

Together with the boundaries, the $\dot{C}=0$ and $\dot{K}=0$ loci determine four regions. Each of these regions has a characteristic direction for possible movements of capital and consumption, as indicated by the arrowed paths in fig. 1. We forego a detailed discussion, since it would be along conventional lines.

Although a continuum of paths is compatible with (26) and (27), from each direction there is only one path, the 'stable branch', leading to the steady-state point. It can be shown that only this stable branch is compatible with an intertemporal general equilibrium. It is assumed for this purpose that the initial capital stock of the economy, K_0, is large enough to allow for a consumption level above the subsistence minimum and that it is lower than that capital stock where the stable branch intersects the boundary $f(K,\bar{L})$.

Consider the paths above the stable branch.[27] If initially the conditions for an interior solution on the firms' part are met, these paths will start at or below the boundary $f(K,\bar{L})$, but, left of the $\dot{C}=0$ locus, they will eventually lead to this boundary. If initially the conditions for an interior solution are not met, the paths will start at the boundary, and although they may temporarily deviate, they must finally also lead back to the boundary $f(K,\bar{L})$ to the left of the $\dot{C}=0$ locus. Once there, it follows from (25) and (26) that the way back to the interior solution space is blocked, and so, in finite time, the economy ends up in a situation where gross production is just

[27]Note that, because of (18), C is a continuous function of time.

enough to satisfy the subsistence minimum consumption. Since, however, in this situation the capital stock must continue to decline so that gross production falls short of the subsistence minimum, the plans of the households will be violated. Thus all paths above the stable branch are incompatible with market equilibrium.

Consider now the paths below the stable branch. Since $U'(0)=\infty$ ensures an interior solution for households and since the boundary $\underline{C}$, faced by the firms, cannot be binding, the possible paths have to satisfy eq. (25), starting with a consumption level above the subsistence minimum. As time goes to infinity, all these paths lead towards the point $(C=M, K=K^*)$ where the $\dot{K}=0$ locus intersects the lower boundary of the solution space. If account is taken of (12), then this property ensures that there is a t^*, $0 \leqq t^* < \infty$, such that $\int_0^{t^*} r(t)(1-\tau)\mathrm{d}t < \infty$ and $\int_{t^*}^{t} r(t)(t-\tau)\mathrm{d}t < 0$. This, however, violates the transversality condition (11) of the producers of the normal commodity — because of (8) and $K=n_{\mathrm{N}}k$, $\lim_{t\to\infty} K=K^*>0$ implies that

$$\lim_{t\to\infty}\left(\exp\left(-\int_0^{t^*} r(s)(1-\tau)\,\mathrm{d}s\right)\exp\left(-\int_{t^*}^{t} r(s)(1-\tau)\,\mathrm{d}s\right)k(t)\right)>0. \qquad (29)$$

So all paths below the stable branch are also incompatible with market equilibrium.

What remains is the stable branch itself. In the appendix it is shown that it satisfies the transversality condition of the producers of the normal commodity. It is also proved that the stable branch satisfies the transversality condition of households, provided that the transversality condition of the resource owners is met. We shall see that the latter is the case under a rather weak assumption.

3.2. *The resource market*

The properties of an equilibrium in the market for the natural resource are analyzed in a (G,t) diagram (fig. 2). The solution space in this diagram has two boundaries. The upper one is given by $\bar{G}\equiv n_{\mathrm{R}}\bar{g}$ and the lower one by the abscissa.

From the decision problem of the resource firm it is known that eq. (5) must hold in the case of an interior solution. Insert $\hat{p}$, as given by this equation, into (23) which is derived from the decision problem of the household and note that in market equilibrium $G=n_{\mathrm{H}}g^{\mathrm{d}}$ and $\dot{G}=n_{\mathrm{H}}\dot{g}^{\mathrm{d}}$. Then

$$\dot{G}=-\frac{G}{\eta_V}\left(\rho-r(1-\tau)\frac{\mu/p(1-\varepsilon)-\Omega}{1-\Omega}\right).$$

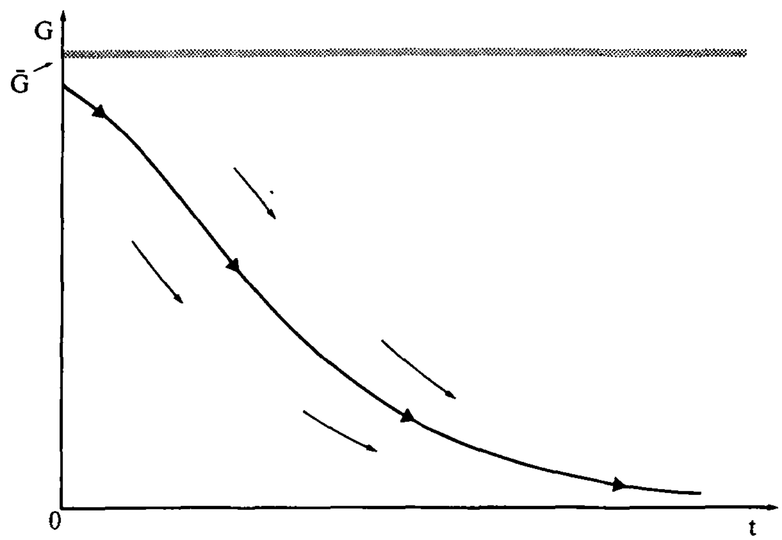

Fig. 2. The intertemporal equilibrium in the resource market.

Here, of course, r cannot be chosen arbitrarily, but is determined by the capital stock in the normal-good sector. By substituting r from (12) we have

$$\dot{G} = -\frac{G}{\eta_V}\left[\rho - (f_1 - \delta)(1-\tau)\frac{\mu/p(1-\varepsilon) - \Omega}{1-\Omega}\right], \tag{30}$$

with

$$\eta_V = \eta_V(G/n_{\mathrm{H}}), \qquad f_1 = f_1(K, \bar{L}),$$

which is a necessary condition for an interior market equilibrium and describes a continuum of possible paths in (G, t) space.

An important property of these paths is that they cannot intersect. Note that

(i) $\eta_V = \eta_V(G/n_{\mathrm{H}})$, $f_1 = f_1(K, \bar{L})$ and, from (18) and (19), $p = V'(G/n_{\mathrm{H}})/U'[(C-M)/n_{\mathrm{H}}]$;

(ii) K and C are continuous functions of time, since, as shown in the previous section, the economy moves along the 'stable branch' in the (C, K) diagram.

This ensures that, at a given point in fig. 2, the variables η_V, f_1, p, and thus $\dot{G}$ are uniquely defined. It is impossible, therefore, for two paths to intersect — an intersection implies at least two different values of $\dot{G}$ in eq. (30).

From (30), generally, we cannot derive the sign of $\dot{G}$ at the beginning of the planning problem. It is clearly negative if $(f_1-\delta)\ (\mu/p(1-\varepsilon)-\Omega)\leqq 0$, but it can be positive if this condition does not hold. Note, however, that the limit for $f_1-\delta$ is given by the steady-state condition (28) and that for $f_1-\delta>y>0$ it follows from (5) and (12) that $\hat{p}>z>0$, for some positive constants y and z. Thus, eventually, the extraction volume G must clearly fall, with the relative time change of this volume approaching

$$\lim_{t\to\infty}\dot{G}/G=-\left(\lim_{G\to 0}\eta_V(G/n_{\mathrm{H}})\right)^{-1}[\rho/(1-\Omega)]<0. \tag{31}$$

Although there is a continuum of extraction paths compatible with the differential eq. (30), only one path leads to an exhaustion of the resource stock exactly as $t\to\infty$. If a lower path is chosen, then some of the resource will never be used up. If a higher path is chosen, the resource will be exhausted in finite time. Since we know that the possible paths cannot intersect, this is immediately apparent if we recognize that the macroeconomic resource constraint $Q\geqq 0$ can be written in the form

$$\int_0^\infty G(t)\,\mathrm{d}t\leqq Q_0 \tag{32}$$

with $Q_0\equiv n_{\mathrm{R}}q_0$, and that $\int_0^\infty G(t)\,\mathrm{d}t$ can be graphically represented by the area under an extraction path.

Provided that, as is assumed, the upper boundary $\bar{G}$ of the solution space is everywhere above the path exhausting the resource at $t=\infty$, it can easily be shown that the latter is the only one compatible with a market equilibrium. A path exhausting the resource in finite time is not feasible since the properties of the felicity function $V(.)$ ensure in connection with (19) that no finite price exists reducing household demand to zero, which is the level of supply after exhaustion. This argument holds regardless of whether the path in question coincides with the upper boundary over some interval. A path not leading to complete exhaustion as $t\to\infty$, which, as shown above, must always be above the lower boundary, is also infeasible. The reason is that it violates the transversality condition of the resource owners, (4): if we write $\lambda_{\mathrm{R}}(t)=\lambda_{\mathrm{R}}(0)\exp(\int_0^t\hat{\lambda}_{\mathrm{R}}(s)\,\mathrm{d}s)$ where, from manipulating (2), (3), and (5),

$\hat{\lambda}_R(s) = r(s)(1-\tau)/(1-\Omega)$, then this condition becomes

$$\lim_{t\to\infty} \lambda_R(0)\, q(t) \exp\left(\int_0^t r(s)\,[\Omega(1-\tau)/(1-\Omega)]\,\mathrm{d}s\right) = 0. \tag{33}$$

Here the limit of the exponential expression is strictly positive since, from (28) and (29),

$$\lim_{t\to\infty} r(t) = \rho/(1-\tau). \tag{34}$$

Thus $\lim_{t\to\infty} q(t)$ cannot also be strictly positive.

Hence the path which exhausts the resource exactly as $t\to\infty$ is the only one remaining. In the appendix it is shown that the transversality conditions of the resource firm and of the household are satisfied if

$$\lim_{G\to 0} \eta_V(G/n_H) < 1/\Omega. \tag{35}$$

In the light of psychophysical measurements of sensation functions this condition seems to be rather weak.[28] Note that Fechner's Law implies $\eta_V = 1$ [and thus $V(.) = a + b\ln(.)$, $b > 0$] and the currently even more popular law of Stevens (1975) $0 < \eta_V < 1$, $\eta_V = \text{const.}$ [and thus $V(.) = a + b(.)^{1-\eta_V}$, $b > 0$].

4. Intertemporal allocation

Having studied the conditions for an intertemporal market equilibrium we can now move on to the economically more interesting discussion of its properties. For this purpose it is sufficient to consider eqs. (27), (28), and (30).

4.1. Laissez faire

If the government does not levy any taxes we obtain from (28) for the steady-state equilibrium of the normal-commodity sector the golden-utility rule

$$f_1 - \delta = \rho, \tag{36}$$

and from (27) an expression for the slope of the path leading to the steady-state point,

$$\mathrm{d}C/\mathrm{d}K = (C-M)(f_1 - \delta - \rho)/[(f - \delta K - C)\eta_U]. \tag{37}$$

[28]For an overview of these measurements, see Sinn (1982, ch. III A 1.3).

In addition, (30) gives an expression for the relative time change of the resource extraction flow:

$$\hat{G} = -\rho/\eta_V. \tag{38}$$

The laissez faire allocation represents a social optimum, if the individual preferences underlying this result are accepted.[29] This can easily be shown by solving the following optimization problem from the viewpoint of a central planner:

$$\max_{\{C,G\}} \int_0^\infty e^{-\rho t} [U((C-M)/n_H) + V(G/n_H)]\, dt, \tag{39}$$

$$\text{s.t.} \quad K(0)=K_0, \quad \dot{K}=f(K,L)-\delta K - C, \quad Q(0)=Q_0, \quad \dot{Q}=-G,$$
$$K, C-M, Q, G \geqq 0.$$

The problem (39) is obviously separable into the subproblems

$$\max_{\{C\}} \int_0^\infty e^{-\rho t}\, U((C-M)/n_H) dt \quad \text{and} \quad \max_{\{G\}} \int_0^\infty e^{-\rho t}\, V(G/n_H)\, dt.$$

This implies that there is an intertemporal misallocation in a particular sector whenever taxes cause a deviation from the laissez faire path, regardless of whether the other sector is optimized or not.

4.2. Ad valorem consumption taxes

Set $\Omega=\tau=\mu=\varepsilon=0$, $\Theta>0$, or $\Omega=\tau=\mu=\Theta=0$, $\varepsilon>0$, or $\Omega=\tau=\mu=0$, $\Theta>0$, $\varepsilon>0$. Then we find from (27), (28), and (30) in comparison with (36)–(38) that the intertemporal allocation is not changed for either of the two commodities.[30] This reaffirms the allocative efficiency of expenditure taxes as stated by I. and H.W. Fisher (1942) and Kaldor (1957). The reason for this result is that the intertemporal supply behavior of both the normal-commodity firm and the resource firm does not depend on the levels of producer prices as such but only on their time profiles.

An interesting corollary stated by Gray (1914) is that the incidence of an ad valorem tax on resource consumption is completely on resource firms. In

[29] A discussion of the normative implications of various kinds of intertemporal welfare criteria is given by Page (1977a, b).

[30] Note that if one of the two commodities is not taxed, the other should be uniformly taxed over time. This result is due to the separability of the utility function. It is in line with a well-known proposition of Atkinson and Stiglitz (1972) according to which, if one commodity (leisure) cannot be taxed, all other commodities should be uniformly taxed when the utility function is separable with respect to the non-taxable good.

the light of Ricardo's theorem of the impossibility of shifting a tax on pure rent this is not too surprising.[31]

4.3. Unit consumption taxes

Assume that $\Omega=\tau=\varepsilon=\Theta=0$, $\mu>0$, or $\Omega=\tau=\varepsilon=0$, $\Theta>0$, $\mu>0$, where unlike in the previous section, Θ now represents a unit tax on the normal commodity.[32] Then obviously there is no change in (27) and (28) so that the intertemporal allocation of the normal commodity remains optimal. But instead of (38) we obtain from (30)

$$\hat{G}=-[\rho-(f_1-\delta)(\mu/p)]/\eta_V. \tag{40}$$

Eq. (40) indicates a clear change in the resource consumption path. From the properties of the growth path of the normal-good sector we know that the marginal productivity of capital is a function of time which stays positive if it is already positive at the planning date zero. For this realistic case, (40) implies that the 'shrinking rate' of resource extraction is lower than in the laissez faire equilibrium at each point in time. Because $\int_0^\infty G(t)\,dt=Q_0$, this implies an extraction path which initially lies under the laissez faire path, cuts it at some point, and thereafter lies above it. Thus, initially, too much of the resource is conserved for the future. Fig. 3 illustrates these relationships.

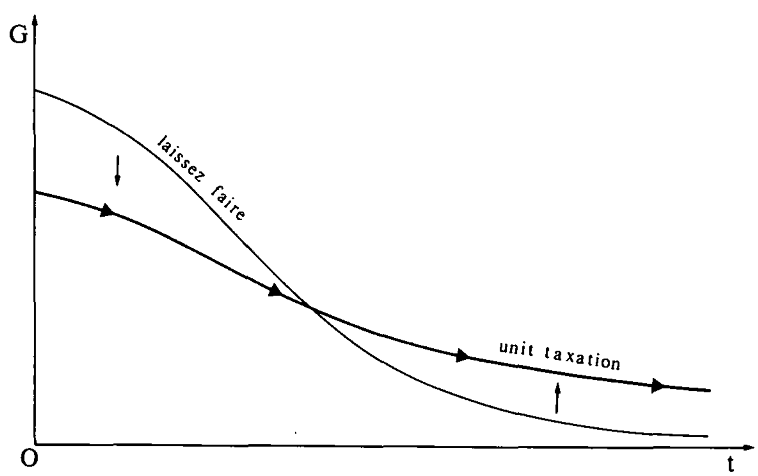

Fig. 3

[31]Note however that the result is not likely to show up in an overlapping-generations model; see Feldstein (1977). As shown by Calvo, Kotlikoff and Rodriguez (1979), the absence of a bequest motive in the overlapping-generations model seems to be responsible for the possibility of shifting the tax.

[32]Recall that the normal commodity was assumed to be the numéraire for the unit taxes.

The effect of a unit tax on resource extraction, which again was correctly predicted by Gray (1914), is easily understood if we interpret this tax as an ad valorem tax with a falling rate.[33] The result that such a tax provides an incentive to shift sales into the future, is quite plausible.

Another implication of (40) is worth noting. Whereas resource extraction under laissez faire is always a falling function of time, (40) shows that for a sufficiently high initial value of $f_1-\delta$, i.e., for a sufficiently low initial capital stock K_0, resource extraction can increase over some initial time span so that households are better off in the near future than in the present. Since resource extraction frequently incurs unit taxes, this result might explain why currently extraction rates are still increasing for most natural resources.

4.4. A general income tax

If we set $\Omega=\mu=0$ but assume $\tau>0$ and $\varepsilon, \Theta \geqq 0$, i.e., a positive income tax and possibly ad valorem taxes on the consumption of the two commodities, then, instead of (36) and (37), (28) and (27) yield

$$(f_1-\delta)(1-\tau)=\rho, \tag{41}$$

and

$$\mathrm{d}C/\mathrm{d}K=(C-M)\,[(f_1-\delta)(1-\tau)-\rho]/[(f-\delta\,K-C)\eta_U], \tag{42}$$

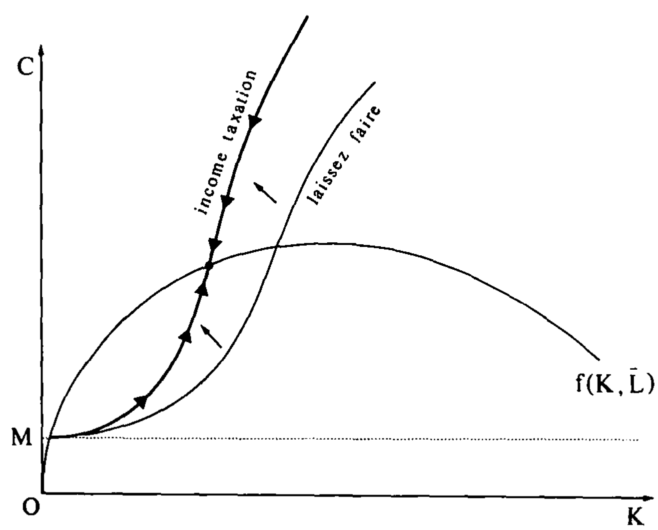

Fig. 4

[33]Note that (5) ensures a steadily rising resource price.

respectively. Eq. (30), however, still implies the steady-state solution (38). Thus only the time path of the consumption of the normal commodity is distorted.

Eq. (41) indicates a violation of the golden-utility rule (36). The reason is that the income tax drives a wedge between the gross and the net rates of interest. The gross rate of interest is equal to the marginal productivity of capital but, in the steady state, the net rate of interest is equal to the given subjective rate of discount ρ. So the steady-state marginal productivity of capital is higher and the capital stock lower than in the laissez faire situation.

From (42) it can be seen that the path leading to this new steady-state point must everywhere lie above the old path. Suppose this were not true, so that the new path intersects or touches the old path at some point. Then at this point the slope of the new path would have to be lower than or equal to that of the old path if $f_1-\delta-\rho<0$ and $f-\delta K-C>0$; and the slope would have to be greater than or equal to this if $f_1-\delta-\rho>0$ and $f-\delta K-C<0$. This, however, is clearly incompatible with (42). Thus the imposition of an income tax will always increase present, and reduce future, consumption.

The previous discussion is summarized in fig. 4, which demonstrates, in a growth setting, the old Fisher/Kaldor argument against an interest income tax.

4.5. A capital-gains tax on the resource stock

If we have $\Omega>0$ and $\Theta,\varepsilon\geqq 0$, but $\mu=\tau=0$, i.e., a capital-gains tax plus possibly ad valorem consumption taxes on the two commodities, then according to (27) and (28) the intertemporal allocation of the normal commodity is still given by the laissez faire eqs. (36) and (37). However, for

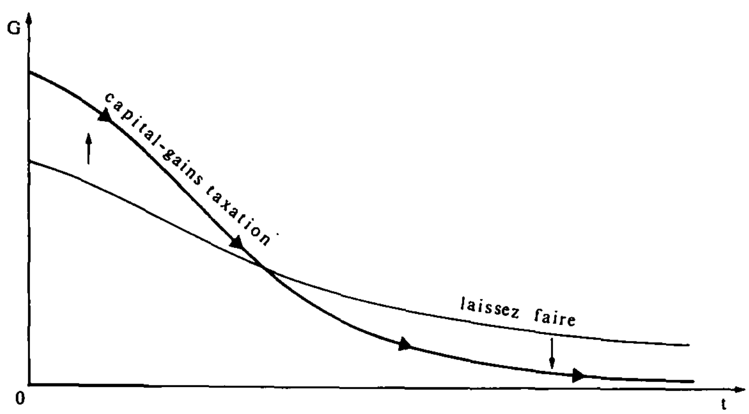

Fig. 5

the time path of resource extraction, (30) yields the condition

$$\hat{G} = -(1/\eta_V)[\rho + (f_1 - \delta)\Omega/(1-\Omega)]. \tag{43}$$

In comparison with the laissez faire formula (38), eq. (43) shows for the realistic case $f_1 - \delta > 0$ that the 'shrinking rate' of resource extraction is higher at each point in time. Because $\int_0^\infty G(t)\,dt = Q_0$ we thus get an extraction path which initially is above and later below the laissez faire path, intersecting the latter only once.[34] This is illustrated in fig. 5.

The effect of the capital-gains tax is opposite to that of a unit tax on the resource. In comparison with the optimal extraction path initially too little of the resource is conserved for the future.

4.6. Income taxation and capital-gains taxation: A second-best problem

The previous analysis shows that the income and the capital-gains tax bring about welfare losses if levied alone. It may be asked, however, whether a capital-gains tax should be introduced as a second-best solution if a given taxation of interest income is regarded as inevitable. The right answer has already implicitly been given in section 3, but, before recalling it, we want to consider two mistakes which could easily be made in attempting to solve this problem.

The first mistake could be made if the ability-to-pay approach to taxation was applied. It could be argued that capital gains increase the personal ability-to-pay of resource owners in the same way as regular interest payments do and should, for the sake of justice, be taxed in the same way. This argument is only partly right for it overlooks the fact that the market mechanism provides the desired justice by equalizing the net rates of return on different assets, regardless of how large the gross rates of return are. In our model this is shown by eqs. (5) and (12), which imply (for $\mu = 0$) that

$$\hat{p}(1-\Omega) = (f_1 - \delta)(1-\tau). \tag{44}$$

Whether $\Omega > 0$ or $\Omega = 0$ — an asset holder who buys shares of a resource firm and another who buys shares of a normal-commodity firm make the same profits when they sell their shares after a given period of time. A capital-gains tax, therefore, cannot reasonably be motivated by the ability-to-pay principle.

Another possible argument in favor of the capital-gains tax is the following. Eq. (44) implies for the welfare maximizing laissez faire case that

[34]Since the consumption path for the normal commodity stays unchanged this means that the introduction of a capital-gains tax leads to an instantaneous fall in the price level and a subsequent rise in its growth rate. A similar result was stated by Timm (1973) for the case of a capital-gains tax on land.

$\hat{p}=f_1-\delta$. If an income tax alone is levied then this equation becomes $\hat{p}=(1-\tau)(f_1-\delta)$, so that for a given capital stock the relative change of the resource price is less than before. Thinking in terms of a partial analytical model with a given time pattern of demand functions, one might be tempted to suppose that the time change in the extraction volume must be smaller than before so that a lower level of present consumption would be necessary to prevent the resource from being exhausted in finite time. Consequently a capital-gains tax at a rate equal to that of the capital-income tax would seem to be necessary in order to ensure that, with $\hat{p}=f_1-\delta$, the extraction path is optimal again. This line of reasoning, however, commits the second mistake.[35]

If we set $\mu=0$, but allow for $\Omega\geqq 0$ and $\tau\geqq 0$ (and possibly $\varepsilon,\Theta\geqq 0$), then, according to (30), the resource extraction path is determined by the rule

$$G=-(1/\eta_V)[\rho+(f_1-\delta)(1-\tau)\Omega/(1-\Omega)], \tag{45}$$

whereas the allocation in the normal-commodity sector is described by (41) and (42). Eq. (45) shows that we cannot hope to reach the laissez faire path with $\Omega=\tau$. With the income tax the distortion, which the resource extraction path shows in comparison with the laissez faire eq. (38), is less than in the case where a resource tax alone is levied [cf. (43)]. But however, the same extraction path as under laissez faire can, despite the income tax, only be reached if there is no capital-gains taxation at all.[36]

The mistake made in the above reasoning can now easily be determined. It is not the supposition that an income tax leads to a lower rate of increase in the resource price. This is of course the case. What is false, however, is the supposition that a change in the price path of the resource requires a change in the path of resource extraction. Since $p=V'(G/n_H)/U'[(C-M)/n_H]$, a change in the consumption path of the normal commodity, or, in other words, a change in the time pattern of resource demand functions could also do the job. Indeed, as shown by eq. (25), which is derived from its microeconomic counterpart (22), $\{C\}$ does change in the desired way. After a fall in the net rate of interest, households plan to distribute the consumption of the normal commodity more evenly over time, reducing $\hat{C}$ just enough to bring about the required change in $\hat{p}$ given the path of resource consumption. The desire to distribute consumption more evenly over time in principle also holds for the natural resource, but, as shown by (23), here the effect of a fall in the net rate of interest is fully compensated by the fall in $\hat{p}$, which by itself gives the incentive of consuming less in the present and more in the future.

[35]Since the original publication of this paper I have seen that the preceding reasoning was put forward by Dasgupta and Heal (1979, pp. 365–366, 368–371). Cf. also Sinn (1981a, p. 187).

[36]This statement would certainly have to be modified for non-separable utility functions. But there is no systematic bias for such a modification. It could either be desirable to have a capital-gains tax or a capital-gains subsidy.

In the previous discussion we find no supporting evidence for the supposition that an existing income tax should be supplemented by a capital-gains tax on the natural resource. If, however, for the sake of symmetry some resource taxation is desired, then our model recommends an ad valorem tax on resource consumption: As was shown in section 4.4, for the case $\tau > 0$, $\varepsilon > 0$, but $\Theta \geqq 0$, $\mu = \Omega = 0$, the resource sector would then still choose the optimal laissez faire path (38).

5. Concluding remarks

The general equilibrium approach of this paper provides a number of clear-cut results concerning the effects of various taxes on the intertemporal market allocation of produced and natural resources. Furthermore, it enables us to evaluate these taxes from a welfare point of view.

The analysis shows that ad valorem taxes on the consumption of one or both of the commodities are allocatively neutral, and hence have no associated welfare losses. However, a unit tax on the extraction of the natural resource, a capital-gains tax on the stock of the natural resource, and a general income tax cause clear welfare losses. In the case of the unit tax on resource extraction, the loss is caused by a distortion towards a *lower* present consumption of the natural resource, while in the case of the capital-gains tax, the loss is caused by a bias towards a *higher* present resource consumption. In the case of the income tax, it results from a distortion towards a *higher* present consumption of the normal commodity.

An important question treated in the paper is whether the capital-gains tax is desirable, given that an income tax already exists. From the viewpoint of the ability-to-pay principle of taxation, the public finance literature has traditionally tended to give a positive answer to this question. Even from the more modern allocative point of view, one could easily come to this conclusion, using a partial equilibrium model of the resource market with a demand function and a gross rate of interest given for each point in time. Yet, approaching the problem in a general equilibrium framework, we do not find any evidence for the usefulness of a second-best taxation of capital gains. Instead our analysis suggests that it would be much better to supplement the income tax by an ad valorem tax on resource consumption. Fortunately, this is how tax laws are typically designed. By defining proceeds from resource extraction as taxable income, they unintentionally impose an ad valorem tax on resource consumption, with a tax rate equal to the income tax rate.[37,38]

[37]Of course this statement is only correct in the absence of extraction costs. Yet, if there are extraction costs then probably a tax on the net revenue from the resource extraction would be desirable from a welfare point of view, so that, once again, present tax laws do the right thing.

[38]If there are percentage depletion allowances, we would still effectively have an ad valorem tax, though at a lower rate.

The most questionable assumption in the present analysis is that the plans of all market agents are compatible in all future periods. Future markets are in fact incomplete. Capital markets exist up to 20–40 years into the future, markets for natural resources up to 10–20 years or so, but most other markets extend over an even shorter period of time. Compatibility of plans, however, does not necessarily require the availability of well-organized markets. Instead of having such markets, it might well be cheaper but still adequate to have informal tâtonnement processes through the trading of information by private agents.[39] For example, the present scientific and public discussion about the natural resource problem can be interpreted as a desire on the part of market agents to exchange information, thereby eliminating deviations in their plans for the future.[40] But, whatever one's view about the degree of approximation, the assumption of a perfectly operating economy has at least the virtue of isolating the allocative effects of taxes. It rules out the possibility of confusion between welfare losses from tax distortions and welfare losses from the imperfection of markets.

There is a number of directions in which the present approach might usefully be pursued. The resource could be treated as a factor of production rather than as a consumption commodity. Technological progress and population growth could be allowed for, the desired employment level could be introduced as a third control variable for households, and the role of inflation in the intertemporal allocation effects of the tax system could be studied. It would be interesting to know how these extensions affect the results.

Appendix

In order to ensure that the market equilibrium we found is compatible with individual optima of all agents this appendix checks the corresponding transversality conditions.

A.1. The producers of the normal commodity

Combining (8) and (11), we can write the transversality condition of the single producer of the normal commodity as

$$\lim_{t\to\infty}\left\{\exp\left(-\int_0^t r(s)(1-\tau)\,\mathrm{d}s\right)(K(t)/n_{\mathrm{N}})\right\}=0. \tag{A.1}$$

[39]Shell and Stiglitz (1967), and Stiglitz (1974) who are rather sceptical about the long-run stability of capitalistic growth, overlook this point entirely.

[40]One could argue that the informal tâtonnement process, though extending further into the future than organized markets, does not include the whole time span up to infinity, as assumed in our model. But this is not important since discounting implies that infinity gets precisely the weight zero.

Because of (34) and $\lim K(t)=K_{\infty}>0$ this condition is clearly satisfied.

A.2. The resource firms

The transversality condition of the resource firm, (33), can be written in the form

$$\lim_{t\to\infty}\left[\lambda_{\mathrm{R}}(0)q_0\exp\left(\int_0^t[\hat{q}(s)+r(s)\,\Omega(1-\tau)/(1-\Omega)]\,\mathrm{d}s\right)\right]=0, \tag{A.2}$$

where $\hat{q}$ is the relative time change of the resource stock q. This condition will be met if for $t\to\infty$ the integral in the exponent approaches $-\infty$, which is the case if

$$\lim_{t\to\infty}\hat{q}(t)<-[\Omega(1-\tau)/(1-\Omega)]\lim_{t\to\infty}r(t),$$

or, after inserting (34), if

$$\lim_{t\to\infty}\hat{q}(t)<-\rho\Omega/(1-\Omega). \tag{A.3}$$

In order to interpret the latter condition note that, because $\dot{Q}=-G$, we have

$$-\dot{G}/G=\dot{G}/\dot{Q}=(\dot{G}/n_{\mathrm{R}})/(\dot{Q}/n_{\mathrm{R}})=\mathrm{d}g^{\mathrm{s}}/\mathrm{d}q\equiv\phi(q), \tag{A.4}$$

where $\phi(q)$ is the slope of the equilibrium extraction path in a (g^{s},q) diagram. Since this path leads to the origin of the diagram it obviously holds that

$$\hat{q}=-g^{\mathrm{s}}/q=-\left(\int_0^q\phi(z)\,\mathrm{d}z\right)\bigg/q,$$

and thus

$$\lim_{t\to\infty}\hat{q}(t)=-\lim_{q\to 0}\left(\int_0^q\phi(z)\,\mathrm{d}z\right)\bigg/q.$$

Note furthermore that

$$\lim_{q\to 0}\left(\int_0^q\phi(z)\,\mathrm{d}z\right)\bigg/q=\lim_{q\to 0}\phi(q),$$

and, because of (A.4) in connection with (31),

$$\lim_{q\to 0}\phi(q) = -\left(\lim_{G\to 0}\eta_V(G/n_{\mathrm{H}})\right)^{-1}(\rho/(1-\Omega)).$$

Thus we can replace the left-hand side of (A.3) by the latter expression and obtain

$$\lim_{G\to 0}\eta_V(G/n_{\mathrm{H}}) < 1/\Omega. \tag{A.5}$$

That this inequality holds is sufficient to satisfy the transversality condition of the resource firm.

A.3. The households

Because of (18) and $1+\Theta = \text{const.} > 0$, the transversality condition of the households, (21), is

$$\lim_{t\to\infty}(\mathrm{e}^{-\rho t}U'[c^{\mathrm{d}}(t)-m]x(t)) = 0. \tag{A.6}$$

Since here

$$\lim U'[c^{\mathrm{d}}(t)-m] = U'[(C_{\infty}/n_{\mathrm{H}})-m] = \text{const.} > 0,$$

(A.6) can be reduced to

$$\lim_{t\to\infty}[\mathrm{e}^{-\rho t}x(t)] = 0. \tag{A.7}$$

Inserting (1) and (7) into (14) we calculate that $x(0)$ is $1/n_{\mathrm{H}}$ of the present value of the sales paths of all resource firms and producers of the normal commodity. Generalizing this result for $x(t)$ we can write (A.7) as

$$\lim_{t\to\infty}\left(\mathrm{e}^{-\rho t}\int_t^{\infty}\exp\left\{-\int_t^{t^*}r(s)(1-\tau)\,\mathrm{d}s\right\}\right.$$
$$\left.\times[p(t^*)G(t^*)/n_{\mathrm{H}}+(1+\Theta)C(t^*)/n_{\mathrm{H}}]\,\mathrm{d}t^*\right) = 0. \tag{A.8}$$

Note that from (12) and (28) it follows that at the steady state

$$\rho = r(1-\tau). \tag{A.9}$$

This ensures that

$$\lim_{t\to\infty}\left(e^{-\rho t}\int_t^\infty \exp\left\{-\int_t^{t^*} r(s)(1-\tau)\,ds\right\}((1+\Theta)\,C(t^*)/n_H)\,dt^*\right)$$

$$=\lim_{t\to\infty}\left(e^{-\rho t}\int_t^\infty e^{-(t^*-t)\rho}((1+\Theta)\,C_\infty/n_H)\,dt^*\right)$$

$$=\lim_{t\to\infty}(e^{-\rho t}(1/\rho)(1+\Theta)C_\infty/n_H)=0.$$

So we can eliminate $(1+\Theta)\,C(t^*)/n_H$ from (A.8). If, in addition, we substitute

$$p(t^*)=p(0)\exp\left\{\int_0^t \hat{p}(s)\,ds\right\}\exp\left\{\int_t^{t^*}\hat{p}(s)\,ds\right\},$$

and

$$G(t^*)=G(0)\exp\left\{\int_0^t \hat{G}(s)\,ds\right\}\exp\left\{\int_t^{t^*}\hat{G}(s)\,ds\right\},$$

and drop the constants $p(0)$, $G(0)$, and n_H, then (A.8) changes to

$$\lim_{t\to\infty}[\alpha(t)\,\beta(t)]=0, \tag{A.10}$$

where

$$\alpha(t)\equiv\exp\left\{\int_0^t[\hat{p}(s)+\hat{G}(s)-\rho]\,ds\right\}, \tag{A.11}$$

and

$$\beta(t)\equiv\int_t^\infty \exp\left\{\int_t^{t^*}[\hat{p}(s)+\hat{G}(s)-r(s)(1-\tau)]\,ds\right\}dt^*. \tag{A.12}$$

From (5), (31), and (A.9) we derive

$$\gamma\equiv\lim_{s\to\infty}[\hat{p}(s)+\hat{G}(s)-r(s)(1-\tau)]=\lim_{s\to\infty}[\hat{p}(s)+\hat{G}(s)-\rho], \tag{A.13}$$

and eventually

$$\gamma=\rho\left[(1-\Omega)^{-1}\left(1-\left(\lim_{G\to 0}\eta_V(G/n_H)\right)^{-1}\right)-1\right].$$

With the aid of this expression it can easily be verified that $\gamma<0$ if, and only if, (A.5) is satisfied. Assuming the latter, we get

$$\lim_{t\to\infty} \beta(t) = -1/\gamma > 0.$$

So β can be eliminated from (A.10), and the transversality condition becomes

$$\lim_{t\to\infty} \alpha(t) = 0.$$

With respect to (A.11) and (A.13) it is clear that this condition is met if $\gamma<0$, i.e., again if (A.5) holds.

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