Capital Income Taxation and Resource Allocation

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Chapter 2: Fisher, Solow, and the General Intertemporal Equilibrium

Chapter 2

FISHER, SOLOW, AND THE GENERAL INTERTEMPORAL EQUILIBRIUM

The purpose of Chapter 1 was to recall some fundamentals of Fisher's theory of allocation. The second chapter has the task of generalizing Fisher's theory by combining it with the neoclassical growth model. This provides the basis for the analysis of tax effects in the subsequent chapters. The behavior of households and firms is derived from microeconomic optimization approaches and the consequent implications for the growth path of the economy are examined. Special attention will be given to the question of to what extent markets are necessary to coordinate intertemporal economic plans.

2.1. The Integration Problem

Fisher's approach provides a number of important insights into the intertemporal allocation process. But the analytical problems involved in using this approach are as great as its attractiveness, at least in the realistic case of many periods. Although general existence proofs and the results reported in the previous chapter can be derived, it is quite difficult to acquire specific information on the time paths of the model's variables. One reason for this difficulty is that the problem of capital accumulation from current production is not explicitly treated in Fisher's approach. For growth problems the approach lacks structure.

To analyze capital accumulation it seems therefore promising to borrow from the neoclassical theory of economic growth founded by Solow (1956). It is true that a direct application of usual growth theoretic approaches is out of the question, for these approaches are either normative central planning models or positive models of the growth of a decentralized economy that have little in common with Fisher's idea of intertemporal general equilibrium. Nevertheless the theory of economic growth does provide those structural elements, in particular the equation of motion of the capital stock and the clear distinction between stocks and flows, that are missing in Fisher's theory. This chapter attempts to integrate the two approaches.

In the light of the second main theorem of welfare economics[1] that was first proved convincingly by Arrow (1951), it seems useful to look for the model of a decentralized economy that produces the neoclassical optimal growth path as the laissez-faire solution. Such a model would be the natural generalization of Fisher's approach and could serve as a basis for a theory of the dynamic allocation effects of taxation.

There is no doubt that such a model exists, for Malinvaud (1953, 1962) already proved the second main theorem of welfare economics explicitly for an intertemporal market equilibrium with an unlimited time horizon.[2] However, the knowledge of the existence of the model does not help much in actually carrying out a dynamic tax analysis. For such an analysis a much more structured model is necessary than for the mere existence proof.

Certain elements of such a model can be found towards the end of the book by Arrow and Kurz (1970, Chapters VII and VIII), where the abstract question of which taxes can be used to make the growth path of an economy with a perfect capital market "controllable" is discussed, or in Hall (1971), who interprets a turnpike growth model from the point of view of a decentralized economy and studies tax announcement effects. Unfortunately, however, these approaches also lack important elements necessary for a complete model of a general intertemporal equilibrium in a decentralized economy. For example, the investment or financial decisions of the firm are not derived from intertemporal optimization, and the role played by markets in the intertemporal coordination of economic plans is completely unclear.

Similar remarks apply to a number of excellent recent studies including Chamley (1981), Abel and Blanchard (1983), and Becker (1983, 1985) that

[1]The theorem says that each Pareto optimum can be represented as a competitive equilibrium provided the price vector, the initial endowments of the individuals, and a number of further technical assumptions are chosen appropriately. It has its origin in Lange (1942).

[2]Malinvaud allows for an infinite horizon but he assumes that all consumption paths under consideration are identical beyond some arbitrarily choosable finite point in time T. This limitation is unimportant with regard to the neoclassical optimal growth path, since its existence requires that the present value of the capital stock vanish for $t \to \infty$ (transversality condition). For a discussion and a generalization of Malinvaud's proof compare Cass and Majumdar (1979) and the literature cited there. A numerical intertemporal general equilibrium model for an economy with a finite time horizon can be found in Chao (1979, Part 2).

investigate the role of taxation in a growth setting with infinitely lived agents. In these studies no explicit capital market is modelled, and the firms cannot choose between different financial instruments. It is assumed that the households own the real capital and rent it to the firms. Only households solve dynamic decision problems. Firms are atemporal, static agents just maximizing current profits. While this specification is appropriate when the effects of a uniform tax on all kinds of capital income are studied, it cannot be used for an analysis of *systems* of capital income taxation where there are different marginal tax rates on different kinds of capital income such as capital gains, retained earnings, dividends, or interest earnings. For such an analysis, an *explicit* intertemporal general equilibrium model that follows Fisher's approach more closely but is nevertheless the decentralized counterpart of the neoclassical model of optimal economic growth seems necessary. Such a model is presented in the subsequent sections. The presentation is based on previous papers by the author on this subject (Sinn 1980b, 1981).[3]

2.2. The Structure of the Model

The basic structure of a simple model that combines the approaches of Fisher and Solow is more or less obvious. There are households and legally

[3]There have been other reinterpretations of Fisher's theory that are quite different from the one presented here. The most prominent example is Hirshleifer (1970). Hirshleifer has provided important insights into the process of intertemporal allocation, however, with the attempt to integrate the problem of capital accumulation explicitly into the model (Chapter VI F, pp. 171–180), the limitations of his analysis become obvious. "Because of the intractability of the problem in full generality" (pp. 171 n.) Hirshleifer limits his analysis to the case of steady states, and, despite the adoption of essential assumptions of growth theory, the relationship between his results and those of this theory remain obscure. Moreover, the author (p. 171) assumes for simplicity that the multi-period decision problem of the household can be represented through repeated and overlapping two-period decision problems. This assumption implies, as Hirshleifer (pp. 179 n.) concedes, that the household is continuously revising its plans and the intertemporal allocation predicted by the model deviates from the "true accumulation path". Approaches similar to Hirshleifer's have recently been combined with the Scarf–Shoven–Whalley algorithm in order to calculate the dynamic welfare losses of different taxes; see Fullerton et al. (1983) or Fullerton et al. (1981). Chapter 9 will provide the opportunity of discussing the results achieved by these authors. It is also worth noting a paper by Sidrauski (1967) where the decentralized version of a neoclassical growth model with households and firms with infinite lives is presented. Because of the assumption of adaptive expectations, this paper does not contain an intertemporal general equilibrium model that can be considered as the analog to the static model of general equilibrium. Finally, a paper by Kemp and Long (1979) should be mentioned which contains an intertemporal general equilibrium model of trade in natural resources. This important paper is in the spirit of Fisher's model, but does not include the possibility of capital accumulation.

independent firms. All agents derive their behavior for intertemporal optimization without direct coordination with other agents. All plans are formulated in continuous time up to an infinite horizon. The agents behave competitively, that is, they accept the time paths of the market prices as exogenous in their planning problems. Nevertheless these paths are endogenous in the market equilibrium. Since there is a sufficient number of credit and futures markets,[4] or since everyone is endowed with perfect foresight, the price paths are determined in such a way that all private plans are compatible with one another. Over time, no one has any interest in revising his plans even if this is possible. There is a time-consistent general intertemporal equilibrium.

Firms produce investment and consumption goods with the aid of labor and capital and the latter can appear as equity or debt. The marginal rate of transformation between the consumption and the investment goods is constant and, because of a suitable choice of units, has a value of one. The households are the shareholders of the firms and provide the firms with equity capital, loans, and labor. Accordingly, the income of a household consists of dividends, interest earnings, and wages.[5] It is used for consumption, for buying new equity, and for providing the firms with new loans. The investment or consumption good at each point in time is a "current numeraire", that is, the undiscounted commodity price is unity for all points in time.[6] The endogenous prices of the model are the interest and wage rates.

In order to economize on notation, the sector of firms is depicted by one single representative firm and the sector of households by one single representative household. This is essentially the same as if there were arbitrarily many identical agents of each type.[7] The assumption of representative agents by no means implies that everyone behaves as if he were

[4]As shown in Section 2.5, futures markets for commodities are not necessary.

[5]Implicitly, there are also capital gains on the existing company shares. See Equation (3.21) for an explicit description of this item.

[6]Because of the explicit consideration of time, there is a continuum of commodities in the model. A complete formal analogy to the static theory would require the price of just one dated commodity to be equal to unity. However, this would not result in a model that can easily be interpreted, since the rate of interest that is defined in terms of this commodity would be zero and the discounting would only indirectly appear through the fact that the price of a commodity is a falling function of the date at which it is delivered.

[7]In the case of homothetic preferences it is moreover admissible that households have different endowments of the various kinds of wealth that are yet to be described. Compare the discussion in Chapter 10.2. A version of the model where the sector of firms is split up into separate sectors is contained in Chapters 6, 7, and 11. For an intertemporal general equilibrium model with an explicit treatment of many firms and households see Sinn (1980b).

alone in the market place. The contrary is the case. In the model of perfect competition, the representative agent behaves as if he were infinitesimally small, too small at any rate to affect prices or other market parameters.

2.3. The Decision Problem of the Firm

The representative firm determines its optimal policy given a number of exogenous constraints. Variables that can be manipulated by the firm are indicated by a superscript "u".

The firm operates under a given production function of the Jorgenson type.[8] Gross output is a linearly homogeneous, strictly quasi-concave function $f(K^u, L^u)$, $f: R_+^2 \rightarrow R_+$, where K^u indicates the employment of capital and L^u the employment of labor. The production function has the properties

$$\lim_{K^u \to \infty} f_K(K^u, L^u) = \lim_{L^u \to \infty} f_L(K^u, L^u) = 0$$

and

$$\lim_{K^u \to 0} f_K(K^u, L^u) = \lim_{L^u \to 0} f_L(K^u, L^u) = \infty.$$

Since a labor-augmenting, Harrod-neutral technological progress is to be allowed, L^u is measured in efficiency units. Accordingly the wage rate $w > 0$ is the price of such an efficiency unit. Economic depreciation that has to be subtracted from gross output in order to calculate net output is a fixed proportion $\delta > 0$ of the capital stock; it is hence geometrically declining.[9]

If D^u is the outstanding stock of debt of the firm and r the market rate of interest, the current accounting profit of the firm is

$$\Pi^u = f(K^u, L^u) - \delta K^u - rD^u - wL^u, \tag{2.1}$$

and

$$\Pi^{du} = \Pi^u + S^u - I^u, \tag{2.2}$$

denotes the corporate distributions to shareholders or the money withdrawn by the owner of a non-corporate firm. Here S^u indicates the flow

[8] See Jorgenson (1967).

[9] According to Jorgenson (1967, pp. 139 n.) it may be useful in highly aggregated models to assume geometrically declining depreciation even if different depreciation rules hold on the micro level.

of new loans taken by the firm and I^u its real net investment. If the distributions are negative, $-\Pi^{du}$ can be interpreted as a flow of funds resulting from issuing new shares or new money injected by the owner of the firm.[10]

The present value of all distributions made after some point in time t is the market value M of equity at this point in time:

$$M^u(t) \equiv \int_t^{\infty} \left[\exp \int_t^{v} - r(s)\,\mathrm{d}s \right] \Pi^{du}(v)\,\mathrm{d}v. \tag{2.3}$$

In the case of a corporate firm, this value is the market value of its shares.[11]

The aim of the firm which is derived from the preferences of its owners is, according to Fisher's separation theorem (see Chapter 1), the maximization of the market value at the beginning of the planning problem ($t = 0$):[12]

$$\max_{\{L^u, S^u, I^u\}} M^u(0). \tag{2.4}$$

Control variables of the planning problem are the employment of labor $\{L^u\}$, the net increase in debt $\{S^u\}$, and the real net investment $\{I^u\}$.[13] The optimization is carried out under the assumption of given continuously differentiable factor price paths $\{w\}$ and $\{r\}$, where the non-negativity constraints

$$K^u \geq 0, \quad L^u \geq 0, \tag{2.5}$$

the historically given initial conditions

$$K^u(0) = K_0 > 0, \quad D^u(0) = D_0 < K_0, \tag{2.6}$$

[10] No clear distinction between corporate and non-corporate firms is necessary at this stage of the analysis. Henceforth the terminology of the corporate firm will be used without, however, precluding an interpretation of the formulas from the viewpoint of a non-corporate firm.

[11] The expression for $M^u(t)$ is merely assumed at this stage. For an explicit derivation of the market value function from an arbitrage condition that must hold in a capital market equilibrium see Equations (3.19)–(3.24).

[12] Note that the market value is defined with regard to *all* future payments that result from the ownership of shares. Sometimes it is assumed that the firm maximizes the sum of the market value of equity and its debt. The resulting definition of the market value of equity implies the exclusion of initial distributions to shareholders that result from a substitution of debt for equity. This maximization will not always lead to correct results. (The reader who has advanced to Chapter 6 should calculate the "optimal" initial stock of debt for the case $\theta_d^* < \theta_r^* = \theta_p$ from Equation (6.4) and compare this result with the result described in Chapter 4.3.3 that the optimal debt–equity ratio of the firm is indeterminate in this case.

[13] The curly brackets characterize time paths. For example, $\{X\}_\alpha^\beta$ denotes the time path of X in the closed interval from $t = \alpha$ until $t = \beta$. $\{X\}$ is equivalent to $\{X\}_0^\infty$.

and the equations of motion[14]

$$\dot{K}^{u} = I^{u}, \quad \dot{D}^{u} = S^{u}, \tag{2.7}$$

for the two state variables have to be taken into account. For the sake of realism, a number of financial constraints should be added to the decision problem of the firm. Such constraints are discussed in detail in Chapter 4 where their role in the firm's investment decision is considered in the context of a tax system that interferes with the firm's financial decisions. For the time being, financial constraints can be disregarded since they are not binding and hence do not have any implications for the firm's real decisions.

The current-value Hamiltonian of the problem is[15]

$$\mathscr{H}^{u} = \Pi^{du} + \lambda_K I^{u} + \lambda_D S^{u}, \tag{2.8}$$

with

$$\Pi^{du} = f(K^{u}, L^{u}) - \delta K^{u} - I^{u} + S^{u} - rD^{u} - wL^{u},$$

from (2.1) and (2.2). The variables λ_K and λ_D are the shadow prices of the total stock of capital employed and of the stock of debt, respectively. With M^* as the optimal market value M^{u} of the firm, these shadow prices are defined by

$$\lambda_K \equiv \left.\frac{\mathrm{d}M^{u}}{\mathrm{d}K^{u}}\right|_{M^{u}=M^*}, \quad \lambda_D \equiv \left.\frac{\mathrm{d}M^{u}}{\mathrm{d}D^{u}}\right|_{M^{u}=M^*}. \tag{2.9}$$

Thus they measure the changes in market value brought about if the respective stocks are increased by one unit each in a situation where the firm has chosen its optimal policy. Among other things it is necessary for an optimal intertemporal policy[16] that the Hamiltonian is maximized and that the condition $\dot{\lambda}_K - r\lambda_K = -\partial\mathscr{H}^{u}/\partial K^{u}$ is satisfied.

The condition for an optimal employment of labor can quite easily be derived. Obviously a maximization of the Hamiltonian with regard to L^{u} requires that $\partial\mathscr{H}^{u}/\partial L^{u} = 0$ and hence

$$f_L(K^{u}, L^{u}) = w. \tag{2.10}$$

Given K^{u}, the employment level is chosen such that the marginal value product of labor equals the wage rate. (Note that the output price is unity by assumption.)

[14]In this book the notation $\dot{X} \equiv \mathrm{d}X/\mathrm{d}t$ is used, where t indicates the time index.

[15]For the technical aspects of dynamic optimization see Arrow (1968), Intriligator (1971), Kamien and Schwartz (1981), or Leonard and Long (1987).

[16]Cf. Footnote 18.

Consider now the debt policy. From (2.1), (2.2), and (2.3) we get the differential quotient[17]

$$\frac{\mathrm{d}M^{\mathrm{u}}(t)}{\mathrm{d}D^{\mathrm{u}}(t)} = \int_t^{\infty} - r(v)\left[\exp\int_t^{v} - r(s)\,\mathrm{d}s\right]\mathrm{d}v$$

$$= \left[\exp\int_t^{v} - r(s)\,\mathrm{d}s\right]_{v=t}^{v=\infty}$$

$$= -1. \tag{2.11}$$

In order for (2.11) to exist, it is necessary that $\lim_{t\to\infty} r(t) > x$ where x is some arbitrarily small, strictly positive constant. In anticipation of a property of the intertemporal general equilibrium that is yet to be derived [see (2.52)], it is assumed that this is the case. The differential quotient says that one additional unit of debt, given the real stock of capital, reduces the market value of shares by one unit. Since (2.11) has been calculated independently of any variables endogenous to the planning problem, it holds also of course in the optimum. Thus, because of (2.9), it is true at each point in time that

$$\lambda_D = -1. \tag{2.12}$$

In the absence of financial constraints, a necessary condition for optimal financial decisions on the part of the firm[18] is $\partial\mathscr{H}^{\mathrm{u}}/\partial S^{\mathrm{u}} = 0$. Because of (2.12) this condition is satisfied with

$$\partial\mathscr{H}^{\mathrm{u}}/\partial S^{\mathrm{u}} = 1 + \lambda_D = 0. \tag{2.13}$$

Since (2.13) holds for arbitrary values of D^{u} *and* arbitrary values of S^{u}, the optimal debt policy of the firm is indeterminate. Suppose the firm decides at a particular point in time t to increase its stock of debt in order to finance additional dividends. This will not affect the market value of shares before this point in time since the repayment of the debt will require dividend reductions in the post-t future whose present value is exactly the same as the present value of the additional dividends paid out in t. In a perfect market, shareholders will obviously be indifferent with regard to such a policy. The result is well known in the theory of finance under the name of

[17] Without loss of generality, the differentiation assumes $\partial D^{\mathrm{u}}(v)/\partial D^{\mathrm{u}}(t) = 1$ for all $v \geq t$.

[18] Another necessary condition for an optimum is $\dot{\lambda}_D - r\lambda_D = -\partial\mathscr{H}^{\mathrm{u}}/\partial D^{\mathrm{u}}$. Since $\partial\mathscr{H}^{\mathrm{u}}/\partial D^{\mathrm{u}} = -r$, $\dot{\lambda}_D = 0$, and $\lambda_D = -1$, this condition is satisfied. It is neglected here since it does not yield any useful information.

the Modigliani–Miller theorem and has been shown to hold even for the case of uncertainty.

In deterministic models of the firm it is sometimes assumed that the rate of interest the firm has to pay on its debt depends on the size of debt so that an optimal debt–equity ratio can be calculated. This procedure is not convincing however. It is true that one can observe in reality that the rate of interest a debtor pays is an increasing function of the amount of debt he takes up. However, this observation does not necessarily imply that there is a unique optimal debt–equity ratio. If the increase in the rate of interest is merely a compensation for a growing probability of default and hence a growing probability of creditors suffering capital losses then it may well be accepted by shareholders as a compensation for a reduction of the risk they have to bear themselves. At any rate, the mere observation that the rate of interest is an increasing function of the level of debt does not indicate at all that the debt loses its attractiveness for shareholders. That there are indeed conditions under which the optimal debt-equity ratio is indeterminate even with the possibility of bankruptcy is one of the results shown by Hellwig (1981) in his generalization of the Modigliani–Miller theorem.

The most important aspect of the firm's policy is its real investment planning. The necessary conditions for an optimum are

$$\partial \mathscr{H}^{u}/\partial I^{u} = \lambda_K - 1 = 0 \tag{2.14}$$

and

$$\dot{\lambda}_K - r\lambda_K = -\partial \mathscr{H}^{u}/\partial K^{u} = -[f_K(K^{u}, L^{u}) - \delta]. \tag{2.15}$$

Obviously they imply that

$$r = f_K(K^{u}, L^{u}) - \delta. \tag{2.16}$$

With the interpretation of λ_K as given in (2.9), (2.14) says that, in the optimum, an additional unit of real capital that is financed with new injections of funds from outside the firm (new issues of shares for example) increases the market value of equity by just one unit. If the additional unit of real capital is financed with an increase in debt, then it follows from (2.11) that the market value stays unchanged.

Condition (2.16) is the analog of Condition (1.10) from Fisher's two-period approach. Capital is optimally employed if its marginal product net of depreciation equals the market rate of interest. The only difference is that in the present case the depreciation rate δ can obtain arbitrary values while in the two-period model it was implicitly assumed to be 100%. Given the paths of the market rate of interest $\{r\}$ and the employment of labor $\{L^{u}\}$, Condition (2.16) determines one and only one path of capital $\{K^{u}\}$ and thus one and only one path of the firm's real net investment $\{I^{u}\}$. Note that the path $\{I^{u}\}$ is well defined although (2.14) by itself does not uniquely determine a level of investment. This is the well-known phenomenon, already discussed by Haavelmo (1960, Chapters 25, 28, and 29), that, in the absence of adjustment costs, it is not the rate of interest itself, but its change over time, that determines the level of investment.[19]

As additional requirements for an optimal plan of the firm, the two transversality conditions

$$\lim_{t\to\infty}\left\{\left[\exp\int_0^t - r(v)\,\mathrm{d}v\right]\lambda_K(t)K^{u}(t)\right\} = 0$$

and

$$\lim_{t\to\infty}\left\{\left[\exp\int_0^t - r(v)\,\mathrm{d}v\right]\lambda_D(t)D^{u}(t)\right\} = 0,$$

have to be met. Since λ_K and λ_D are finite constants these conditions are satisfied if[20]

$$\lim_{t\to\infty}[\hat{K}^{u}(t) - r(t)] < 0 \tag{2.17}$$

and

$$\lim_{t\to\infty}[\hat{D}^{u}(t) - r(t)] < 0; \tag{2.18}$$

that is, if, in the limit, the growth rates of the state variables fall short of the discount rate. Anticipating the properties of intertemporal general equilibrium it is assumed that the limits exist.

2.4. The Decision Problem of the Household

The representative household does not only plan for its current members, it also takes into account the well-being of its possibly growing number of

[19] To incorporate adjustment costs might be a useful complication for short-term investment models that are concerned with trade cycle phenomena. The present model, however, is designed to investigate the long-term allocation effects of taxation, and for this purpose it seems complicated enough.

[20] In this book the symbol "^" indicates growth rates: $\hat{X} \equiv \dot{X}/X = (\mathrm{d}X/\mathrm{d}t)/X$.

descendents. At each point in time t, the household evaluates the flow of consumption $\{C^h\}_t^\infty$ that it can enjoy after this point in time in terms of the utility or welfare function

$$\bar{U}(t) = \int_t^\infty e^{-\rho(v-t)} N(v) U(C^h(v)/N(v))\, dv. \tag{2.19}$$

Here $\rho > 0$ is a subjective rate of discount, N the size of the family, and U, $U: R_+ \to R$, a twice differentiable, strictly concave, monotonically increasing function that indicates the level of instantaneous felicity per capita. By analogy with the procedure of the previous section, variables that can be manipulated by the household are characterized by the superscript "h".

The utility function (2.19) is well known and hence does not have to be discussed in detail here. With $N = \text{constant}$ and $\rho = 0$ it first appeared in Ramsey (1928). Samuelson (1936/37) introduced discounting, Strotz (1955/56) provided an argument for the constancy of the subjective rate of discount ρ that seems particularly valid in the multiple generations context, Koopmans (1960) studied the axiomatic basis of the function for $N = \text{constant}$, and with their "island example" Arrow and Kurz (1970, pp. 13 n.) popularized the version with a variable N that was utilized first by Mirrlees (1967). The utility function has at least the one advantage that its implications are well known for growth theoretic central planning models. This significantly simplifies the interpretation and evaluation of the allocative result brought about by market forces. In addition, it will be shown in Section 2.7 that the use of this function for the decision problem of the household is close to being a cogent necessity once it is accepted as the central planner's goal.

For the sake of simplicity, it is assumed that the felicity function $U(\cdot)$ is characterized by a constant elasticity of its first derivative:

$$\eta \equiv -\frac{U''(C^h/N)}{U'(C^h/N)} C^h/N = \text{constant} > 0. \tag{2.20}$$

With $\eta = 1$, this assumption implies $U(C^h/N) = a + b \ln(C^h/N)$, $b > 0$. It thus includes the special case of a logarithmic function that, from the point of view of psychophysical laws, can be attributed a certain degree of relevance.[21]

The number of efficiency units of labor L that the household supplies is the product of the size of the population N and an *efficiency factor* G, where

[21] See Sinn (1980a, Chapter III. A).

it is assumed that both quantities follow exogenous time paths.[22]

$$L = NG,$$
$$N(t) = N_0\,\mathrm{e}^{nt}; \quad G(t) = G_0\,\mathrm{e}^{gt}; \quad N_0,\, G_0 > 0; \quad n,\, g \geq 0. \tag{2.21}$$

Given the time paths of the wage rate $\{w\}$ and the market rate of interest $\{r\}$, the present value of labor income – that is, the human capital – is

$$A(t) = \int_t^{\infty} \left[\exp \int_t^u - r(v)\,\mathrm{d}v\right] w(u)L(u)\,\mathrm{d}u. \tag{2.22}$$

The sum of human capital, the value M^{u} of company shares as defined in (2.3), and the value of the stock of bonds D^{h} issued by the firm is the total wealth of the household:

$$V^{\mathrm{h}} = A + M^{\mathrm{u}} + D^{\mathrm{h}}. \tag{2.23}$$

The total wealth is the state value in the optimization problem of the household. The corresponding equation of motion is

$$\dot{V}^{\mathrm{h}} = \dot{A} + \dot{M}^{\mathrm{u}} + S^{\mathrm{h}}. \tag{2.24}$$

Here $S^{\mathrm{h}} = \dot{D}^{\mathrm{h}}$ is the supply of savings that the household offers in the credit market. This supply is the difference between the household's earnings and its consumption C^{h}, where earnings comprise wages wL, dividends Π^{du}, and interest income rD^{h}:

$$S^{\mathrm{h}} = wL + \Pi^{\mathrm{du}} + rD^{\mathrm{h}} - C^{\mathrm{h}}. \tag{2.25}$$

The variable $\dot{A}$ measures the change in human capital per unit of time. Differentiating (2.22) one obtains

$$\dot{A} = rA - wL. \tag{2.26}$$

Similarly, the change $\dot{M}^{\mathrm{u}}$ of the market value of company shares can be calculated from (2.3):

$$\dot{M}^{\mathrm{u}} = rM^{\mathrm{u}} - \Pi^{\mathrm{du}}. \tag{2.27}$$

Thus the equation of motion (2.24) becomes

$$\dot{V}^{\mathrm{h}} = rV^{\mathrm{h}} - C^{\mathrm{h}}. \tag{2.28}$$

[22] It seems tempting to think about the possibilities of endogenizing labor supply. The labor–leisure choice, the investment in human capital, and the decision about the optimal rate of birth are certainly, within limits, results of economic considerations. The tremendous analytical problems that an explicit consideration of such effects would encounter in a model with technological progress are bypassed in this study. Compare, however, Chapter 9.6.

If the household consumes nothing, total wealth rises by the (partially fictive) returns it generates. If there is consumption, then the rise is correspondingly less.

The household can freely manipulate the time path $\{V\}$ but, like the firm with (2.6), it has to take into account the historically given initial condition

$$V^h(0) = M^u(0) + A(0) + D^h(0) > 0, \quad D^h(0) = D_0. \tag{2.29}$$

In addition, there is a liquidity constraint

$$V^h - \beta A \geq 0, \quad 0 < \beta \leq 1. \tag{2.30}$$

In the case $\beta = 0$ this constraint allows the household to borrow against future labor income including that of future generations. Practically all societies, however, allow a bequest to be declined and thus prevent a net debt being inherited. If only for this reason, more severe debt constraints have developed in the market. If any borrowing against labor income is excluded, then $\beta = 1$ and the constraint becomes $M^u + D^h \geq 0$. The realistic case, however, seems to be $0 < \beta < 1$. For the derivation of the optimality conditions of the household's planning problem, Constraint (2.30) will not yet be considered. Instead, it will be discussed in the context of the intertemporal general equilibrium in Section 2.6. There it will turn out that the constraint is not binding even in the case where $\beta = 1$.

Given these specifications, the decision problem of the household is

$$\max_{\{C^h\}} \bar{U}(0)$$

$$\text{s.t. (2.28), (2.29), and (2.30).} \tag{2.31}$$

The current-value Hamiltonian is

$$\mathscr{H}^h = N\, U(C^h/N) + \lambda(rV^h - C^h). \tag{2.32}$$

Here, λ denotes the shadow price of wealth measured in utility units, which is defined as

$$\lambda \equiv \mathrm{d}\bar{U}/\mathrm{d}V^h|_{\bar{U}=\bar{U}^*}, \tag{2.33}$$

where $\bar{U}^*$ is the level of $\bar{U}$ achieved in an optinum.

A necessary condition of an optimum is the maximization of the Hamiltonian with regard to C^h. Obviously, a maximum is achieved if

$$U'(C^h/N) = \lambda, \tag{2.34}$$

that is, if the utility gain from the immediate consumption of one unit of wealth just balances the utility loss from foregone future consumption. A further condition for an optimum is

$$\dot{\lambda} - \rho\lambda = -\partial\mathscr{H}^{\mathrm{h}}/\partial V^{\mathrm{h}} = -\lambda r, \tag{2.35}$$

and hence

$$\gamma = r, \tag{2.36}$$

where

$$\gamma \equiv \rho - \hat{\lambda} \tag{2.37}$$

is defined as the household's subjective *rate of time preference*. Condition (2.36) is the analog of Condition (1.11) in Fisher's two-period model. The analogy is obvious if one notes that, according to Definition (1.5), the rate of time preference equals the percentage decline in marginal utility between the two periods.

Equation (2.37) shows that, given the utility function $\bar{U}(t)$ from (2.19), the relative decline in marginal utility consists of two components. One is the subjective rate of discount ρ that reflects von Böhm–Bawerk's second reason for interest, the "underestimation of future wants".[23] The other, $-\hat{\lambda}$, is the decline in instantaneous felicity that, following from a logarithmic differentiation of (2.34) and Definition (2.20), is explained through the growth rate $(\hat{C}^{\mathrm{h}} - n)$ of per capita consumption:

$$\hat{\lambda} = -\eta(\hat{C}^{\mathrm{h}} - n). \tag{2.38}$$

This component reflects von Böhm–Bawerk's first reason which he calls the "difference in the relationships between demand and supply".[24]

Because of (2.38) and (2.37), the marginal condition (2.36) obviously requires that

$$\hat{C}^{\mathrm{h}} - n = \frac{r - \rho}{\eta}. \tag{2.39}$$

Hence the level of consumption per capita stays constant if the rate of interest just equals the subjective rate of discount ρ. If the rate of interest is higher, then it pays to consume less in the near future but more in the far distant future; that is, it pays to choose a positive rate of growth of per capita consumption. If the rate of interest is lower, the reverse is true.

An additional requirement for the optimum of the household is described

[23] See Von Böhn–Bawerk (1888, pp. 332–338).
[24] Ibid. pp. 328–331. Cf. also Frisch (1964, pp. 421 n.).

through the transversality condition

$$\lim_{t \to \infty}[e^{-\rho t}\lambda(t)V^{h}(t)] = 0.$$

Because of (2.36) and (2.37), this condition is equivalent to the condition

$$\lim_{t \to \infty}\left(\left[\exp \int_0^t - r(u)\,du\right]\lambda(0)V^{h}(t)\right) = 0,$$

or, with an existing limit, also equivalent to the condition

$$\lim_{t \to \infty}[\hat{V}^{h}(t) - r(t)] < 0. \tag{2.40}$$

2.5. Conditions of an Intertemporal Market Equilibrium: The Irrelevance of Futures Markets

The factor price paths $\{r\}$ and $\{w\}$ that are exogenous to the individual planning problems are endogenous to the market equilibrium. Without addressing the stability problem, it is assumed here that the paths are determined such that the plans of all market agents are compatible with one another. The compatibility is assured if the individual optimization conditions of the market agents are satisfied and if, in addition, at each point in time the labor, the commodity, and the capital markets are clearing.

The condition for a labor market equilibrium is that the flow of efficiency units of labor that is inelastically supplied by the household sector is demanded by the sector of firms:

$$L^{u} = L. \tag{2.41}$$

The commodity supply of the sector of firms is $f(K^{u}, L^{u})$. Commodity demand consists of the firms' own demand for investment commodities, $\delta K^{u} + I^{u}$, and the consumption demand C^{h} of the household sector. The equilibrium condition for the commodity market is therefore

$$f(K^{u}, L^{u}) = \delta K^{u} + I^{u} + C^{h}. \tag{2.42}$$

The condition for an equilibrium in the capital market is

$$D^{u} = D^{h}; \tag{2.43}$$

it requires an equality of the stocks of credit supplied and demanded by the household sector and the firm sector, respectively.

At first glance it might be expected that, because of Walras' law, one of these three equilibrium conditions is redundant, that it is already implied by the other two. But this expectation is too hastily arrived at. It is important to realize that Conditions (2.41), (2.42), and (2.43) do not refer to three markets in the usual sense but, since they apply to all points in time from zero to infinity, to three continua of markets, that is, no less than three times uncountably infinite single markets. Seeing it this way, one should expect that one of the three equilibrium conditions is redundant for just one single point in time but not for the total continuum.

Even this expectation is misleading, however. The truth is that "Walras' law" was never intended for a general intertemporal equilibrium in a world with bond markets. In Walras' model there are markets for goods, labor, and the services of specific capital goods but no markets for financial assets. Walras (1874, §255, p. 269) considered such markets important but theoretically superfluous.[25] In general, even the more recent interpretations of the intertemporal general equilibrium in the tradition of Malinvaud (1953) exclude financial assets.[26] Walras' law, at least in its usual formulation, is therefore not applicable.

The truth is more subtle. Remarkably, the first intuitive expectation approaches it quite closely. One of the three equilibrium conditions is indeed redundant for *all* points in time from zero until infinity although not *because* of Walras' law. This can easily be shown if (2.43) is replaced with the equivalent condition

$$S^u = S^h, \quad D^u(0) = D^h(0), \tag{2.44}$$

where $S^u \equiv \dot{D}^u$ is the net increase in debt desired by the firms and $S^h \equiv \dot{D}^h$ is the supply of savings planned by the households. If S^h is substituted according to (2.25) in Equation (2.44) and Π^{du} from (2.1) and (2.2) is used, then it follows immediately that (2.44), and hence also (2.43), imply the condition

$$0 = w[L - L^u] + [f(K^u, L^u) - \delta K^u - I^u - C^h]. \tag{2.45}$$

This proves the contention.

The model developed above is based on the assumption that there is a sufficient number of markets for a perfect coordination of economic plans

[25] "Le marché du capital numéraire, qui est un avantage pratique n'étant ainsi qu'une superfétation théorique, nous le laisserons de côté ...".

[26] Cf. Cass and Majumdar (1979) and the literature cited by these authors. An exception is the contribution of Malinvaud (1966) but no reference is made there to the role of Walras' law.

or a perfect foresight of the development of all market data. This assumption is a strong idealization. While intertemporal contracts with a certain, sometimes significant, depth are settled in the labor and especially in the capital markets,[27] the commodity markets are typically organized as spot markets. Futures markets for commodities are more an exception than a rule. In the light of this fact, Condition (2.45) is quite illuminating for it shows that, in the model presented here, forward markets are not necessary for an intertemporal general equilibrium. Perfect capital and labor markets are sufficient for coordinating private plans in *all* markets of the model.

The reason for this result is what could be called the *intertemporal linking function of the capital market.* To illustrate this, assume that, starting from a situation of general equilibrium, the household sector is subject to an exogenous change in preferences and plans to reduce its consumption at a point in time t in order to finance an increase in consumption at a later point in time t^*. If the capital market operated in a way similar to a commodity market in a static equilibrium model, then neither it nor the labor market would be affected at any point in time by the change in the excess demands in the two consumption goods markets in t and t^*. Intertemporal equilibria in the labor and capital markets would be compatible with disequilibria in the commodity markets. In fact, however, provided the labor markets are in equilibrium, an excess demand in the commodity market at a point in time t^* can appear only if for the time span between t^* and t there is also an excess demand for bonds or an excess supply of loans. Conversely, this implies that, where there is equilibrium in both, capital and labor markets, there can be no excess demand in the commodity markets.

It could be objected that the household has opportunities for an intertemporal resource transfer in addition to the capital market so that the desire to shift consumption between the two points in time does not have to affect this market. However, this objection is not valid, for the essence of Fisher's separation theorem is that these possibilities are employed *independently* of the preferences of the household up to the point where the marginal return on the capital used for them equals the market rate of interest.[28] Thus the change in preferences assumed in the example would have the same consequences as before.

[27]Note that, in addition to formal contracts, the labor market is usually characterized by implicit contracts that cover a much longer time span. Legal or contractual periods of notice are therefore not appropriate indicators of the true length of employment contracts.

[28]See Chapter 1.3.

Another objection could be directed towards the limitation of the number of commodities in the model. There seem to be difficulties associated even with a model that has more than one consumption goods market at each point in time, but, in fact, this is not necessarily so. Consider the simple case of constant marginal rates of transformation between the consumption goods that can be produced at a particular point in time and assume the transformation can be carried out instantaneously. Then it is again necessary that the equilibrium conditions (2.41) through (2.43) hold and, as before, (2.42) is implied by (2.41) and (2.43). The only peculiarity is that, in (2.42), the quantity of consumption C^h planned by the household has to be reinterpreted as a desired expenditure *sum*, while the investment commodity may act as numeraire. The fact that only the size, but not the allocation, of this sum to the different commodities is determined shows the limits of the coordination function of the capital market. However, contrary to first appearances, this does not mean that commodity futures markets are required for an intertemporal general equilibrium. Under the assumptions made, commodity spot markets will be sufficient even in the multi-commodity case. However households want to allocate their consumption budget, when the time of decision comes, the sector of firms will succeed in matching all consumption plans with the given factor endowments available at this point in time by simply reallocating the production structure. The level of investment is not affected. Thus the growth path of the economy satisfies the conditions of an intertemporal general equilibrium although a significant part of the necessary coordination of plans is carried out in spot markets only.

Admittedly, reality is even more complicated than this scenario. In particular, it seems quite obvious that completely constant marginal rates of transformation and instantaneous transformation possibilities cannot plausibly be assumed. Nevertheless, the scenario provides a picture of the intertemporal allocation mechanism that could approach reality quite closely. The capital market brings about a rough and approximate coordination of plans in that it determines the time paths of the stock of capital and hence the allocation of future production capacities to consumption and investment uses. When the time comes, this approximate coordination is supplemented by a kind of fine tuning of private plans on commodity spot markets or comparatively short run futures markets.

In practice the possibility that the fine tuning turns out to be insufficient and that – contrary to the idea of intertemporal general equilibrium – planning or coordination mistakes show up cannot be excluded. If these or other market mistakes are a central element of capitalistic growth, as some economists believe, then the approach chosen in this book is not a useful idealization of reality. However, there seems to be good reason for optimism. Mistakes in coordination mean a violation of rules of Pareto efficient allocation and hence utility and/or profit losses. In an economy with well defined property rights and selfish maximizing agents, there are strong incentives for removing these coordination mistakes through a creation of new markets.[29] There are, in fact, futures markets for *some* commodities. It is by no means surprising, in the light of what has been said before, that these commodities are typically characterized by relatively large price changes. Commodities whose prices do not follow the general trend cannot be connected with other goods through constant marginal rates of transformation. But whatever the truth, the previous considerations of the two-step coordination process in capital and commodity spot markets have been able to show at least that the mere fact that futures markets are absent is not in itself a sign of market failure. Instead, it seems that, in principle, such markets are so rare because there is simply no need for them.

2.6. The Laissez-faire Allocation and the Social Optimum

The basic structure of the model of intertemporal allocation has been set up, the individual optimization approaches of households and firms have been solved, and central conditions of the market equilibrium are known. Now the question is which growth paths will result from the interaction of autonomous market forces.

Before an answer to this question can be given, two technical remarks are appropriate. (1) The superscripts "h" and "u" are omitted in the following since all variables of the model that differ only by these superscripts have the same values in a market equilibrium. (2) According to (2.21), the employment of efficiency units of labor is continuously growing at a constant rate $n + g$. In the hope of achieving constant steady-state values, it is stipulated that the flow of consumption, the stock of capital, and the production level are defined relative to these efficiency units:

$$c \equiv \frac{C}{L}, \quad k \equiv \frac{K}{L}, \quad \varphi(k) \equiv f\left(\frac{K}{L}, 1\right) = \frac{f(K, L)}{L}. \tag{2.46}$$

Using these definitions, two important differential equations can be

[29] It is true that the costs of installing and running markets are obstacles to a complete coordination of plans. On the other hand, these costs are the upper bound of the losses that can arise from the non-existence of markets.

achieved that represent possible time paths of the economy in a (c, k) diagram. The first of these differential equations is

$$\dot{k} = \varphi(k) - (\delta + n + g)k - c. \tag{2.47}$$

It follows from the equilibrium condition (2.42) if it is noted that, according to (2.7), $\dot{K} = I$ and that, because of $\hat{k} = \hat{K} - (n + g)$, it holds that $\dot{k} = \dot{K}/L - (n + g)k$.

This is a crucial technological relationship that describes the time change of the normalized stock of capital or, as we want to say, of the *capital intensity* k. With $\dot{k} = 0$ Equation (2.47) implies the concave curve of Figure 2.1. On this curve, k is constant since the gross investment per efficiency unit of labor, $\varphi(k) - c$, is sufficient to compensate for the reduction in k that, ceteris paribus, would be brought about through the depreciation of capital and the increase in efficient labor. Below the curve, investment is higher and k grows over time; above, the reverse is true.

At the maximum of the $(\dot{k} = 0)$ curve, it holds that $\varphi' - \delta = n + g$. This condition is the *Golden Rule of Accumulation* derived by Phelps (1961), Meade (1962, pp. 110 n.), and von Weizsäcker (1962) that characterizes the highest possible steady-state growth path of consumption. In a primitive

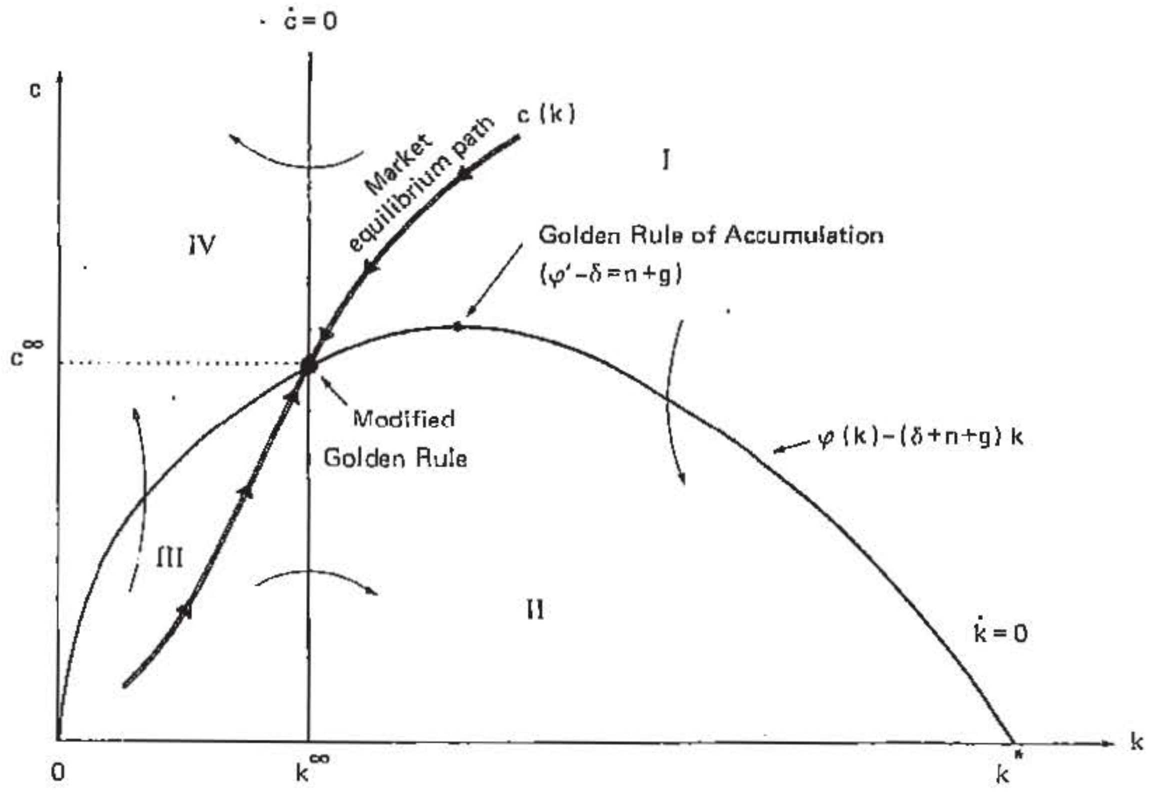

Figure 2.1. The general intertemporal equilibrium in a decentralized economy.

capitalist economy, where all capital income is invested and all wages are consumed, it is automatically satisfied in the long run, for in such an economy the growth rate of the capital stock will always be equal to the market rate of interest ($\hat{K} = r = \varphi' - \delta$) and will approach the natural rate of growth as time goes to infinity.[30] However, an economy with intertemporally optimizing agents and perfect capital markets exhibits a different growth pattern.

In order to recognize this pattern, the second differential equation is necessary. Note first that, because of $\hat{c} = \hat{C} - \hat{L}$ and $\hat{L} = n + g$ from (2.39), it follows that

$$\hat{c} = \frac{r - \rho}{\eta} - g.$$

If Condition (2.16) is utilized and the equation $f_K = \varphi'$ that follows from the definition of $\varphi(k)$ is taken into account we obtain

$$\dot{c} = \frac{c}{\eta}[\varphi'(k) - \delta - \gamma^{\infty}] \tag{2.48}$$

with

$$\gamma^{\infty} \equiv \rho + \eta g. \tag{2.49}$$

The quantity γ^{∞} is the *steady-state rate of time preference* of the representative household for, according to (2.37) and (2.38), the rate of time preference is generally defined as

$$\gamma = \rho + \eta(\hat{C} - n), \tag{2.50}$$

and in the steady state it holds, we hope, that $\hat{C} = n + g$.

According to Equation (2.48), the sign of the time derivative of c depends on k and γ^{∞}, but not on c itself. Thus the equation implies that the geometrical locus of the points characterized by $\dot{c} = 0$ is a vertical straight line in the (c, k) diagram. Because of $\varphi'' < 0$, to the left of this line it holds that $\dot{c} > 0$, to the right that $\dot{c} < 0$. In order to find out which position the line obtains relatively to the $(\dot{k} = 0)$ curve note that, according to (2.47), the slope of the $(\dot{k} = 0)$ curve has the value $\varphi'(k) - \delta - (n + g)$. If, for example, $\gamma^{\infty} = n + g$, then this straight line passes exactly through the maximum of the $(\dot{k} = 0)$ curve. For existence reasons that will be explained below it must be assumed, however, that the steady-state rate of time preference exceeds

[30]Note that, because of the linear homogeneity of the production function, there are no economic profits.

the natural rate of growth of the economy:

$$\gamma^{\infty} > n + g. \tag{2.51}$$

This condition ensures that the straight line is located to the left of the maximum of the ($\dot{k} = 0$) curve and hence implies a steady-state capital stock that is smaller than the one characterizing the Golden Rule of Accumulation.

At the point of intersection of the ($\dot{c} = 0$) and the ($\dot{k} = 0$) curves there is a steady-state point of the model where, by construction, c and k stay constant over time. The steady-state values c^{∞} and k^{∞} are implicitly given by

$$\varphi'(k^{\infty}) - \delta = \rho + \eta g = \gamma^{\infty} \tag{2.52}$$

and

$$c^{\infty} = \varphi(k^{\infty}) - (\delta + n + g)k^{\infty}. \tag{2.53}$$

The ($\dot{k} = 0$) and the ($\dot{c} = 0$) curves delimit four regions denoted by I through IV in Figure 2.1. According to the previous considerations, for each of these regions there is a typical direction of motion. On the border lines between the regions (outside the steady-state point) the motion is either horizontal or vertical. The possible shapes of the paths are illustrated by the curved arrows in Figure 2.1. If the economy is in Region II or in Region IV it can no longer move into the other regions. In Regions I and III, however, different developments are possible. The paths can pass across the ($\dot{k} = 0$) and the ($\dot{c} = 0$) curves into Regions II or IV, but they can also go to the steady-state point. The latter case is illustrated through the *market equilibrium path* which is the heavy line in Figure 2.1. Paths that deviate from the market equilibrium path anywhere have this property everywhere. It is true that they can stay for a very long time in its neighborhood. However, eventually they will glide off either towards Region II or Region IV. That there is one and only one market equilibrium path follows, among other things, from the fact that $\dot{c}$ and $\dot{k}$ are strictly monotonic functions of c and k in Regions I and III. We forego a proof of this property.

Appendix C shows that only the market equilibrium path is compatible with the conditions of an intertemporal general equilibrium. Paths above this path will, in finite time, lead to infinite factor prices. Paths below approach the point with the coordinates $(0, k^*)$ and thus at the very least violate the transversality condition (2.17) of the representative firm. The market equilibrium path, on the other hand, is compatible with all the optimization conditions and constraints on the decision problems of the representative household and the representative firm.

The liquidity constraint (2.30) is not considered in the appendix. It generalizes the model to the case of an imperfect capital market and is interesting enough to be treated here. In the extreme case $\beta = 1$, it excludes any borrowing against human capital and hence requires material wealth to be positive for each point in time: $M + D \geq 0$. To find out whether this condition is satisfied, note that the linear homogeneity of the production function and the exogeneity of the factor price paths in the planning problem of the firm imply that, at each point in time, the market value of shares is a linearly homogeneous function of the two state variables K and D. Using Euler's theorem, the definitions of the shadow prices λ_K and λ_D, given in (2.9), and Equations (2.12) and (2.14) one thus obtains

$$M = \lambda_K K + \lambda_D D = K - D, \tag{2.54}$$

and it follows that the material wealth equals the value of the capital stock:

$$M + D = K > 0. \tag{2.55}$$

As the capital stock is strictly positive everywhere on the market equilibrium path this implies that the liquidity constraint (2.30) is never binding. The representative household has no incentive to borrow against its own future labor income, let alone against the labor income of its descendants.

Thus, despite all complexity, the intertemporal general equilibrium model considered here yields a remarkably simple allocation result. The crucial aspect is that the consumption behavior of the households that shows up in market equilibrium can be described with a simple consumption function of the kind

$$c = c(k), \quad c'(k) > 0, \quad c(k)\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}\varphi(k) - (\delta + n + g)k \Leftrightarrow k\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}k^{\infty}, \tag{2.56}$$

where k^{∞} is the steady-state value of the capital intensity defined in (2.52). The graph of the consumption function is the market equilibrium path from Figure 2.1. With a given historic starting value $k(0) = K_0/L(0)$ of the capital intensity, this consumption function uniquely describes the growth path of the economy. Later, our goal will be to find out how this consumption function, and with it the growth path of the economy, changes under the influence of taxation.

After studying the behavior of the decentralized model of an economy with households and firms that act on an individual basis, independently of one another, the question arises of how this result is to be evaluated from a welfare theoretic point of view. Many possibilities are conceivable. For example one could accept the *Golden Rule of Accumulation* as a normative precept. In this case, the market economy is guilty of an inherent deficiency in growth, and government action to overcome this deficiency seems appropriate. Alternatively, some intertemporal welfare function could be arbitrarily defined and then used as a yardstick for measuring the performance of the market economy. The only problem with such a procedure is that the evaluation has a merit-good component. A wise and benevolent central planner is assumed, who knows better what is good for people than they themselves do. Here this route will not be taken.

The alternative which is chosen consists of accepting the preferences of the representative household described in (2.19). Thus the following optimization problem from the point of view of a central planner has to be solved in order to find out how the laissez-faire market allocation is to be evaluated:

$$\begin{aligned}
&\max_{\{C\}} \int_0^\infty e^{-\rho t} N(t) U(C(t)/N(t))\,dt \\
&\text{s.t. } \dot{K} = f(K, L) - \delta K - C, \quad L = NG, \\
&N(t) = N_0 e^{nt}, \quad G(t) = G_0 e^{gt}, \quad N_0 > 0, \quad n \geq 0, \quad G_0 > 0, \quad g \geq 0, \\
&K(0) = K_0 > 0, \quad K, C \geq 0. \qquad (2.57)
\end{aligned}$$

The solution of this problem is well known from the work of, for example, Arrow and Kurz (1970, Chapter III). It corresponds fully to the intertemporal general equilibrium that was here derived from separate individual optimization approaches and a number of market clearing conditions. There are no differences in either the steady-state condition (2.52) known from the central planning literature as the *Modified Golden Rule* or the path leading to the steady-state point, as described by (2.56).

For the solution of the central planning problem to exist it has to be assumed that $\rho + \eta g > n + g$; that is, that the steady-state rate of time preference exceeds the economy's natural rate of growth.[31] With (2.51), this inequality was assumed here too, and it was shown to imply a steady-state to the left of the Golden-Rule point. In Appendix C it is proved that the inequality is a necessary condition for the existence of an intertemporal general equilibrium. If it is not satisfied, then, at the very least, the transversality condition of the representative household is violated. Thus, given the preferences and technology assumed, no dynamically inefficient steady state, and not even the Golden-Rule solution, can result from market equilibrium.

[31] See Arrow and Kurz (1970, pp. 71 n.).

2.7. Alternative Approaches and the Problem of Built-in Interventionism

A perfect congruence between the laissez-faire allocation and the social optimum is the main characteristic of the model of decentralized economic growth developed here. Based on non-trivial individual intertemporal optimization problems of households and firms, the approach produces the well-known neoclassical central planning solution for optimal economic growth including the steady-state point that satisfies the Modified Golden Rule and the adjustment path towards this point. This characteristic implies that the laissez-faire allocation can be used as a yardstick for evaluating tax distortions and it supports the goal of *taxation neutrality* frequently recommended.

One of the reasons for the congruence between the social optimum and the laissez-faire solution is the assumption that the preferences of households are the same as those from which the central planner derives the socially optimal growth path. This assumption creates a sharp contrast between the present approach and a large number of other decentralized growth models that were constructed to investigate the dynamic effects of government activity. Some of these models, including those in the text book literature,[32] follow the tradition of Solow (1956) and use more or less ad hoc Keynesian-type behavior functions to explain the supply of savings capital by households. The models of Krzyzaniak (1966), Sato (1967), Feldstein (1974a), Grieson (1975), and Friedlaender and Vandendorpe (1978) have to be mentioned in this context.[33] The intertemporal allocation that is generated by these models when taxes are absent cannot, in general, be considered to be socially optimal regardless of the details of the concept of optimality.

Another group of models consists of the so-called overlapping-generations approaches like those of Diamond (1965, 1970), Pestieau (1974), Ihori (1978), Ordover and Phelps (1979), Atkinson and Sandmo (1980), King (1980), and Rose and Wiegard (1983). It is true that in these models the accumulation behavior of the economy is explained using intertemporal optimization approaches for households. However, the preference structures assumed for the households are not compatible with the social welfare function of the neoclassical growth model although this welfare function, or

[32] Cf. e.g. Atkinson and Stiglitz (1980, pp. 230–242).

[33] Cf. the comments made on these studies in Sections 10.1 and 10.4.2.1. The reproach that, in the next paragraph, is made against the overlapping-generations literature does not affect these studies as the authors are solely concerned with the purely positive analysis of tax incidence and abstain from studying the welfare-theoretic implications of taxation.

slightly modified versions of it, are explicitly accepted by the authors and used as a benchmark for evaluating the allocation.[34] Even in this group of models the laissez-faire behavior of the economy cannot in any meaningful sense be considered as socially optimal. Implicitly or explicitly *government intervention* is recommended in order to compensate for the differences between private and social preferences.

The crucial point at which, in the cited contributions, the preferences of the households deviate from the social welfare function of the neoclassical growth model is the treatment of future generations. In the social welfare function the well-being of future generations is taken into account, albeit with declining weights. In the preference function of the households on the other hand, future generations are completely neglected. No approach allows for a bequest motive. The only reason – and this is very important in the overlapping-generations literature – that households save and the stock of capital is accumulated at all is to ensure a sufficient retirement consumption.[35]

The importance of the role played by the bequest motive in the intertemporal allocation mechanism is well known from Barro's (1974) analysis. If each generation is concerned for the happiness of its own children then the utilities of *all* generations are interlinked and the decisions of today's generations will indirectly take into account the preferences of all future generations. Formally, this utility interdependence between the generations means that the allocation mechanism operates as if the households deciding today were infinitely long lived. Thus it legitimates the model specification chosen here.

The merit-good component that is implicit in the different roles the bequest motive plays in the preferences of the household and those of the central planner was deliberately avoided in the previous section so as not to mix up efficiency considerations with value judgements in the analysis of government intervention. This is not necessarily the only sensible procedure. Possibly the authors mentioned above consider the divergences between private and social preferences so obvious that they see no purpose in separating efficiency considerations from value judgements. If this is the case, one might be inclined to defend the neglect of the inheritance motive

[34]Some of the authors take the Golden-Rule path as the social optimum, which follows from the central planning problem (2.57) as a limiting case for $\rho \rightarrow 0$.

[35]A central result of the literature is that this reason for capital formation vanishes to the extent that government bonds are introduced into the models. Cf. Diamond (1965) where the previous contentions by Modigliani (1961) and Vickrey (1961) are supported in a formal model.

as a mere idealization that overstates reality but correctly reflects observable tendencies. Unfortunately, however, the problem does not seem to be quite so trivial.

Barro (1974, pp. 103–106), Carmichael (1982, pp. 204–206), and Burbidge (1963) show that the assumption of a *corner solution* with regard to the optimal level of bequest is of decisive importance for the way the traditional overlapping-generations model functions. If the representative household plans to leave a bequest to its heirs, even a very small one,[36] then basic results of the model disappear.

One example here is the possibility that the laissez-faire allocation in the overlapping-generations model is inefficient, something first pointed out by Diamond (1965); that is, the fact that, under certain parameter constellations, a steady-state capital intensity above that of the Golden Rule will prevail.[37] The investigations of Carmichael and Burbidge show that, if such a possibility exists at all, it can only be compatible with the existence conditions of a generation's intertemporal optimization problem if the preferences of this generation exclude a bequest to future generations.[38] However, if such single generation has an altruistic concern for its descendants and leaves a strictly positive bequest, then a solution of the intertemporal optimization problem will only exist with a steady-state capital intensity below the Golden-Rule value; that is, in a range where, as with the steady state of Figure 2.1, the marginal product of capital *exceeds* the natural rate of growth.

This confirms results previously achieved without an explicit modelling of overlapping generations by Koopmans (1965, Section 6) and Arrow and Kurz (1970, p. 71 n.) for a central planning economy.[39] Moreover, it now becomes obvious that a properly specified overlapping-generations model

[36]In fact it seems that a significant part of aggregate capital formation can be attributed to the bequest motive. See Kotlikoff and Summers (1981) for an empirical analysis of the problem.

[37]Such a situation is inefficient since it would be possible to increase consumption permanently to the Golden-Rule level. In Figure 2.1, efficient steady states are situated on the $(\dot{k}=0)$ curve to the left of the maximum. In this region, a permanent increase in consumption is impossible, and a temporary increase can only be achieved if it is "paid" for by a reduction in consumption during other periods of time.

[38]This was first conjectured by Barro (1974, Footnote 12). Carmichael even shows that an inefficient solution must obtain if people favor their parents over their own children. This is a theoretical possibility that is not in line with the evolutionary adjustment of our preferences towards a maximization of the survival probability of our genes and hence lacks a biological foundation. Compare the discussion in Chapter 11.1.

[39]The corresponding proof for a decentralized economy can be found in Sinn (1981, p. 300 in connection with Appendix B) or in the appendix of a 1979 Mannheim discussion paper (132–79) that contains an extended version of Sinn (1980b). The discussion paper is available from the author upon request.

is much more closely related to the approach used in this book than the casual reader of the overlapping-generations literature might expect.

In the light of the results of Carmichael and Burbidge, it seems justified to doubt the recommendations of the overlapping-generations literature. Does the laissez-faire economy really need government intervention in order to produce an efficient intertemporal allocation? In particular, is it true that the laissez-faire economy tends to over-accumulate real capital, thus requiring the introduction of a social insurance scheme or the issue of new government bonds as a remedy, as many authors contend? The doubts persist even if one neglects the problem of merit goods. There is the risk that a model simplification that seemed quite innocent at first sight has produced a discontinuity that was not intended but nevertheless is fraught with grave consequences. It is possible that, on efficiency grounds alone, certain kinds of government intervention are recommended that, with a realistic model specification, turn out to be superfluous or even detrimental.

These remarks on the connections between the current approach and the existing literature suffice for the time being. Later, in Chapter 11, the problem of the optimality of the laissez-faire allocation is taken up once again in order to scrutinize the desirability of certain proposals for tax reform from points of view other than the one proposed here.