

Capital Income Taxation and Resource Allocation

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North Holland: Amsterdam, New York, Oxford and Tokio 1987

Chapter 1: Introduction to the Theory of Intertemporal Allocation
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Chapter 1

INTRODUCTION TO THE THEORY OF INTERTEMPORAL ALLOCATION

The first two chapters of this book are preparatory to the analysis of tax effects that will be carried out in the later chapters. This chapter presents a simple basic model of intertemporal allocation that is essential for reading this book. The reader who is familiar with Fisher's theory can skip this chapter.

1.1. Preliminary Remarks

The well established *static* theory of taxation can provide a useful focal point for formulating its *dynamic* counterpart. The static theory makes use of the microeconomic general equilibrium model. There are profit-maximizing firms and utility-maximizing households that exchange goods and factors. A flexible price system takes care of the compatibility of all exchange plans. With perfect markets, the laissez-faire allocation of private commodities is Pareto optimal. However, taxes, levied for financing public goods or in order to redistribute income, will usually distort the allocation. They drive wedges between the marginal rates of commodity substitution and transformation and induce behavior changes on the part of private agents that imply losses in utility or profit (excess burden) in addition to the direct financial burden of taxation. The goal of the static allocation theory of taxation is to find tax systems that minimize such losses. It is tempting to construct the dynamic theory of taxation on similar lines. This, at any rate, is the route taken here.

For didactic purposes, the analysis starts with Fisher's (1907, 1930) theory of intertemporal equilibrium. This theory, which was brought back to the attention of the economic discipline by Hirshleifer (1958, 1970), is the

dynamic counterpart of the static theory of competitive equilibrium.¹ As will be shown below, Fisher's theory needs certain amendments if it is to serve as a basis for a dynamic theory of taxation. Even as it stands, however, it yields some fundamental insights into the essentials of intertemporal allocation.²

1.2. Intertemporal Allocation without a Capital Market

As is frequently the case, it is useful here to start with a Robinson Crusoe economy where there are no markets. Crusoe plans for two years. Nothing edible grows on his island but fortunately he could bring wheat of quantity Y from the ship. He can eat the wheat in the first year or he can use it as seed for the second year's wheat harvest. If Crusoe consumes the quantity C_1 , the amount of wheat remaining for sowing is

$$K = Y - C_1, \quad (1.1)$$

and, according to the production function $f(K)$, $f(0) = 0$, $f' > 0$, $f'' < 0$, the quantity of wheat

$$C_2 = f(K) \quad (1.2)$$

is available for second-period consumption. Crusoe's preferences can be described with the utility function $U(C_1, C_2)$ that satisfies the usual neo-classical assumptions (strict quasi-concavity, $U_1 > 0$, $U_2 > 0$). His problem therefore is

$$\begin{aligned} &\max_{C_1} U(C_1, C_2) \\ &\text{s.t. (1.1), (1.2), } 0 \leq C_1 \leq Y. \end{aligned} \quad (1.3)$$

Under the assumption of an interior solution, the equation

$$U_1(C_1, C_2)/U_2(C_1, C_2) = f'(K), \quad (1.4)$$

¹Walras (1874), too, includes the problem of intertemporal allocation explicitly in his equilibrium model. Fisher's merit is, that he concentrates on one aspect of Walras' general mathematical approach and subjects this aspect to a problem-oriented economic analysis. The analogies between the static and the dynamic allocation problem are nicely illustrated by Malinvaud (1961).

²No attempt is made here to present a fully authentic outline of Fisher's theory. The aim instead is to provide an introduction into the problem of intertemporal allocation with the aid of his ideas. Fisher avoids an explicit use of the production function described in (1.2). Instead, he prefers to use the more general but also more abstract assumption of given transformation possibilities between the consumption levels of both periods.

is a necessary condition for an optimum. Because of $U_1/U_2 = -dC_2/dC_1|_U$ this condition says that the absolute marginal rate of substitution of C_2 for C_1 equals the absolute marginal rate of transformation of C_1 into C_2 where the marginal rate of transformation in this intertemporal context can also be identified with the gross marginal product of capital, i.e. of the seed.

Instead of the marginal rate of substitution, intertemporal models usually utilize the so-called subjective rate of time preference. The subjective rate of time preference γ is that percentage by which the increase of C_2 has to exceed unity in order to just compensate the reduction in C_1 by one unit from the viewpoint of the household:

$$\gamma(C_1, C_2) \equiv -\frac{dC_2}{dC_1}\bigg|_{U(C_1, C_2)} - 1 = \frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} - 1. \quad (1.5)$$

Equation (1.4) can therefore also be interpreted so that an optimal decision is characterized by equality between the rate of time preference and the net marginal product of capital:

$$\gamma(C_1, C_2) = f'(K) - 1. \quad (1.6)$$

Figure 1.1 illustrates the optimization problem. On the transformation curve $f(K)$, Robinson Crusoe chooses the optimal point R, which is the point of tangency between the indifference curve and the transformation

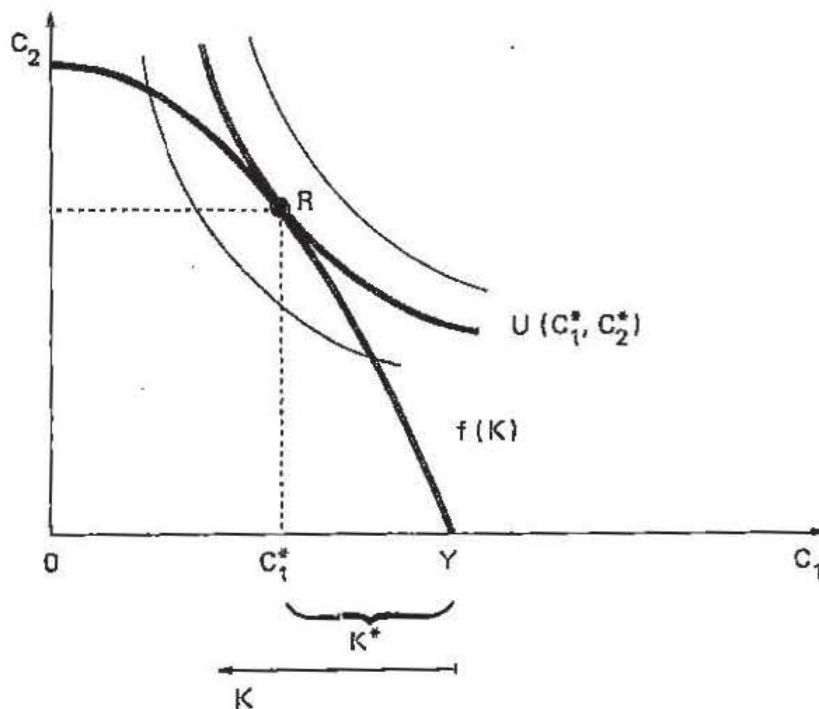


Figure 1.1. The intertemporal optimization problem without a capital market.

curve. The quantities marked with an asterisk are results of the optimization process.

1.3. The Role of the Capital Market

In contrast to the previous assumption, this section assumes the existence of a capital market. There are many neighboring islands with one castaway each. These castaways are in a similar position to Robinson Crusoe, but it is assumed that they have different initial endowments of wheat and different preferences. Thus they are engaged in intertemporal contracts where wheat is borrowed and lent at the rate of interest r . If Crusoe borrows to the extent D , (1.1) and (1.2) change to

$$K = Y - C_1 + D \quad (1.7)$$

and

$$C_2 = f(K) - (1 + r)D. \quad (1.8)$$

His maximization problem therefore is

$$\max_{C_1, D} U(C_1, C_2)$$

$$\text{s.t. (1.7), (1.8), } C_1 \geq 0, C_2 \geq 0.$$

It has been enriched by another control variable and has to satisfy other constraints. The optimality conditions for the case of an interior solution are now

$$f'(K) - 1 = r \quad (1.10)$$

and

$$\gamma(C_1, C_2) = r; \quad (1.11)$$

that is, the net marginal product of capital and the rate of time preference both have to equal the market rate of interest r that in Crusoe's calculations is assumed exogenous.

The optimization problem is illustrated in Figure 1.2. The quantities marked with an asterisk are the result of optimization. Without a capital market, the opportunity locus available to Crusoe was given by the transformation curve $f(K)$. With a capital market, this opportunity locus grows to the size of the shaded area. The reason is that a market transformation line with a constant slope $-(1 + r)$ passes through each

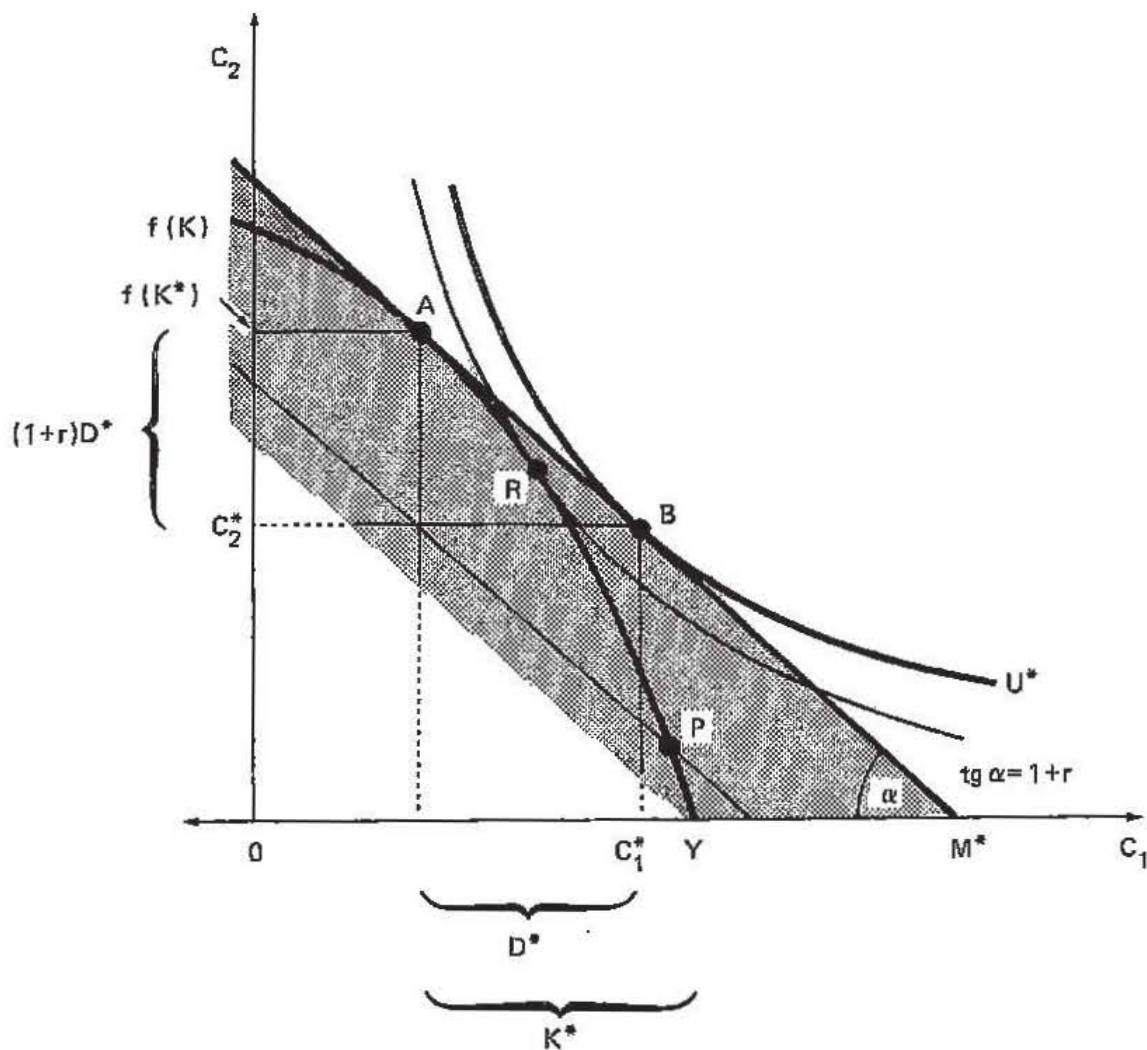


Figure 1.2. The separation of optimal consumption and investment decisions.

point of the technological transformation curve $f(K)$. If Crusoe chooses the production point P for example, and lends a certain quantity of wheat, then after one period, he can afford an additional consumption of $(1 + r)$ of the amount he lent. And if he borrows additional wheat in the first period, the repayment of his debt reduces his consumption in the second period by $(1 + r)$ times the amount he borrowed. The market line that passes through P is not attractive for Robinson Crusoe though. Whatever the particular shape of his indifference curves, as long as his utility is an increasing function of C_1 and C_2 he will always choose the outmost market line. In Figure 1.2 this is the line which is tangent to curve $f(K)$ at point A where the investment of seed capital is K^* . With a choice of this line and a loan of size D^* , Robinson Crusoe succeeds in reaching the indifference curve U^* that is tangent to the market line at point B . This indifference curve incorporates the highest utility level that is possible if a capital market is

present. The level is significantly higher than the highest feasible level without a capital market which is determined by the indifference curve that passes through point R. The tangential solutions at points A and B represent the marginal conditions (1.10) and (1.11).

Optimization problems similar to Robinson Crusoe's are solved by all other castaways, and everyone finds an optimal value of his credit demand like D^* . If the sum of all demand quantities exceeds (falls short of) zero, then the market rate of interest r rises (falls) and the market transformation lines "roll" along the technological transformation curve $f(K)$ downwards (upwards) and alter the horizontal distance between the points of tangency A and B. Thus, a new sum of the credit demands is determined with a new market rate of interest. Possibly this leads again to changes in the interest rate and so on. If the process is stable, an interest level will emerge where the sum of all net credit demands equals zero. With this interest level an intertemporal equilibrium is achieved and the exchange contracts are concluded.

Among the implications of Fisher's theory, which was briefly sketched here, there are four which are particularly interesting.

(1) The production decision is independent of Crusoe's preferences and independent also of his initial capital endowment. This follows from (1.10) and is also immediately obvious from Figure 1.2 if one notes that the position of point A relative to point Y on the abscissa (and with it the magnitude of the optimal capital investment K^*) depends only on the slope of the market transformation line. Thus it is possible to split up Robinson Crusoe's activities analytically into the function of an entrepreneur and the function of a household. The behavior of the "firm Crusoe" would not change, if it were sold to another castaway. An exchange of the islands among the castaways would not affect the optimum amount of seed on a particular island even if the preferences of the owners differed. This result is the famous *separation theorem* of Irving Fisher.

(2) The optimal production decision requires maximization of the market value M of Robinson Crusoe's initial wealth. The initial wealth consists of the present value of future returns plus the withdrawals for consumption in the first period and minus the first-period debt,

$$M = \frac{f(K)}{1+r} + C_1 - D. \quad (1.12)$$

It is measured by the distance between the origin and the point where the chosen market transformation line intersects the abscissa, that is, by the distance OM^* . Crusoe's initial wealth can also be expressed as the sum of

his initial seed capital Y and the net present value of his enterprise. The net present value is the value of the right to utilize the production possibilities described by $f(K)$; that is, it is the value of the island itself, without the seed capital. It is the difference between the present value of the future returns $f(K)/(1+r)$ and the invested seed capital K and is represented by the distance YM^* in Figure 1.2. Since the maximization of this distance is identical with the maximization of the distance OM^* , there is no meaningful difference between the maximization of wealth and the maximization of the net present value.

(3) The problem of an intertemporal optimization of the consumption plan is affected by the real investment opportunities only in so far as these opportunities determine the maximum achievable wealth M^* . Given this wealth, the optimal consumption plan is, according to Equation (1.11), determined solely by the preferences and the market rate of interest. It is useful to imagine Crusoe initially borrowing against all his future returns and then distributing the resulting cash among the consumption for the current period and the capital market investment. With the same preferences Crusoe's consumption plan will therefore not differ from that of another castaway who had the misfortune of landing on an uncultivable, rocky island but who was lucky enough to save a correspondingly higher initial stock of wheat.

(4) In a market equilibrium all marginal rates of transformation and all marginal rates of substitution with regard to the consumption levels of both periods have the same value $(1+r)$. Thus, analogously to the static model, the allocation satisfies three basic conditions for a Pareto optimum:

- (1) Because of the equality of the marginal rates of substitution there is no further scope for mutually advantageous credit contracts between any two households.
- (2) The equality of the marginal rates of transformation ensures that the invested stock of capital is allocated to the different production opportunities in a way that maximizes the value of aggregate production.
- (3) Thanks to the equality of the marginal rates of transformation and substitution, it is impossible to increase anyone's utility through a variation of the total stock of capital invested without at the same time decreasing someone else's utility.

So much for the basic elements of Fisher's model. Several extensions seem obvious. For example, there can be more than two periods, or different capital and consumption goods, or a joint ownership of various households

in a firm, or various other changes. None of the four results reported is affected by such alterations.³

1.4. Tax Analyses in Fisher's Model: A Remark

The previous sections have shown the analogies between the static and the dynamic allocation problem and make Fisher's model appear an attractive candidate for an analysis of intertemporal taxation effects. Indeed, almost all the literature on this topic makes use of basic elements of Fisher's model.⁴ But with a few exceptions that are cited in the next chapter,⁵ nearly all contributions are partial analyses. They study how economic agents react to taxation if the time paths of the market rate of interest and possibly of other factor prices are unchanged.

A partial analysis is not necessarily a shortcoming. Any good equilibrium theory must be based on a proper partial analytic foundation and there are many questions, for example in connection with the international and intersectoral aspects of taxation treated in Chapters 6 and 7, that can be answered on the basis of partial analytic tax models with only minor amendments. However, for an analysis of the *intertemporal* allocation effects, an economy-wide, general model that describes the interactions of firm and household decisions is indispensable. In particular, the endogenous change of the market rate of interest that is completely neglected in partial analyses is of decisive importance for the mechanism of intertemporal allocation.

³Cf. Hirshleifer (1970).

⁴For a random sample of studies that discuss the influence of taxation on the firm's investment behavior see Smith (1963), Samuelson (1964), Hall/Jorgenson (1967), Sandmo (1974), Stiglitz (1976), Schneider/Nachtkamp (1977), Boadway/Bruce (1979), and Boadway (1980). Less frequent are studies that treat the role of taxation in the intertemporal optimization problem of the household. See, however, Feldstein/Tsiang (1968), Levhari/Sheshinski (1972), Feldstein (1978a), and Summers (1981). A good overview of the present state of the discussion can be found in Atkinson/Stiglitz (1980, Lectures 3 and 5).

⁵See Section 2.1. Of course, the list of names cited there cannot claim to be all inclusive.