Capital Income Taxation and Resource Allocation

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Appendix

APPENDIX A

It was assumed with (3.33) that

$$K_0 P_K > D_{f0}, \tag{A.1}$$

which, according to (6.2) is equivalent to postulating a strictly positive initial market value of shares:

$$M(0) > 0. \tag{A.2}$$

This appendix shows that this condition together with the constraints on the firm's financial decisions that were assumed in Chapter 4 imply that the market value of equity and the flow of retainable net profits will always be strictly positive when the firm's real net investment is non-negative ($I \geq 0$):

$$M(t) > 0 \quad \forall t \geq 0, \tag{A.3}$$

$$\Pi_n^*(t) > 0 \quad \forall t \geq 0. \tag{A.4}$$

Because of the privilege of limited liability, the first of these two conditions is a self-evident requirement for a sensible solution to the decision problem of a corporate firm. The second condition ensures the existence of a non-degenerate solution space for the firm's financial decisions. It implies the weaker condition $\Pi_n^* \geq 0$ that was assumed with (4.10).

Because of Euler's theorem and the fact that, according to (3.38), the wage rate equals the marginal product of labor, it follows from (4.6) that

$$\begin{aligned}\Pi_n^* &= \theta_r[(f_K - \delta - \tau_k)K - rD_f] + \tau_r r(\alpha_2 \dot{K} - \alpha_3 D_f) \\ &= r\frac{\theta_r}{\theta_d}\left[\theta_d(1-\alpha_3) + \frac{\theta_d}{\theta_r}\alpha_3\right][KX - D_f],\end{aligned} \tag{A.5}$$

where

$$X \equiv \frac{\theta_r \dfrac{f_K - \delta - \tau_k}{r} + \tau_r \alpha_2}{\theta_r(1-\alpha_3) + \alpha_3}. \tag{A.6}$$

Using (5.6), it can easily be shown that $X = P_K$ where P_K is the general expression for the effective price of capital as given in (6.3). Thus a comparison between (A.5) and (6.2) reveals that

$$\Pi_n^* = r\frac{\theta_r}{\theta_d}M. \tag{A.7}$$

As $f_K - \delta - \tau_k > 0$ holds by assumption [cf. Chapter 3.1.1], as (5.6) and (8.39) indicate that $r = (f_K - \delta - \tau_k)/(\theta_p \tilde{P}_K)$, and as it was shown with (8.55) and (8.57) that $\tilde{P}_K > 0$, the market rate of interest r is always strictly positive. Thus (A.3) and (A.4) are clearly satisfied if

$$\dot{M}(t) \geq 0 \quad \forall t \geq 0 \tag{A.8}$$

which, because of (6.2), is equivalent to the condition

$$\dot{D}_f \leq P_K \dot{K}. \tag{A.9}$$

As $\dot{D}_f \equiv S_f$ and $\dot{K} \equiv I$, it follows from the financial constraints (4.1) and (4.2), or equivalently from (4.7) and (4.9), that

$$\dot{D}_f \leq (1 - \alpha_1 \tau_r)\dot{K}. \tag{A.10}$$

For $I \equiv \dot{K} \geq 0$, this condition obviously implies that (A.9) is satisfied if

$$P_K \geq 1 - \alpha_1 \tau_r. \tag{A.11}$$

As (4.8) and (4.9) indicate that

$$\varepsilon^* + \sigma^* = 1 - \alpha_1 \tau_r, \quad \varepsilon^* \geq 0, \tag{A.12}$$

it follows from (6.3) for the case of deductible debt interest that

$$P_K = \frac{\theta_p}{\max(\theta_d^*, \theta_r^*)}\varepsilon^* + \sigma^* \quad (\alpha_3 = 0), \tag{A.13}$$

and for the case of non-deductible debt interest that

$$P_K = \frac{\theta_p \theta_r}{\max(\theta_d^*, \theta_r^*)}\varepsilon^* + \sigma^* \quad (\alpha_3 = 1). \tag{A.14}$$

Together with (A.12), both of the two latter expressions imply (A.11). Equation (A.13) does so because $\theta_p \geq \max(\theta_d^*, \theta_r^*)$ results from the two basic assumptions (3.14) and (3.15), and Equation (A.14) does so because, to ensure the existence of an optimal financial decision of the firm, it had been assumed already with (4.25) that $\theta_p \theta_r = \theta_r^*$ (as $\theta_r^* \equiv \theta_r \theta_c$ and $\theta_p = \theta_c$) and that $\theta_r^* > \theta_d^*$. This completes the proof.

APPENDIX B

Assuming that debt interest is deductible ($\alpha_2 = \alpha_3 = 0$), that retentions dominate new share issues ($\theta_r^* > \theta_d^*$), and that the economy converges to a steady state it is shown that, for arbitrary ε^* in the range $0 < \varepsilon^* \leq 1 - \alpha_1 \tau_r$, a situation with

$$\Pi_n^*(t) > \varepsilon^* I(t), \tag{B.1}$$

(1) will always prevail ($t \leq 0$) if the firm's initial stock of debt is small enough,
(2) will prevail after some finite period of time ($t \geq t^* > 0$) even if $\Pi_n^*(0) < \varepsilon^* I(0)$, and
(3) is self-perpetuating in the neighborhood of the steady state.

In the light of Appendix A, all three statements trivially hold true in an economy that converges to a stationary state with a zero growth rate and hence a zero level of net investment. The analysis is therefore confined to the case where the economy converges to a steady state with a strictly positive growth rate, i.e. a situation where I is strictly positive and grows in proportion to the capital stock.

The proofs for statements (1), (2), and (3) use steady-state properties of the growth model set up in Chapter 8. One property needed is that the steady-state rate of time preference is above the steady-state growth rate:

$$\gamma^\infty > \lim_{t \to \infty} \hat{K}(t). \tag{B.2}$$

This property is stated with (8.49) and (8.51), and it is proved in Appendix C. (It is a fundamental existence requirement that is not limited to the particular assumptions on the household's utility function made in this book, nor to special assumptions on the kind of tax system that applies). The second property needed is simply that, as stated in (8.50), the net rate of interest equals the rate of time preference which implies that

$$\lim_{t \to \infty} \theta_p r(t) = \gamma^\infty. \tag{B.3}$$

Using (A.5) with $\alpha_2 = \alpha_3 = 0$, (B.1) can also be written as

$$\theta_r[(f_K - \delta - \tau_k)K - rD_f] - \varepsilon^* I \geq 0 \tag{B.4}$$

or, dividing by $K\theta_r$ and noting that $\sigma \equiv D_f/K$ and $\hat{K} \equiv I/K$, as

$$r\left(\frac{f_K - \delta - \tau_k}{r} - \sigma\right) - \frac{\varepsilon^*}{\theta_r}\hat{K} > 0. \tag{B.5}$$

It follows from (A.12) and (5.6) that

$$\frac{f_K - \delta - \tau_k}{r} = \frac{\theta_p}{\theta_r^*}\varepsilon^* + \sigma^*. \tag{B.6}$$

Inserting this into (B.5) gives

$$\frac{\varepsilon^*}{\theta_r}\left(r\frac{\theta_p}{\theta_c} - \hat{K}\right) + r(\sigma^* - \sigma) > 0. \tag{B.7}$$

As $\mathrm{d}S_f/\mathrm{d}I \leq \sigma^*$ for $\theta_p = \theta_r^*$ (Type 4) and $\mathrm{d}S_f/\mathrm{d}I = \sigma^*$ for $\theta_p > \theta_r^*$ (Type 1), it holds that

$$\lim_{t \to \infty} \sigma(t) \leq \sigma^*. \tag{B.8}$$

Together with $\theta_c \leq 1$ [from 3.14)], (B.2), and (B.3), this implies that (B.7) will permanently be satisfied at, or in the neighborhood of, a steady state as contended with (3). Note that this property holds regardless of the economy's growth rate and regardless of the size of the minimum marginal equity–asset ratio.

Statement (2) can be confirmed along similar lines. Assume that, contrary to (B.1),

$$\varepsilon^* I \geq \Pi_n^* \quad \forall t \geq 0. \tag{B.9}$$

In this case, firms are forced to use an inferior source of equity finance at the margin so that

$$\frac{f_K - \delta - \tau_k}{r} = \lambda\frac{\theta_p}{\theta_r^*}\varepsilon^* + \sigma^*, \quad \lambda \geq 1. \tag{B.10}$$

Inequality (B.9) is therefore equivalent to

$$\frac{\varepsilon^*}{\theta_r}\left(r\frac{\lambda\theta_p}{\theta_c} - \hat{K}\right) + r(\sigma^* - \sigma) < 0 \quad \forall t \geq 0. \tag{B.11}$$

As $\mathrm{d}S_f/\mathrm{d}I = \sigma^*$ (and not $\leq \sigma^*$) when $\theta_p \geq \theta_r^* > \theta_d^*$ and $\Pi_n^d = 0$, we have

$$\lim_{t \to \infty} \sigma(t) = \sigma^*, \tag{B.12}$$

and so, in order for (B.10) to be true, it is necessary that

$$r\frac{\lambda\theta_p}{\theta_c} \leq \hat{K} \quad \forall t \geq 0. \tag{B.13}$$

As $\lambda/\theta_c \geq 1$ this contradicts (B.2) and (B.3). Thus, although $\varepsilon^* I \geq \Pi_n^*$ might be possible in the short run, it is definitely impossible in the long run when the economy has converged to the neighborhood of a steady state, as stated under (2).

To see that (1) is true assume for a moment that the lower constraint to the firm's financial solution space is absent or, in other words, that negative dividend payments (with algebraic application of the dividend tax formulas) are possible. Under these circumstances, (B.6) describes the firm's investment policy regardless of whether or not $\varepsilon^* I \leq \Pi_n^*$, so that the sign of $\Pi_n^* - \varepsilon^* I$ is definitely the same as that of the lefthand side of (B.7). Suppose, with some given value of the initial stock of debt, D_{f0}^*, this sign is negative for some time span. Since

$$\sigma(t) \leq [D_f(0) + \sigma^* \int_0^t I(u)\mathrm{d}u]/K(t), \tag{B.14}$$

and since the steady-state properties derived ensure that this time span is finite, it is then always possible to choose a value of $D_f(0)$ sufficiently far below D_{f0}^* such that the sign will be positive for all points in time and the firm will permanently be able to pay dividends.

APPENDIX C

Assuming that

$$(n+g)(\theta_c \tilde{P}_K - 1) + \tau_k \geq 0 \tag{C.1}$$

and

$$\gamma^\infty \equiv \rho + \eta g > n + g, \tag{C.2}$$

this appendix shows that:

(1) those paths in (c, k) space that are compatible with the differential equations (8.36) and (8.45), but do not lead to the steady-state point defined by (8.46) and (8.47) cannot represent an intertemporal market equilibrium, and that
(2) the paths leading to this steady-state point satisfy the transversality conditions (7.36), (3.37), and (8.20) of the individual decision problems of the representative household and the representative firm.

The proofs are given for the general model. They include the four classes of tax system defined in Chapter 3.1.4 as well as the laissez-faire model from Chapter 2. The analysis refers to Figure 8.1 from Chapter 8. In the discussion, the term "stable branch" will be used to characterize the market equilibrium path as the equilibrium property is yet to be derived.

C.1. The Transversality Conditions

To prepare for the analysis, this section first transforms the transversality conditions of the model agents' decision problems into expressions that are somewhat easier to manage.

Because of the constancy of the shadow prices λ_K and λ_D that follows from (5.5) and (4.13) the firm's transversality conditions (3.36) and (3.37) can be written in the form

$$\lim_{t\to\infty}\left[\hat{K}(t)-\frac{\theta_p}{\theta_c}r(t)\right]<0 \tag{C.3}$$

and

$$\lim_{t\to\infty}\left[\hat{D}_f(t)-\frac{\theta_p}{\theta_c}r(t)\right]<0. \tag{C.4}$$

The household's transversality condition (8.20) refers to the growth rate of wealth ($\hat{V}$), but implicitly it characterizes a relationship between the growth rates of consumption ($\hat{C}$) and the net-of-tax market rate of interest ($\theta_p r$). To see this, note that, at each point in time t, the wealth of the representative household equals the present value of the future flow of consumption evaluated at its gross price including the value-added tax:

$$V(t)=\int_t^\infty C(u)(1+\tau_v)\left[\exp\int_t^u -\theta_p r(s)\,ds\right]du. \tag{C.5}$$

This equation follows by integrating Equation (8.10), but it is also possible to derive it directly from (8.4). Because of $\lim_{t\to\infty}k(t)=$ constant, a path in the (c,k) diagram that leads to a steady-state point with $c>0$ is characterized by

$$\lim_{t\to\infty} r(t)=\text{constant}, \tag{C.6}$$

and, because of $\lim_{t\to\infty}c(t)=$ constant, by

$$\lim_{t\to\infty}\hat{C}(t)=n+g. \tag{C.7}$$

Together, these two equations imply that

$$\lim_{t\to\infty}\hat{V}(t)=n+g, \tag{C.8}$$

and therefore the transversality condition (8.20) becomes

$$\lim_{t\to\infty} r(t)\theta_p>n+g. \tag{C.9}$$

If a path is to be evaluated that leads to a steady-state point with $c>0$, Condition (C.9) is stronger than Conditions (A.3) and (A.4). On the one hand, the financial constraint

$$\dot{D}_f\equiv S_f\le(1-\alpha_1\tau_r)I,\quad I\equiv\dot{K},\quad 0\le\alpha_1\tau_r<1, \tag{C.10}$$

from (4.1) and (4.2) ensures that (C.3) implies (C.4). On the other hand,

$$\lim_{t\to\infty}\hat{K}(t)=n+g, \tag{C.11}$$

implies (C.3) in conjunction with $\theta_c \leq 1$ and (C.9). To show that all transversality conditions are satisfied it is therefore necessary and sufficient to show that (C.9) is valid.

C.2. Paths that Diverge from the Stable Branch

As can be seen from (8.36) and (8.45), paths below the stable branch must all lead to the point with coordinates $(0, k^*)$, where the $(\dot{k} = 0)$ curve enters the abscissa to the right of its maximum. Since the paths have the property (C.11), satisfying the transversality condition (C.3) requires

$$\lim_{t \to \infty} r(t) \frac{\theta_p}{\theta_c} > n + g. \tag{C.12}$$

Using (3.39), this condition can be transformed to

$$\frac{\varphi'(k^*) - \delta - \tau_k}{\tilde{P}_K} > \theta_c(n + g), \tag{C.13}$$

or, equivalently, to

$$\varphi'(k^*) - \delta - (n + g) > (n + g)(\theta_c \tilde{P}_K - 1) + \tau_k. \tag{C.14}$$

As k^* is located at the righthand side of the maximum of the $(\dot{k} = 0)$ curve it holds that $\varphi'(k^*) - \delta - (n + g) < 0$. However, (C.1) indicates that $(n + g)(\theta_c \tilde{P}_K - 1) + \tau_k \geq 0$. Obviously, therefore, the transversality condition is not met, and paths below the stable branch cannot represent a market equilibrium.

Paths above the stable branch are not feasible as they involve negative net investment and an exhaustion of the capital stock with infinite factor prices in finite time. Such paths therefore cannot represent a market equilibrium either.

C.3. The Stable Branch

Among all conceivable paths, only the stable branch remains. In Figure 8.1, it leads to the point (c^∞, k^∞) with $c^\infty > 0$, $k^\infty > 0$. To check whether this point satisfies the three transversality conditions of the individual decision problems, it suffices to consider (C.9). Because of (8.38), (8.44), and the fact that the steady state is characterized by $\dot{c} = 0$, it holds that

$$\gamma^\infty = \lim_{t \to \infty} \mathrm{r(t)}\theta_p. \tag{C.15}$$

In conjunction with (C.9) this equation indicates that all transversality conditions are met if, and only if, $\gamma^{\infty} > n+g$, as was assumed with (C.2). This completes the proof of statements (1) and (2) and shows that the stable branch is the path that represents the intertemporal general equilibrium in a competitive economy with taxation.

APPENDIX D

This appendix derives expressions for the change in the market value of equity that results from using accelerated tax depreciation methods given the time path of real investment and given the time path of the sum of interest and depreciation deductions. The analysis is carried out separately for the different types of tax system classified in Figure 4.2.

Let $L(t)$ denote the stock of hidden reserves (i.e., the difference between the market value of assets and their book value) and $\Delta X(t)$ a perturbation of the time path of a variable X. Consider a perturbation $\{\Delta L\}_0^T$, $0 < T \leq \infty$, of the time path of hidden reserves that satisfies the conditions

$$\begin{cases} \Delta\dot{L}(t) > 0 & \text{for} \quad 0 < t < t^*, \\ \Delta\dot{L}(t) < 0 & \text{for} \quad t^* < t < T, \end{cases} \tag{D.1}$$

and

$$\int_0^T \Delta\dot{L}(t)\,\mathrm{d}t = 0. \tag{D.2}$$

It is assumed that the variation in $L(t)$ is compensated by a variation in deductible debt interest rD_f such that

$$r(t)\Delta D_f(t) = -\Delta\dot{L}(t) \quad \forall t \geq 0. \tag{D.3}$$

The time paths of $r(t)$ and $\Delta\dot{L}(t)$ are chosen such that $\Delta D_f(t)$ is differentiable for all $t \geq 0$. As the time derivative $\Delta\dot{L}(t)$ is the change in the flow of tax depreciation at point in time t, Conditions (D.1) define an acceleration of tax depreciation. Condition (D.2) ensures that the variation implies a change only in the time pattern of depreciation, not a change in its volume. Condition (D.3) implies that the firm adjusts its debt policy so that the time path of the corporate tax base is unaffected by the choice of accelerated depreciation. In anticipation of the result derived in the subsequent analysis of Chapter 5.2, it is implicitly assumed with Condition (D.3) that a *permanent* rivalry between rD_f and $\dot{\Delta}L$ – even for $t > t^*$ when $\Delta L < 0$ – is possible.

The perturbations will be evaluated through the change in the market value of equity at $t = 0$ which they induce. According to (3.24), this change is generally given by

$$\Delta M(0) = \int_0^T \left[\frac{\Delta \Pi_n^d(t)}{\theta_c} - \Delta Q(t) \right] \left[\exp \int_0^t -\frac{\theta_p}{\theta_c} r(s)\,ds \right] dt. \tag{D.4}$$

If $\Delta M(0) > 0$, the firm does have an incentive to make use of accelerated depreciation schemes even when it has to reduce its volume of debt finance in exchange.

Type 1 $(\theta_d^* < \theta_r^* < \theta_p)$

As shown in Chapter 4, it holds with this type that $Q = 0$. Thus, only $\Delta\Pi_n^d$ matters for calculating $\Delta M(0)$. Consider first the potential change in internal funds available for financing investment projects. This change consists of a change in the tax savings from accelerated depreciation, $\tau_r \dot{\Delta}L(t)$, and a change in the funds obtained through new issues of debt, $\Delta S_f(t)$, minus a change in net interest costs $\theta_r r(t) \Delta D_f(t)$. "Grossing up" these items by dividing by the corporate tax factor for retained profits and multiplying with the combined tax factor for dividends, the following is obtained:

$$\Delta \Pi_n^d(t) = \frac{\theta_d^*}{\theta_r} \tau_r \dot{\Delta}L(t) + \frac{\theta_d^*}{\theta_r} \Delta S_f(t) - \theta_d^* r(t) \Delta D_f(t). \tag{D.5}$$

If (D.3) is used, $\Delta S_f \equiv \dot{\Delta}D_f(t)$ is noted, and (D.5) is inserted into (D.4), then

$$\Delta M(0) = A + \frac{\theta_d^*}{\theta_c \theta_r^*}(\theta_r^* + \tau_r \theta_c - \theta_p)B, \tag{D.6}$$

with

$$A \equiv \frac{\theta_d^*}{\theta_r^*} \int_0^T \left[\dot{\Delta}D_f(t) - \Delta D_f(t) \frac{\theta_p}{\theta_c} r(t) \right] \left[\exp \int_0^t -\frac{\theta_p}{\theta_c} r(s)\,ds \right] dt \tag{D.7}$$

and

$$B \equiv \int_0^T \dot{\Delta}L(t) \left[\exp \int_0^t -\frac{\theta_p}{\theta_c} r(s)\,ds \right] dt > 0, \tag{D.8}$$

where the inequality sign follows from (D.1) and (D.2) under the assumption $r(t) > 0 \, \forall t \geq 0$. The solution of (D.7) is

$$A = \frac{\theta_d^*}{\theta_r^*}\left[\Delta D_f(t) \exp \int_0^t -\frac{\theta_p}{\theta_c} r(s)\,ds \right]_{t=0}^{t=T} = 0, \tag{D.9}$$

where the differentiability assumption on $\Delta D_f(t)$ is used and it is assumed that

$$\lim_{t\to\infty}\left[\Delta D_f(t) \exp \int_0^t -\frac{\theta_p}{\theta_c} r(s)\,ds \right] = 0 \quad \text{if } T = \infty. \tag{D.10}$$

It follows from (D.6) and (D.9) that

$$\Delta M(0) = (\tau_p - \tau_c)\frac{\theta_d^*}{\theta_c \theta_r^*} B. \tag{D.11}$$

As $B > 0$, $\Delta M(0) > 0$ if, and only if, $\tau_p > \tau_c$.

Type 3 ($\theta_r^* < \theta_d^* < \theta_p$)

With Type 3, the firm distributes all profits and chooses new issues of shares (Q) as the source of equity finance. Thus, similarly to (D.5),

$$\Delta \Pi_n^d(t) = -\theta_d^* r(t) \Delta D_f(t). \tag{D.12}$$

However, unlike before, variations in tax savings through accelerated depreciation, $\tau_r \Delta \dot{L}$, and variations in newly issued debt, ΔS_f, will no longer replace retained profits and carry over into changes in dividends. Instead, they bring about one-to-one changes in new issues of shares:

$$\Delta Q(t) = -\Delta S_f(t) - \tau_r \Delta \dot{L}(t). \tag{D.13}$$

Inserting (D.12) and (D.13) into (D.4), instead of (D.6)

$$\Delta M(0) = \frac{\theta_r^*}{\theta_d^*} A + \frac{1}{\theta_c}(\theta_d^* + \tau_r \theta_c - \theta_p) B, \tag{D.14}$$

is obtained, where A and B are defined as in (D.7) and (D.8). The analogous expression to (D.11) that follows from (D.14) and (D.9) is

$$\Delta M(0) = (\tau_p - \tau_c + \theta_d^* - \theta_r^*)\frac{1}{\theta_c} B. \tag{D.15}$$

As Type 3 is characterized by $\theta_d^* > \theta_r^*$ and $B > 0$ it holds that $\Delta M(0) > 0$ when $\tau_p \geq \tau_c$.

Other Types

Type 2 is an intermediate case between 1 and 3 which is characterized by

$\theta_d^* = \theta_r^* < \theta_p$. Since, with this type, new issues of shares are equivalent to retained profits, it is implicitly characterized through both the analyses for Type 1 and Type 3, and indeed Equations (D.11) and (D.15) coincide for it. Clearly, $\Delta M(0) > 0$ if, and only if, $\tau_p > \tau_c$.

For Types 4–6, the upper boundary of the solution space of the firm is not binding and so the firm is indifferent between debt and retentions (Type 4); debt, retentions, and new issues of shares (Type 5); or debt and new issues of shares (Type 6). If the firm nevertheless wishes to maximize its debt financing until deductibility of debt interest and accelerated depreciation become rival possibilities then the above analyses are perfectly valid, where Type 4 can be subsumed under Type 1, Type 5 under Type 2, and Type 6 under Type 3. Thus, with these types too, the firm prefers to make use of accelerated depreciation allowances rather than debt interest deduction provided capital gains are taxed less heavily than interest income on the personal level.

APPENDIX E

This appendix derives an expression for the relative magnitude of the "Harberger triangle" in the Cobb–Douglas case,

$$\xi \equiv \frac{Y_0 - Y_\tau}{Y_0}, \tag{E.1}$$

where Y_0 is the level of aggregate output, net of depreciation, in the absence of taxes and Y_τ the respective level in the case with taxation. It is assumed that the capital market equilibrium has the general form

$$\frac{\partial f}{\partial K^Y} - \delta = \frac{1}{\chi}\left[\frac{\partial f}{\partial K^X} - \delta\right], \tag{E.2}$$

where χ is some strictly positive constant whose magnitude indicates the comparative tax discrimination of a marginal investment in Sector X relative to the discrimination of a marginal investment in Sector Y.

With the Cobb–Douglas specification

$$f(K^i, L^i) - \delta K^i = a(K^i)^{1-\beta}(L^i)^\beta, \quad i = X, Y, \tag{E.3}$$

it holds that

$$\frac{\partial f}{\partial K^i} - \delta = a(1-\beta)\left[\frac{L^i}{K^i}\right]^\beta. \tag{E.4}$$

Let $\kappa^i \equiv K^i/K$ denote the post-tax endowment share of capital in Sector i and $\lambda^i \equiv L^i/L$ the post-tax *and* pre-tax endowment share of labor in Sector i, where $i = X$, Y and

$$\kappa^X + \kappa^Y = \lambda^X + \lambda^Y = 1. \tag{E.5}$$

It follows from (E.2), (E.4), and (E.5) that

$$\kappa^X = \frac{\lambda^X}{\lambda^X + \chi^{1/\beta}\lambda^Y}. \tag{E.6}$$

Moreover, (E.1), (E.3), and (E.5) imply

$$\zeta = 1 - (\kappa^X)^{1-\beta}(\lambda^X)^\beta - (1-\kappa^X)^{1-\beta}(\lambda^Y)^\beta. \tag{E.7}$$

After inserting (E.6) into this expression and carrying out a number of straightforward manipulations,

$$\xi = 1 - \frac{\lambda^X + \chi^{(1-\beta)/\beta}\lambda^Y}{(\lambda^X + \chi^{1/\beta}\lambda^Y)^{1-\beta}} \tag{E.8}$$

is obtained.