Capital Income Taxation and Resource Allocation

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Chapter 4: Taxation and the Financial Decisions of the Firm

Chapter 4

TAXATION AND THE FINANCIAL DECISIONS OF THE FIRM

It hardly seems possible to understand the process of capital formation in an economy and the way tax systems can affect it if the effect of taxation on firms' financial decisions is not known. If only for this reason the present chapter is essential. In addition, however, the financial decisions of the firm are interesting in their own right. For example, policy makers and economic advisors are frequently afraid of excessive debt financing. Too much financing through debt creation is considered to be unsound and to make the economy vulnerable to economic crises. That the tax system should be financially neutral is a generally accepted requirement.

Three financial instruments are available to the firm: retained profits, new issues of shares, and debt. In the absence of taxes, all three instruments are equivalent; that is, a change in the firm's planned financial policy does not affect today's market value of its shares. However, taxes create clear preferences for particular financial instruments and, in addition, alter the financial constraints on the firm which, as a result of these preferences, begin to play an important role in decision making. The influence on the financial decisions of the firm that results from alternative systems of capital income taxation is the subject of this chapter. The problem is discussed in the context of the general optimization problem of the firm presented in Chapter 3.2.

A vast theoretical literature exists which illuminates many facets of the financing problem but it seems that the last word on the impact of taxation on financial decisions has not yet been said and, of course, it cannot be said here either. For example, the effects of double taxation of dividends, the determinants of the relative desirability of debt financing versus financing through retained earnings, and the role of taxation in explaining a finite debt–equity ratio are quite unclear. Moreover, the role of institutional constraints on financial decisions has not received much attention. As a result, there is no developed theory of the real economic distortions caused by alternative systems of capital income taxation that incorporates the firms' financial decisions. Extending King's (1974a, 1977) pioneering work on the firm's financial decisions in the presence of taxation this and the following chapters try to help close the gap.[1]

Studies of the impact of taxation on the firm's real investment behavior and of the resulting economic distortions typically bypass the problem of finance. The "Harberger literature" that accuses the corporate income tax of causing huge welfare losses is a prominent example which will be criticized in the subsequent chapters. Typically it is (implicitly) assumed that 100% of the firm's real net investment is financed with new issues of shares notwithstanding the fact that, under the classical system of capital income taxation considered in this literature, this is both the most expensive of the three elementary sources of finance and also the one that is, in reality, least frequently used. It is true that dutiful lip service is often paid to the possibility of debt finance but this possibility is then quickly dismissed as "unrealistic". Even worse, the Harberger literature hardly ever gives attention to the possibility of financing through retained profits. Profit retentions and new issues of shares are lumped together under the heading of "equity finance" as if they incurred the same capital cost for marginal investment projects and hence brought about identical real distortions in the economy. Referring to the analysis of this chapter it will be demonstrated at various places in this book how misleading this view is.

The analysis of the financial decisions of the firm is begun in this chapter and is continued in the following Chapter 5 where, in connection with the firm's real investment decision, a hypothesis for the choice of an interior debt–equity ratio is developed. This chapter is arranged in four sections. The first section studies the solution space within which the financial instruments can be chosen. The second section compares pairs of alternative financial instruments. The third section points out the implications of the paired comparisons for an overall optimal financial decision and offers, in addition, some critical remarks on differing opinions about the problem of optimal finance. The fourth and last section considers special problems that result in the case where the firm is not allowed to deduct debt interest.

[1]For other extensions of this work see Auerbach (1979a, 1983) and Bradford (1981). An overview of the literature on optimal finance including the taxation problem is provided in Swoboda (1981). Cf. moreover Stepan and Swoboda (1982) for a review of the control theoretic approaches to the problem of finance (where, however, taxation problems are not in the foreground).

4.1. The Constraints on the Firm's Financial Decisions

In the laissez-faire model it was not necessary to consider the various institutional constraints on the firm's financial decision that are relevant in practice. Since, in the absence of taxes, firms are indifferent between all financial instruments, none of the constraints would have been binding. However, if there are different tax burdens on different types of capital income, the indifference will no longer hold and boundary solutions may prevail. For this reason, a careful modelling of the financial constraints that were assumed in a general form with (3.34) is essential. Attention will be limited to the constraints that are typical for corporations. Since it will turn out that the constellation $\tau_d = \tau_r = \tau_p$ and $\tau_c = 0$ that characterizes non-corporate firms implies an equivalence of all financial instruments, as in the case without taxes, we need not discuss the constraints that are valid for these firms. Without any loss of generality with regard to the firms' real investment choices, it can be assumed that the constraints must hold for all types of firm.

The analysis of the constraints is carried out by means of the diagram in Figure 4.1. The ordinate of this diagram shows the net increase in debt S_r and the abscissa measures the value of new issues of shares Q. The four borderlines represent constraints that have yet to be explained. The diagram holds for given levels of capital stock, employment of labor, real net investment, and stock of debt. For the time being, only the decision on the structure of the current flow of finance is considered as a variable.

The lefthand constraint that coincides with the ordinate excludes negative values of new issues of shares:

$$Q \geq 0. \tag{4.1}$$

This constraint forbids the firms to repurchase their own shares. Share repurchases are illegal in most countries to prevent firms making distributions to their shareholders that are exempt from personal income tax. Even for the United States that seems to be very generous in this regard and does not formally forbid repurchases it would be wrong to allow for negative values of Q since here the tax law treats regular repurchases as dividends and taxes them.[2] Personal tax on dividends could also be

[2]A useful discussion for the situation of the United States can be found with Auerbach (1979a, p. 439). For West Germany, to take another example, see Aktiengesetz (1965, §§57 and 71).

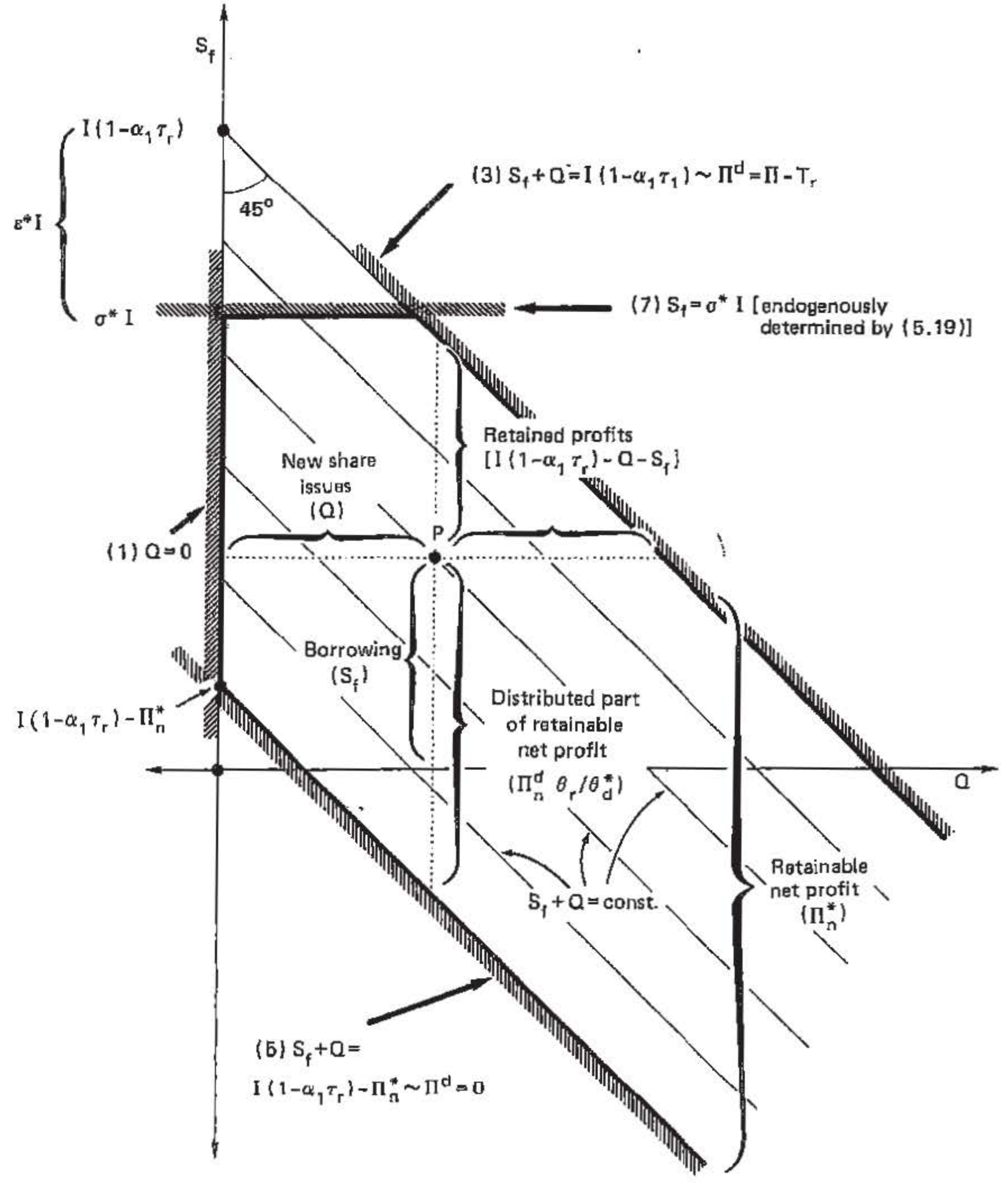

Figure 4.1. The financial constraints of the firm.

circumvented through mergers where firms buy other firms' shares in the capital market. This possibility is excluded for the time being by assuming effective anti-trust regulations. Chapter 6.1.2 gives a detailed analysis of the implications of removing this assumption.

A further, quite important, constraint that approximates a proscription common to all Western industrial countries is that the gross dividends Π^d cannot exceed the accounting profit net of the tax on retentions:[3]

$$\Pi^d \leq \Pi - T_r. \tag{4.2}$$

The purpose of this constraint is to keep the equity base of corporations intact. To see how the constraint shows up in the (S_f, Q) diagram of Figure 4.1 it seems useful to transform (4.2) using (3.26) into the equivalent inequality

$$S_f + Q \leq I(1 - \alpha_1 \tau_r) \qquad [\sim (4.2)]. \tag{4.3}$$

This inequality says that, at most, only that part of net investment not covered by tax savings through accelerated depreciation can be financed with new issues of debt or shares or, in other words, that negative retentions $[I(1 - \alpha_1 \tau_r) - S_f - Q < 0]$ are forbidden. The figure represents this inequality by the upper of the two borderlines inclined to the right. Since on this line (4.3) holds with equality, the line has obviously a slope of -1 and intersects both the ordinate and the abscissa at a distance from the origin of $I(1 - \alpha_1 \tau_r)$.

As a further constraint, exclusion of negative dividends is assumed:

$$\Pi^d \geq 0 \qquad \text{(will occasionally be removed)}. \tag{4.4}$$

If this constraint did not hold, government would in fact allow a kind of equity finance where funds injected into the firm are subsidized in the same terms as dividends are taxed. In the discussion of reform systems in Chapters 5 and 11 such a possibility will indeed be considered and the constraint will be removed for this purpose. In the normal case, however, its validity will be assumed.

If (3.18), (3.25), and (3.26) are used, then (4.4) can be transformed into the equivalent condition

$$S_f + Q \geq I(1 - \alpha_1 \tau_r) - \Pi_n^*$$
$$[\sim (4.4), \text{ will occasionally be removed}], \tag{4.5}$$

where

$$\Pi_n^* \equiv \theta_r[f(K, L) - \delta K - wL - rD_f - \tau_k K] + \tau_r(\alpha_2 rK - \alpha_3 rD_f) \tag{4.6}$$

is a quantity that will be called *retainable net profit*. Condition (4.5)

[3] A modification of this constraint can result from the fact that the firms pay dividends out of surplus reserves. This possibility is disregarded since it can obviously only be temporarily important. For a discussion of alternative stock constraints see Boadway and Bruce (1979).

indicates that the sum of debt financing and new issues of shares must be at least large enough to ensure that that part of net investment which cannot be financed through retentions (Π_n^*) and deferred taxes ($\alpha_1 \tau_r I$) is covered. If this condition is written in the form $\Pi_n^* \geq I(1 - \alpha_1 \tau_r) - S_r - Q$ then it says that retentions cannot exceed net profit.

In Figure 4.1, Condition (4.5) is represented by the lower of the two inclined borderlines. Since Π_n^* is independent of S_r and Q, this line too has the slope -1 and hence is parallel to the upper inclined borderline. The line intersects both the abscissa and the ordinate at a distance $I(1 - \alpha_1 \tau_r) - \Pi_n^*$ from the origin for, if the equality sign holds in (4.5), then $S_r = I(1 - \alpha_1 \tau_r) - \Pi_n^*$ when $Q = 0$, and $Q = I(1 - \alpha_1 \tau_r) - \Pi_n^*$ when $S_r = 0$. The vertical (and horizontal) distance between the two inclined borderlines is Π_n^*. In the figure it was assumed that $\Pi_n^* < I(1 - \alpha_1 \tau_r)$, but the case $\Pi_n^* \geq I(1 - \alpha_1 \tau_r)$ is also admissible.

In addition to the three constraints described above, which follow directly from legal proscriptions, it is assumed that

$$S_r \leq \sigma^* I. \tag{4.7}$$

Here, σ^* indicates the *maximum marginal debt–asset ratio* or, more precisely, the maximum proportion of net investment that the firm is allowed to finance through new loans raised in the credit market.

The term "debt–asset ratio" has to be interpreted in a very narrow sense, since accounting laws treat deferred taxes as a debt to the government. In a broader sense, including deferred taxes, the maximum marginal debt–asset ratio is $\sigma^* + \alpha_1 \tau_r$. Postulating a maximum marginal debt–asset ratio σ^* is equivalent to postulating a *minimum marginal equity–asset ratio* ε^*. The relationship between these two ratios is given by

$$\varepsilon^* \equiv 1 - \sigma^* - \alpha_1 \tau_r. \tag{4.8}$$

Frequently in this book, reference will be made to ε^* rather than to σ^*.

In the figure the constraint is represented by the horizontal upper borderline that intersects the ordinate between the two inclined borderlines. In general it is assumed that

$$-\alpha_1 \tau_r \leq \sigma^* \leq 1 - \alpha_1 \tau_r \tag{4.9}$$

or, equivalently, that $1 \geq \varepsilon^* \geq 0$. The first parts of these inequalities exclude the possibility that the firm is forced to finance more than 100% of its net investment with equity capital. The other parts ensure that Constraint (4.7) is a potentially more severe limitation to debt financing than Constraint (4.3).

A number of reasons for $\sigma^* < 1 - \alpha_1 \tau_r$ or $\varepsilon^* > 0$ are conceivable. Among them are unwritten laws of sound finance that seem to be important for businessmen, the attempt to avoid bankruptcy, the necessity of building up risk-bearing capital, or simply the attempt to signal successful management. While it seems difficult, if not impossible, to incorporate most of these reasons explicitly into the model, there is at least one hypothesis that can be endogenously derived within the framework of the perfect-foresight world studied in this book. This hypothesis is based on the rivalry between accelerated depreciation and debt interest for tax deductibility in the case where the corporate income tax is endowed with a limited loss-offset. The problem will be discussed in detail in Chapter 5.2. For the time being, σ^* is treated as an exogenous parameter in the model.

It should be noted that $\sigma^* < 1 - \alpha_1 \tau_r$ implies a diminution of the solution space only when $I > 0$ for only in this case $\sigma^* I < (1 - \alpha_1 \tau_r)I$. The financial constraints that become operative in shrinking economies are not discussed here though. Instead it is assumed that the firm's real net investment is non-negative ($I \geq 0$) when $\sigma^* < 1 - \alpha_1 \tau_r$ and the tax system favors debt financing over profit retentions so that the upper horizontal borderline becomes binding (cf. Types 1–3 in Figure 4.2). Since the hypothesis for an endogenous explanation of $\sigma^* < 1 - \alpha_1 \tau_r$ requires long run growth of the firm, this partial relinquishment of an analysis of negative net investment will turn out not to be very restrictive.

The solution space is unlimited in the south-eastern direction. Thus, new issues can be increased at the expense of debt financing without limitation. No law in the world forbids the firm to redeem its debt and, as far as is known, the firm is never forbidden to become a creditor.

In the solution space between the upper and the lower borderlines there are a number of parallels on which $Q + S_f$ is constant. Since I is treated as a given quantity, each of these lines characterizes given levels of retentions and dividends. The retentions are $I(1 - \alpha_1 \tau_r) - S_f - Q$ and are hence represented by the vertical (or horizontal) distance from the upper one of the two inclined borderlines. The distance from the lower inclined borderline is $S_f + Q - [I(1 - \alpha_1 \tau_r) - \Pi_n^*]$ and measures that part of the retainable net profit that is used for distributions to shareholders. This can be seen if this distance is multiplied by θ_d^*/θ_r in order to express it in terms of net-of-all-tax distributions available to the shareholder: using Definition (4.6), the expression for Π_n^d given in (3.28) is obviously achieved. As exemplified for the point P, the magnitudes of all three forms of finance as well as the level of distributions can be inferred from the position of a particular point in the solution space.

Up to now it has been tacitly assumed that the solution space exists; that is, that all constraints can be satisfied simultaneously. This is the case if, and only if, the "upper" of the two inclined borderlines is not situated below the "lower", or in other words, if dividends are not at the same time required to be both strictly negative and strictly positive. Because of (4.3) and (4.5), this condition is formally $I(1-\alpha_1\tau_r) \geq I(1-\alpha_1\tau_r) - \Pi_n^*$ or

$$\Pi_n^* \geq 0. \tag{4.10}$$

Hence the existence is ensured if the retainable net profit is non-negative. Appendix A demonstrates that this condition is equivalent to postulating a non-negative market value of shares ($M \geq 0$) and will be satisfied, for example, if, as assumed with (3.33), the initial market value is strictly positive and if the firm's real net investment is non-negative ($I \geq 0$). Throughout this book it is assumed that (4.10) is satisfied for all $t \geq 0$.

The preceding considerations have shown how narrow the scope for deciding about the structure of financial instruments is. The expense of an investment project is implicitly separated into several parts by the constraints. Without allowing the firm any choice, it is specified first that the liquid funds (albeit not profits) resulting from true economic depreciation (δK) and the current tax savings through accelerated depreciation must be retained. Somewhat more flexible are the possibilities for forming equity capital of size $(1-\sigma^*-\alpha_1\tau_r)I$, for the firm can choose between retentions and new issues of shares. Only on the remaining part of the expense, $\sigma^* I$, can the firm decide freely and choose the optimal financial instrument where, however, once again a constraint has to be respected, namely that sufficient retainable profits are available. If this is not the case (as in the figure), then the decision about how to cover the missing amount will again narrow down to two financial instruments: debt financing and new issues of shares.

To capture the complexity of this multi-stage decision situation analytically may sound quite difficult a priori. However, with the aid of the diagram from Figure 4.1, this problem comes down to the simple task of finding the optimal point in a well specified solution space. If this point is known, the structure of optimal finance, including the decision on the size of corporate distributions, is determined. We now want to start searching for the optimal point.

4.2. Comparison of Financial Instruments

Following the analysis of the institutional *constraints* on the firm's financial decisions, attention will now be focussed on the financial *preferences* of the firm. Preferences meant are not preferences in the sense of mere utility levels. Because of Fisher's separation theorem, such preferences are unimportant in the decision making of the firm. Instead, preferences that follow from the goal of maximizing the market value of company shares are considered.

The following sections compare in pairs the three financial instruments available to the firm in order to determine these preferences. Formally the procedure is such that the solution space shown in Figure 4.1 is crossed in three directions. Horizontally, in order to compare new issues with retentions. Vertically, in order to compare debt and retentions. And along the inclined parallels shown in the diagram in order to compare debt and new issues of shares.

The change in the market value resulting from the substitution of two financial instruments can be seen from the change in the value of the Hamiltonian (3.35) and serves as the criterion for evaluating these instruments. A glance at (3.28) shows that the Hamiltonian is linear in the net increase in debt S_f and in the value of new issues of shares Q. Hence the function describes a plane above the solution space shown in Figure 4.1. This implies that the preference relations between any two financial instruments that hold at some point are equally valid everywhere else in the solution space and that the solutions, provided they exist, are normally boundary solutions.[4] What these general properties of the solutions mean in detail will now be shown.[5]

4.2.1. New Issues of Shares versus Retained Profits

As mentioned above, it is frequently argued that the corporate tax discriminates against equity finance as such, and, implicitly, equity capital is seen as a homogeneous entity. This view seems highly misleading since equity capital can be formed in two ways, via retained profits $[I(1-\alpha_1\tau_r)-S_f-Q]$ and via new issues of shares (Q). Both possibilities are equivalent in a world without taxes but, with the existing tax laws, they are not. As these laws exclude the possibility of netting out corporate distributions and new issues of shares in order to calculate the tax liability

[4]Note that a solution on the upper horizontal boundary is not a solution with full debt finance, but one with partial equity finance, and that, moreover, the position of this boundary will be *endogenously* explained in Chapter 5.2.

[5]The analysis makes use of the definitions $\theta_r^* \equiv \theta_c\theta_r$ and $\theta_d^* \equiv \theta_p\theta_d$ from (3.10) and (3.11).

of the firm and its shareholders, retentions and new issues of shares have to be well distinguished. To find out the firm's preferences with regard to these two instruments differentiate the Hamiltonian (3.35), using (3.28) and assuming given values of I and S_{f}, for the control variable Q. The result is

$$\frac{\partial \mathscr{H}^{u}}{\partial Q}=\frac{\theta_{p}\theta_{d}}{\theta_{c}\theta_{r}}-1\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}0 \Leftrightarrow NI\begin{Bmatrix} \succ \\ \sim \\ \prec \end{Bmatrix}RP, \tag{4.11}$$

where NI stands for new issues of shares and RP for retained profits. Given the net investment I and the net increase in debt S_{f}, an increase in the flow of new issues of shares Q means a reduction of retentions and an increase of dividends. Thus $\partial\mathscr{H}^{u}/\partial Q$ measures the marginal advantage of substituting new issues of shares for retentions, or more pointedly: the marginal advantage of a *distribute-and-call-back policy.* According to the sign of this marginal advantage, the different preference relations between the two financial instruments that are indicated in (4.11) result.

For the solution space of Figure 4.1, these preference relations show the kind of horizontal move that the firm finds attractive. With $NI \succ RP$ a movement to the right and with $NI \prec RP$ a movement to the left is preferable. With $NI \sim RP$ all solution points that are situated on a horizontal line are equally attractive.

The particular algebraic form of the formula for the marginal advantage of a distribute-and-call-back policy can easily be explained if this policy is seen from the point of view of the shareholder and if it is assumed that one dollar of net profit is replaced by one dollar of new issues of shares. This replacement does not affect the future time path of the market value of company shares since it does not change the stock of equity capital, but it brings about advantages and disadvantages for the shareholder.

First, there is an advantage in that, according to Equation (3.13), capital gains taxes of the amount $\$\tau_{c}$ are saved. The reason for this saving is that a taxable increase in the market value of shares resulting from retaining profits is replaced by a tax-exempt increase in the market value resulting from new issues of shares.[6] A further advantage is, of course, the distribution which the shareholder receives. Because of the tax saving on retained profits, one dollar of retainable net profit can be transformed into $\$1/\theta_{r}$ of dividends and, after subtracting the corporate and the personal taxes on dividends, $\$\theta_{p}\theta_{d}/\theta_{r}$ remain. The disadvantage of the policy is that, as

[6]In some countries there is an additional tax on the value of new issues. See Hax (1979) for a theoretical discussion of this and other obstacles to new issues.

assumed, one dollar has to be returned to the firm. The net advantage in terms of dollars is therefore $(\theta_p\theta_d/\theta_r) + \tau_c - 1$. This implies that the policy is attractive for the firm if, and only if, $\theta_p\theta_d/(\theta_r\theta_c) - 1 > 0$.

As shown in Chapter 3.1, the classical system practised in the United States, for example, is characterized by $\theta_p\theta_d < \theta_r\theta_c$ since retained and distributed profits bear the same corporate tax burden ($\theta_d = \theta_r$) and since accrued capital gains are effectively taxed less than dividends at the household level ($\theta_p < \theta_c$). In this system, retentions are therefore preferred to new issues of shares.

In an ideal system with perfect integration between personal and corporate taxation (Teilhabersteuer) it holds that $\theta_p\theta_d = \theta_c\theta_r$. Hence, the firms are indifferent between retentions and new issues of shares.

In partial imputation systems like those of Canada, France, and the United Kingdom, the magnitude relationship between $\theta_p\theta_d$ and $\theta_r\theta_c$ is ambiguous. Depending on the representative shareholder's tax bracket, both a preference for retentions and a preference for new share issues are possible.

The full imputation system is typically characterized by $\theta_p\theta_d \geq \theta_c\theta_r$ since, as explained in Chapter 3, the corporate tax rate on distributed profits is zero ($\theta_d = 1$) and $\theta_p \geq \theta_c\theta_r$ is a stylized economic fact. In this system there can therefore be tax incentives to substitute new issues of shares for retained profits. Even here, such incentives will not necessarily prevail though. For shareholders whose marginal personal tax rate is sufficiently high, the case of an equivalence of the two sources of finance is possible, and, as will be argued in Section 4.3.4, a certain degree of plausibility can even be attributed to it.

4.2.2. Debt versus Retained Profits

After an analysis of the distribute-and-call-back policy we now consider what can be called the *borrow-and-distribute policy*. The firm issues new debt in order to finance distributions to its shareholders or, in other words: it substitutes debt financing for retentions. This policy is not frequently discussed in the finance literature, but to understand its effects is essential for the further analysis in this book. Many of the allocative results to be derived depend crucially on the way the tax system affects the firm's choice between debt and retentions as marginal sources of finance.

Given the level of net investment I and the level of new issues of shares Q, the following preference relations can be obtained through differentiating of the Hamiltonian (3.35) subject to (3.28):

$$\frac{\partial \mathscr{H}^{u}}{\partial S_{f}} = \frac{\theta_{d}\theta_{p}}{\theta_{r}\theta_{c}} + \lambda_{D}\begin{Bmatrix} > \\ = \end{Bmatrix}0 \Leftrightarrow DF\begin{Bmatrix} \succ \\ \sim \end{Bmatrix}RP. \tag{4.12}$$

Here *DF* stands for debt financing and *RP* again for retained profits. Since S_f varies with given I and Q the level of retentions $[I(1-\alpha_1\tau_r) - Q - S_f]$ has to vary inversely. The differential quotient $\partial\mathscr{H}^u/\partial S_f$ measures therefore, as required, the marginal advantage of a substitution of debt financing for retentions. Its sign determines in an obvious way the preference relations between *DF* and *RP* indicated in (4.12).

In the solution space of Figure 4.1, the policy considered means a vertical movement. With $DF \succ RP$ the firm prefers an upward movement, and with $DF \sim RP$ it is indifferent with regard to such a movement.

The case $\partial\mathscr{H}^u/\partial S_f < 0$, $DF \prec RP$ is not considered in (4.12), because a solution of the optimization problem of the firm does not exist here. Since, as can be seen from the following equation, (4.13), λ_D is a constant, the inequality $\partial\mathscr{H}^u/\partial S_f < 0$ would have to hold for all points in time if it holds at all and the maximization of the Hamiltonian would *permanently* require a solution at the lower boundary of the solution space. Since this boundary is characterized by a complete absence of any distributions ($\Pi^d = 0$) the market value of shares [cf. (3.24)] would be zero or, if $Q > 0$, negative. That, however, cannot be the result of an optimal policy if, as assumed, there are distributable profits. The problem is hardly diminished if the lower boundary of the solution space is removed, an exception admissible for the analysis of reform systems in the last chapter of this book. In this case, a maximization of the Hamiltonian with $\partial\mathscr{H}^u/\partial S_f < 0$ would require an unlimited "negative dividend" and unlimited capital market investments of the firm. This again implies the nonexistence of a solution to the optimization problem.

For an economic interpretation of (4.12) the shadow price λ_D has to be calculated. Analogously with (2.11) and (2.12), from (3.24) and (3.28)[7]

$$\lambda_D(t) = \frac{\mathrm{d}M(t)}{\mathrm{d}D_f(t)} = \int_t^{\infty} -r(v)\frac{\theta_d^*}{\theta_r^*}(\theta_r + \tau_r\alpha_3)\left[\exp\int_t^{v} -\frac{\theta_p}{\theta_c}r(s)\,\mathrm{d}s\right]\mathrm{d}v$$

[7]In the differentiation it is assumed that an increased interest payment results in a reduction of dividends and is not financed by new share issues. The correctness of this procedure could be doubted for the case where new issues of shares dominate retentions ($\theta_d^* > \theta_r^*$). Note, however, that, in this case, the optimal point is situated on the upper of the two inclined borderlines where all profits are distributed [i.e. $Q = I(1-\alpha_1\tau_r) - S_f$ in (3.28)]. Since an increased interest payment reduces the accounting profit and since not more than this profit can be distributed to shareholders, this means that a reduction of dividends has to be assumed even when issuing new shares would be cheaper.

$$= \left[\frac{\theta_d^* \theta_c}{\theta_r^* \theta_p} (\theta_r + \tau_r \alpha_3) \exp \int_t^v - \frac{\theta_p}{\theta_c} r(s)\, ds \right]_{v=t}^{v=\infty}$$

$$= -\left[\theta_d(1 - \alpha_3) + \frac{\theta_d}{\theta_r} \alpha_3 \right], \tag{4.13}$$

is obtained. If this value is inserted into (4.12) then it follows for the preference relation between retentions and debt that

$$\theta_p \begin{Bmatrix} > \\ = \end{Bmatrix} \theta_r \theta_c (1 - \alpha_3) + \theta_c \alpha_3 \Leftrightarrow DF \begin{Bmatrix} \succ \\ \sim \end{Bmatrix} RP. \tag{4.14}$$

This expression shows that the indifference between debt financing and retentions that prevailed in the laissez-faire model vanishes. The financial instrument that the firm prefers depends on the magnitude of the tax factors for personal interest income (θ_p), capital gains (θ_c), and retained profits (θ_r) as well as on the possibility (α_3) of deducting debt interest from the profit tax base (cf. Chapter 3.1.4).

To interpret the result, it seems useful to multiply the first inequality in (4.14) with $\theta_d \theta_p r/(\theta_r \theta_c)$:

$$\frac{\theta_d \theta_p}{\theta_r \theta_c} r \theta_p \begin{Bmatrix} > \\ = \end{Bmatrix} \theta_d \theta_p r(1 - \alpha_3) + \frac{\theta_d \theta_p}{\theta_r} r \alpha_3 \Leftrightarrow DF \begin{Bmatrix} \succ \\ \sim \end{Bmatrix} RP. \tag{4.15}$$

Consider first the case where debt interest is deductible ($\alpha_3 = 0$) and where the representative shareholder is indifferent with regard to the borrow-and-distribute policy. If the firm decides to take one additional dollar of credit as a substitute for retained profits, it can increase its gross dividend payments by $\$1/\theta_r$. After deducting the corporate and personal taxes on dividends, $\$\theta_d \theta_p/\theta_r$ remain. Through the dividends, the market value of shares changes by, say, $\$\Delta M$, and so the capital gains tax liability changes by $\$\tau_c \Delta M$. Altogether, therefore, the shareholder receives additional funds of size $\$(\theta_d \theta_p/\theta_r - \tau_c \Delta M)$. If the shareholder is, as was assumed, indifferent between retentions and debt, two requirements have to be met. First, immediately after the dividend payment, the market value has to fall to such an extent that selling the shares would create neither a gain nor a loss. Second, the future reduction of dividends made necessary by the additional interest burden must be of the same magnitude as the interest the shareholder can earn when he invests the funds initially received in the capital market. The first requirement says that $\Delta M = -(\theta_d \theta_p/\theta_r - \tau_c \Delta M)$. This implies that the initial funds received by the shareholder, $\$(\theta_d \theta_p/\theta_r - \tau_c \Delta M)$, have the size $\$\theta_d \theta_p/(\theta_r \theta_c)$. The second requirement implies that the current net interest return from investing these funds in the capital market, $\$\theta_p r\theta_d\theta_p/(\theta_r\theta_c)$, equals the interest-induced dividend reduction of size $\$\theta_d\theta_p r$. This explains (4.15) for the case where the equality sign holds and interest on the firm's debt is deductible ($\alpha_3 = 0$).

If debt interest is not deductible ($\alpha_3 = 1$), the reduction in dividends made necessary by the additional interest payments of the firm is higher than $\$\theta_d\theta_p r$. Because of the non-deductibility, the firm loses the full interest cost before paying dividends, but after distributing its profits, exempting the tax on retained profits and levying the two dividend taxes, the net dividend available to the shareholder reduces by the amount $\$r\theta_d\theta_p/\theta_r$. An indifference will therefore prevail if $\theta_p r\theta_d\theta_p/(\theta_r\theta_c) = r\theta_d\theta_p/\theta_r$, as indicated by (4.15) for the case $\alpha_3 = 1$.

If there is no indifference with regard to the borrow-and-distribute policy then the reduction of the market value of shares that this policy brings about is no longer equal to $\$\theta_d\theta_p/(\theta_r\theta_c)$, and a precise *verbal* argument for proving (4.15) becomes difficult. In the light of the previous considerations, it seems plausible though that, starting from a situation of indifference, a reduction of the tax burden on the household's interest income – that is, an increase in θ_p – will create a strict preference for the borrow-and-distribute policy. Obviously, the current net interest return from investing the initial flow of funds received by the shareholder will more than outweigh the interest-induced reduction in dividends. This explains the inequality sign in (4.14) or (4.15), respectively.

It is worth noting that the size of the dividend tax factor θ_d played no role in these arbitrage considerations since this factor was applied to both the initial increase and the future reduction in dividends caused by the borrow-and-distribute policy. Contrary to a frequent belief, it thus turns out that double taxation of dividends with personal and corporate taxes does not discriminate against equity capital as such. It is true, as was shown in the previous section and will be confirmed in Section 4.2.3, that double taxation of dividends discriminates against new issues of shares. But issuing new shares is only one of two methods of equity formation. The desirability of equity formation by retaining profits is not affected at all by a high tax burden on dividends.

Quite plausibly, (4.14) shows that if double taxation discriminates against this method of equity formation, it may be the double taxation of retained profits with corporate and capital gains taxes that does so rather than the double taxation of dividends with corporate and personal income taxes: if $\theta_r\theta_c$ is below θ_p while debt interest is deductible ($\alpha_3 = 0$), the tax system will indeed discriminate against retentions relative to debt financing. Note, however, that this is not a necessary outcome. As normally only part of the capital gains is included in the personal income tax base it holds that $\theta_p < \theta_c$ and hence the case $\theta_r\theta_c > \theta_p$ that characterizes a dominance of retentions over debt cannot easily be dispensed with on purely theoretical grounds.

The firm's preference ordering over debt and retentions as marginal sources of finance appears to be primarily an empirical matter. All existing systems of capital income taxation are characterized by deductibility of debt interest by the firms ($\alpha_3 = 0$). In addition, it has already been made clear with (3.15) that $\theta_r^* \leq \theta_p$, $\theta_r^* \equiv \theta_r\theta_c$, is a stylized empirical fact. Obviously, both pieces of information together imply that retentions are typically either equivalent or inferior to debt financing.

That the case of a strict dominance of retentions over debt financing can be largely ruled out on empirical grounds may not be a matter of pure chance. One potential reason is the presence of a so-called *Miller equilibrium* that will be discussed in Section 4.3.4. Another possible reason is the existence problem mentioned above which arises if $\partial\mathscr{H}^u/\partial S_f < 0$. If, with a deductibility of debt interest ($\alpha_3 = 0$), the overall tax burden on retentions falls short of the tax burden on the interest income of the representative shareholder ($\theta_r\theta_c < \theta_p$), or if this interest income is taxed more heavily than capital gains ($\theta_p < \theta_c$) while the firm cannot deduct its debt interest ($\alpha_3 = 1$), then there is a continuing incentive to retain all available funds within the firm, and the firm would never choose to pay dividends. This strange behavior implied by the non-existence of a mathematical solution may be the reason for such parameter constellations being extremely rare in the existing systems of capital income taxation. Parliaments observing this behavior may have felt strong incentives to close the loopholes in the tax system that induced it.

4.2.3. Debt versus New Issues of Shares

In order to complete the comparison of the financial instruments available to the firm we now consider a substitution of new issues of shares (NI) for debt financing (DF). Formally, this means that the Hamiltonian (3.35) has to be differentiated for Q given I and given the constraint $S_f + Q =$ constant. Using (4.11) and (4.12) the following implications for the firm's financial preferences

$$\left.\frac{d\mathscr{H}^u}{dQ}\right|_{S_f+Q=\text{constant}} = \frac{\partial\mathscr{H}^u}{\partial Q} - \frac{\partial\mathscr{H}^u}{\partial S_f} = -1 - \lambda_D \begin{Bmatrix} = \\ < \end{Bmatrix} 0 \Leftrightarrow NI \begin{Bmatrix} \sim \\ \prec \end{Bmatrix} DF \qquad (4.16)$$

are obtained. Analogously to the previous cases, the expression $\partial\mathscr{H}^{u}/\partial Q|_{S_r+Q=\text{constant}}$ measures the marginal advantage from a substitution of new issues of shares for debt financing and again the sign determines the corresponding preference relation.

In the solution space of Figure 4.1 the substitution corresponds to a movement along one of the inclined parallels downward and to the right. In the case of $NI \sim DF$ the firm is indifferent with regard to this movement, in the case $NI \prec DF$ it prefers a movement in the opposite direction.

The expression (4.16) excludes the case $NI \succ DF$. The reason is again an existence problem. Since the solution space is unbounded downwards to the right, the firm would try to carry out an unlimited substitution of new issues of shares for debt financing. It has already been mentioned that there are no legal restrictions on such an attempt. The question is whether it will be made.

In order to transform (4.16) into a more specific expression, λ_D has to be replaced with the expression given in (4.13). After a few substitutions

$$\theta_p \begin{Bmatrix} = \\ > \end{Bmatrix} \theta_d\theta_p(1-\alpha_3) + \frac{\theta_d\theta_p}{\theta_r}\alpha_3 \Leftrightarrow NI \begin{Bmatrix} \sim \\ \prec \end{Bmatrix} DF \tag{4.17}$$

is achieved. This rule, too, can easily be interpreted.

For this purpose it is useful, similarly to the previous section, to multiply the first inequality in (4.17) with r,

$$\theta_p r \begin{Bmatrix} = \\ > \end{Bmatrix} \theta_d\theta_p(1-\alpha_3)r + \frac{\theta_d\theta_p}{\theta_r}\alpha_3 r \Leftrightarrow NI \begin{Bmatrix} \sim \\ \prec \end{Bmatrix} DF, \tag{4.18}$$

and to consider the net advantage which the following transaction yields for the shareholder household. The household reduces its purchase of bonds by one dollar to purchase new shares issued by the firm, and the firm uses the inflowing funds to reduce its net increase in debt. Through the reduction in the value of bonds owned by the household, its net interest income is reduced by $\$\theta_p r$. This disadvantage has to be subtracted from the advantage that, because of a reduced interest burden, the firm can increase its flow of dividend payments. If the firm is allowed to deduct debt interest from its taxable profit ($\alpha_3 = 0$), then the flow of dividends net of the corporate tax and net of the personal tax can rise by the amount $\$\theta_d\theta_p r$. If a deduction of debt interest is not allowed ($\alpha_3 = 1$), an even higher increase in the flow of net dividends is possible. Since the firm saves the full interest cost before dividends, net dividends after distribution, after exemption from the tax on retained profits, and after imposition of the two taxes on distributed profits can be increased by the amount $\$r\theta_d\theta_p/\theta_r$. The capital gains tax does not affect this result since an increase in the market value that results from new issues of shares is tax-exempt according to (3.13). If the increase in after-tax dividends happens to exactly equal the reduction in after-tax interest income, then the household and its firm are indifferent with regard to the transaction described. If, however, the reduction in after-tax interest income is higher, then the issue of new shares should be reduced as much as possible. In the reverse case, when the increase in after-tax dividends is higher, an unlimited increase in the stock of equity capital would be attractive and, in the absence of legal constraints, the existence problem mentioned above would arise.

Fortunately, a strict preference for new issues over debt is excluded in the tax systems of all OECD countries, for it holds not only that $\alpha_3 = 0$, but, as stated with (3.14), at any rate that $\theta_d\theta_p \leq \theta_p$. In the classical system, in its modified version with a split corporate tax rate, and in the partial imputation systems, it holds that $\theta_d < 1$ and hence $\theta_d\theta_p < \theta_p$. Here new issues are strictly dominated by debt. With the full imputation systems of Norway, Italy, and West Germany as well as with the notional Teilhabersteuer system, $\theta_d = 1$ ensures an indifference between new issues of shares and debt financing. Thus it seems plausible that there are no legal constraints on a substitution of new issues for debt. It was not necessary for parliaments to forbid the firms to do something they did not want to do in any case.

4.3. The Overall Financial Optimum: The Case of Deductibility of Debt Interest

In Section 4.1 the firm's opportunity set of financial instruments was described. In Section 4.2 the financial instruments were compared in pairs. This section and Section 4.4 will combine the results achieved in Sections 4.1 and 4.2 in order to find out what the firm's overall optimal financial choice will be. This section treats only the case of deductible debt interest ($\alpha_3 = 0$) that is relevant for all Western industrial countries. Section 4.4 is devoted to the problem of non-deductible debt interest.

4.3.1. The Optimal Solutions for Alternative Systems of Capital Income Taxation

With (3.14) and (3.15) it was stated that the existing systems are typically characterized by marginal tax rates for the single kinds of capital incomes that satisfy the conditions $\theta_d^* \leq \theta_p$ and $\theta_r^* \leq \theta_p$ where $\theta_d^* \equiv \theta_d\theta_p$ and $\theta_r^* \equiv \theta_r\theta_c$ are the combined tax factors for distributed and retained profits defined in (3.10) and (3.11). Completely independently of the empirical facts, these same conditions were shown in Section 4.2 to be necessary for the existence of a solution of the optimization problem of the firm. For two reasons the following analysis can be limited to those tax systems that satisfy the two conditions.

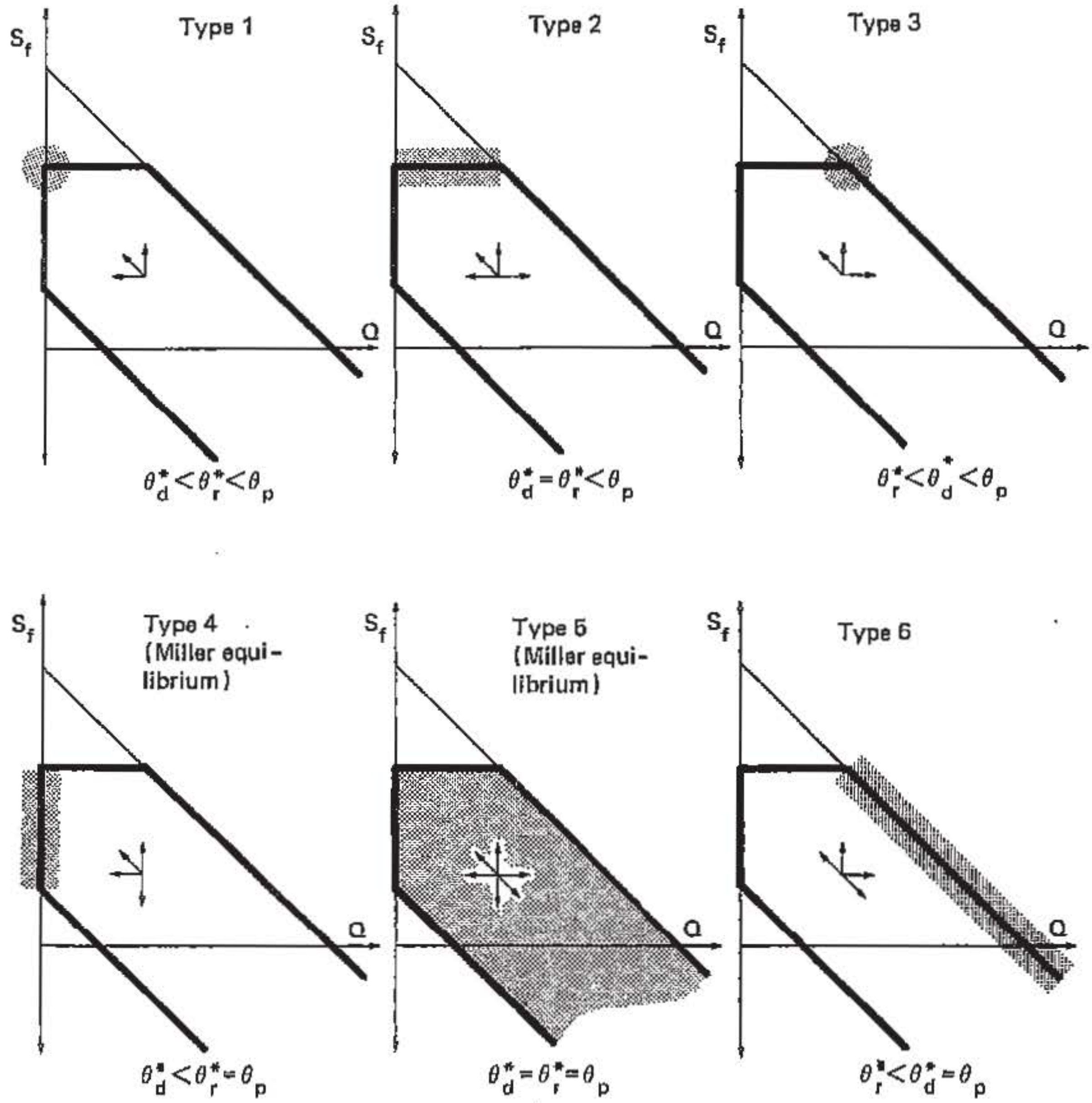

Figure 4.2. The optimal financial choice with deductibility of debt interest*.

*If negative distributions are allowed, the lower boundary of the solution space is irrelevant. With the solutions indicated for Types 4 and 5, points on the lower boundary of the solution space (or below it) can only occur temporarily since otherwise there will be existence problems.

If different assumptions about the value-added tax (τ_v), the capital tax (τ_k), the two taxes on retained profits (τ_r, τ_c), the corporate tax on dividends (τ_d), the personal income tax (τ_p), the wage tax (τ_w), and the tax depreciation rules (α_1) are allowed for, then a large number of different tax systems can be constructed. Fortunately, most aspects of these tax systems turn out not to be relevant for the firm's financial decision. The previous results hold for the full complexity of the general tax system described in Chapter 3.1, and, quite plausibly, they show that in the case of deductible debt interest only the taxation of capital income itself is important for the firm's financial preferences. The form of the tax depreciation rules alone can affect the optimal financial choice via an alteration of the upper horizontal borderline [cf. (4.7)]. This problem, however, is bypassed for the time being. The next chapter will analyze it in detail.

There are precisely six different ordinal constellations of magnitude of the tax factors θ_d^*, θ_r^*, and θ_p that satisfy the conditions $\theta_d^* \leq \theta_p$ and $\theta_r^* \leq \theta_p$. According to the results of the previous section, each of these constellations implies a typical preference pattern for possible movements in the solution space of Figure 4.1. The different cases are illustrated in Figure 4.2 where the arrows indicate the preference directions following from (4.11), (4.14), and (4.17). An indifference is indicated by a two-headed arrow.

Despite the variety of preference patterns there is a very simple rule for memorizing these patterns and reconciling them with the different tax systems. This rule is to consider the tax factors θ_d^*, θ_r^*, and θ_p as ordinal preference indicators where θ_d^* stands for new issues of shares, θ_r^* stands for retained profits, and θ_p stands for debt financing. If, for example, $\theta_p = \theta_d^* > \theta_r^*$ then it can be immediately recognized with the aid of this rule that debt and new issues of shares are equivalent financial instruments that dominate retentions. The preference pattern of Type 6 illustrates this case. It is left to the reader to check the rule for the other five cases.

In order to interpret the financial preferences a look at Figure 3.1 which summarizes the discussion of Chapter 3.1 is useful. Obviously Type 1 corresponds to the classical system of capital income taxation and Types 1, 2, and 3 correspond to the partial imputation systems or the system with a split corporate tax rate. Type 4 is an important case of the classical system or the partial imputation systems that might approximately hold in countries with a low corporate tax burden on retained profits or a high marginal personal tax rate. Type 4 prevails precisely if distributed profits are subject to at least some degree of double taxation and the sum of the direct and indirect marginal tax burdens on retained profits equals the personal marginal tax rate of the representative shareholder household. Type 5 corresponds to the Teilhabersteuer and at the same time represents the case relevant for non-corporate firms. The full imputation systems of countries like Norway, Italy, and West Germany are typically characterized by Type 6, but when the marginal personal tax rate of the representative shareholder is sufficiently high, Type 5 can also be attributed to them.

The optimal total choice of the firm's financing and dividend policy results from the interaction of financial preferences and financial constraints. Hence the optimal choice can be determined by finding those points in the solution space, from where a movement along a single-headed preference arrow is no longer possible. Consider, for example, the financial preferences of Types 1 and 3. For both of them directly upward, and upward to the left, movements are preferable; however, while a horizontal leftward movement leads to an increase in the market value with Type 1, a rightward movement will have this effect with Type 3. Hence, with Type 1, the left and, with Type 3, the right upper corner of the solution space is optimal. This means that the firm tries as far as possible to finance that part of net investment not covered by deferred taxes with debt and that the remaining gap $(1 - \sigma^* - \alpha_1 \tau_r)I$ is closed by a retention of profits (Type 1) or new issues of shares (Type 3). In a similar way, the results can be derived for the other types. In general, the optimal solutions are illustrated with the shaded points, lines, or areas in Figure 4.2.

Compared to Types 1 and 3, Type 2 has an intermediate position where, with a given level of debt financing, the firm is indifferent between closing the financial gap by retentions or new share issues or arbitrary combinations of these two financial instruments. With Type 4, new issues are excluded as with Type 1, but the firm can replace debt financing through a retention of profits without affecting the market value of shares. The properties of Type 6 in comparison to Type 3 are similar to those of Type 4 in comparison to Type 1, the only difference being that no part of profit is retained and that a replacement of debt by new share issues is possible without changing the market value. Only Type 5 is characterized by a complete equivalence of all financial instruments as in the case without taxes. It is true that the firm is forced to retain the funds that originate from the tax reduction through accelerated depreciation $(\alpha_1 \tau_r I)$. However, the part $(1 - \sigma^* - \alpha_1 \tau_r)I$ can arbitrarily be financed with retained profits or new issues of shares, and the way in which the remaining financial needs $\sigma^* I$ are covered is completely irrelevant from the viewpoint of market value maximization.

The role of debt in covering the proportion σ^* of net investment is worth noting. With the first three types, debt financing is strictly preferred to retained profits and new issues of shares. With Types 4 through 6, debt is at least equivalent to one of these financial instruments but never inferior. Thus, in all cases, at least *one* point on the upper horizontal borderline of the opportunity space belongs to the set of optimal solutions; that is, in all cases the proportion σ^* of net investment can be fully financed with credit. The reason for this aspect is the exclusion of the tax constellations $\theta_p < \theta_d^*$ and $\theta_p < \theta_r^*$ that imply a strict dominance of new issues of shares or retained profits over debt and that could be ruled out both for empirical reasons and mathematical existence requirements.[8] Typically, the tax laws of the single countries have been constructed in such a way that there is a weak dominance of debt financing, and this weak dominance is a prerequisite for the existence of a solution to the optimization problem of the firm, given the institutional constraints on its financial decisions.

4.3.2. Retentions as the Marginal Source of Equity Finance

It has been tacitly assumed in the previous analysis that the firm's profit is large enough to be able to provide the required funds for equity formation. In Figures 4.1 and 4.2, this meant that the upper horizontal borderline and the lower of the two inclined borderlines do not intersect [Cf. (4.5) and (4.7)]. If this condition is not met the firm has to issue new shares in order to collect equity capital regardless of whether or not the tax system discriminates against this source of finance. For most of the six types of solution illustrated in Figure 4.2 this would be meaningless, but for Types 1 and 4 the nature of the solutions would change.

This possibility will not be elaborated here. Instead it is assumed for the tax systems of Type 1 and 4 that the firm's existing endowment with equity capital is sufficiently high to ensure that the retainable net profit Π_n^* exceeds the part of net investment that is to be financed with new equity capital:

$$\Pi_n^*(t) > \varepsilon^* I(t) \quad \forall t \geq 0 \qquad (\theta_d^* < \theta_r^* \leq \theta_p). \tag{4.19}$$

It is true that the case $\Pi_n^* \leq \varepsilon^* I$ cannot be ruled out a priori. However, this case does not seem overly important for the long-run allocation problems studied in this book. A concentration on the case of a sufficiently large retainable profit, as defined by (4.19), can be legitimated both on empirical and theoretical grounds.

[8]The case $\theta_p < \theta_r^*$ can also be ruled out in a Miller equilibrium. See Section 4.3.3.

One empirical legitimation is simply the observation that corporations typically pay dividends. With the tax systems of Type 1 and 4, new issues of shares are strictly dominated by retentions. Thus firms will not pay dividends if they are issuing new shares and vice versa. A payment of dividends is a clear indication that profits are large enough to provide for the required amount of equity formation.[9]

Another empirical legitimation is the relatively infrequent occurrence of new share issues that has been observed by many authors. Consider for example the United States, a country that employs the classical system and that can hence be associated with the financial preferences of Types 1 and 4. In the period from 1960 to 1985, on average 67.8% of gross investment by U.S. non-financial corporations was internally financed and 31.0% was debt-financed, but only 1.2% was financed by net share issues.[10] The figures may look somewhat different for other countries, but their tendency clearly describes a general empirical phenomenon. In developed economies, corporations are self-perpetuating enterprises that rarely rely on new equity injections from the household sector. The funds that the household sector injects into the sector of corporate firms are primarily channelled through the bond and credit markets, and the flow of new equity capital needed is typically generated within the corporations themselves.

A theoretical legitimation – based on the fundamental existence requirement of intertemporal growth models that the steady-state rate of time preference is above the economy's natural rate of growth [cf. Chapter 2.6] – is provided in Appendix B. Assuming that the initial market value of equity is strictly positive [$M(0) > 0$] and that the economy converges to a steady-state growth path with $I \geq 0$, the appendix shows for the crucial tax systems of Types 1 and 4 that a situation with

$$\Pi_n^*(t) > \varepsilon^* I(t) \tag{4.20}$$

(1) will always prevail ($t \geq 0$) if the initial stock of debt is small enough,
(2) will prevail after some finite period of time ($t \geq t^* > 0$) even if $\Pi_n^*(0) \leq \varepsilon^* I(0)$, and
(3) is self-perpetuating in the neighborhood of a steady state.

[9] In 1984, for example, U.S. corporations paid out about 54% of their profits as dividends. See Federal Reserve Bulletin (February 1986, Table 1.48).

[10] Calculated from *Survey of Current Business*, Volumes 57 (July 1977, p. 24 n.), 61 (1981, special supplement, p. 10), 63 (July 1983, p. 30), 66 (July 1986, p. 33); and *Federal Reserve Bulletin*, Volumes 55 (November 1969, p. A 71.4), 60 (October 1974, p. A 59.4), 64 (June 1978, p. 433), 65 (December 1979, p. A 44).

The first of these results ensures that (4.19) is a theoretically feasible assumption.

The second result says that the case of an insufficient size of retainable net profits ($\Pi_n^* \leq \varepsilon^* I$) is a transitory phenomenon. Even if, in some initial phase, new issues of shares are necessary to provide the firm with sufficient amounts of equity capital, the retainable net profit (the vertical distance between the two sloped borderlines) will soon exceed the part of net investment that is to be financed with equity capital.

The third result indicates that, even in a growing economy, the retainable net profit will always stay large enough to allow for both the required formation of new equity capital and dividend payments if the economy is in a neighborhood of a steady state. In principle the firm can indeed operate as a self-perpetuating enterprise that, once it is founded with some equity capital, grows indefinitely and maintains a given equity–asset ratio without ever requiring further injections of equity capital from the household sector.

4.3.3. Double Taxation of Dividends and the Alleged Lock-in Effect

While it is often argued that the double taxation of dividends that characterizes the classical system favors equity financing over debt financing – a position that was criticized in Section 4.2.2 – many authors hold the seemingly adverse view that it induces an over-accumulation of equity capital through retentions.[11] Since, unlike the case of distributed profits, there is no direct personal tax on retained profits and only a small capital gains tax, shareholders prefer, it is argued, to receive the firm's profits as capital gains rather than as dividends: the tax system brings about a *lock-in effect*. In order to avoid this effect and to provide an incentive to distribute profits it is recommended that the double taxation be removed by making dividends deductible from the corporate tax base.

The contention of a lock-in effect can only be justified to a very limited extent through the results achieved in the previous sections, for it was shown that a high tax burden on dividends discriminates against new issues of shares but not against dividends. Given the investment volume, more

[11]Compare, taking one example from many, Fullerton et al. (1981, p. 683 and p. 688). The opposite view is held by Bradford (1980, p. 57; 1981) who studies the role of dividend taxation in an overlapping-generations framework. Cf. also Moxter (1976, Columns 1613–1616) who made useful remarks on the problem.

dividends mean more debt financing *or* more share issues. Discrimination against share issues only implies a discrimination against dividends when the firm cannot alter the amount of new loans it takes. Only in this case will the double taxation result in a lock-in effect. If the decision is, however, to retain or to distribute and borrow, the tax burden on dividends is irrelevant since it is the relative advantages of debt financing and retentions that matter and they depend only on the relationship of the magnitudes of the tax factors for personal interest income (θ_p) and the combined tax factor for retained profits and capital gains of company shares ($\theta_r^* \equiv \theta_r \theta_c$). According to the analysis in Section 4.2.2, the size of the combined tax factor for the corporate and personal tax burden on dividends ($\theta_d^* \equiv \theta_d \theta_p$) plays no role whatsoever in this case!

To illustrate this point, it is useful to look at Figure 4.2. With Types 1 and 4, it holds that $\theta_d^* < \theta_r^*$ and hence these types can be identified with the case of double taxation. For both types, new issues of shares are excluded but a lock-in effect, which would require a solution point in the lower left corner of the opportunity space,[12] is definitely absent. With $\theta_r^* = \theta_p$ (Type 4) the firm is indifferent with regard to the size of dividends and with $\theta_r^* < \theta_p$ (Type 1) it even distributes as much as possible given the financial constraints. The problem of a lock-in effect arises only in so far as the firm has to decide how to cover the financial gap that remains after exploiting its scope for debt financing, $S_f \leq \sigma^* I$, and after using the funds resulting from deferring taxes through accelerated depreciation, $\alpha_f \tau_r I$. As the comparison of Types 1 through 3 from Figure 4.2 has already shown, double taxation means, because of $\theta_d^* < \theta_r^*$, that the firm prefers to close this gap by not distributing profits rather than by issuing new shares. Only in this limited sense is there a lock-in effect.

The reason the double taxation does not generally produce a lock-in effect is that retaining profits only appears to avoid double taxation. It is true that a retention means an immediate tax saving. However, if the shareholders are to enjoy the retained funds at some stage, then the burden of double taxation cannot be avoided. Assume, in order to illustrate this result, proved for a more general case in Section 4.2.2, that there is only a corporate tax on dividends ($\theta_d < 1$) and that the firm faces the alternatives of either distributing the amount of gross profit $X(t^*)$ at point in time t^* or investing it in the capital market and distributing principal and interest at later points in time. Let $\{-\dot{X}\}_t^\infty$, $\{X\}_t^\infty$, and $\{rX\}_t^\infty$ denote the time path of distributions financed out of principal, the time path of remaining principal,

[12]Compare the information given in Figure 4.1 for interpreting Figure 4.2.

and the time path of distributed interest earnings. Then the present value of the flow of future net dividends that can be expected from a lock-in policy is

$$Y(t^*) = \int_{t^*}^{\infty} \theta_d[-\dot{X}(t) + r(t)\, X(t)]\left[\exp \int_{t^*}^{t} - r(v)\,dv\right]dt \tag{4.21}$$

or, upon integration,

$$Y(t^*) = \theta_d\left[-X(t) \exp \int_{t^*}^{t} - r(v)\,dv\right]_{t=t^*}^{t=\infty}. \tag{4.22}$$

Provided that the transversality condition

$$\lim_{t\to\infty}\left[X(t) \exp \int_{t^*}^{t} - r(v)\,dv\right] = 0 \tag{4.23}$$

is satisfied, Equation (4.22) becomes

$$Y(t^*) = \theta_d X(t^*). \tag{4.24}$$

As $\theta_d X(t^*)$ is the net dividend that could be paid out to the shareholder at $t = t^*$, this expression shows that there is no net advantage from a lock-in policy. It is true that this policy compensates the shareholders for the initial loss in dividends with a capital gain. However, as shown by (4.24), this capital gain [$Y(t^*)$] is not as large as the retained profit [$X(t^*)$], but falls short of it in proportion to the dividend tax rate. Thus, implicitly, capital gains are subject to the dividend tax, too!

An objection to this view might be that the dividend tax could be permanently avoided if principal and interest from the initial retention $X(t^*)$ were *never* distributed to the shareholders so that $\dot{X}(t) = r(t)X(t)$ for all $t \geq t^*$. While this objection is true, it misses the point. As (4.21) or (4.22) show, such a policy would imply $Y(t^*) = 0$; that is, there would be no capital gains at all! Capital gains in the present and dividends expected for the future are two sides of the same coin. Without the expectation that locked-in profits can somehow and some day be returned to the shareholders, these profits will not be able to generate an increase in the shareholders' wealth today.

The "capital gains argument" is certainly the most frequent argument for the existence of a lock-in effect, but it is not the only one. Other arguments that seem to enjoy some popularity in the discipline refer to particular credit contracts for avoiding the dividend tax between the company and its own shareholders. For example, it is argued that, rather than paying out the amount $X(t^*)$ as dividend and subjecting it to the dividend tax, the company could use this amount to repay a loan that it had received previously from its shareholders. This way, the dividend tax would be permanently saved. This argument cannot be considered as valid since it neglects the fact that, in the act of repaying the loan, shareholders loose all claims on future repayments that they otherwise might have received. If the loan was given at market conditions, the present value of these claims precisely equals the value of the immediate repayment and, by itself, the policy is indistinguishable from the useless policy of *permanently* withholding the profit. Only if the loan was given at more favorable conditions than those offered by the market, for example with the promise never to demand the repayment of principal and interest, will the repayment in itself be able to benefit the shareholders. The question is then, however, why the shareholders should give such a favorable credit to their companies in the first place. Through making a gift to the company and receiving this same gift back from it, the dividend tax can hardly be circumvented.

A related but more significant argument says that the company uses its profit not to pay back a loan that it received *from* its shareholders, but to give a loan *to* them. Clearly, when this is done at market conditions it is meaningless to the shareholders and cannot be a means of circumventing the dividend tax. However, when the conditions are more favorable than those at which the shareholders could borrow and lend in the capital market, then, in economic terms, the company distributes "dividends" but pays no dividend tax. The argument is similar to saying that the company employs the shareholders as over-paid managers or buys commodities or services from them at above-market prices.

Such arguments are certainly valid in principle although their empirical validity may well be called into question. They refer to examples of loopholes in the tax system that allow for a tax-free distribution of dividends and explain why a dividend tax may induce companies to pay fewer dividends. Note, however, that they merely suggest that companies will try to distribute their earnings through *other* channels than those prescribed by law. They do not explain a lock-in effect in the sense that the dividend tax induces a *postponement* of corporate distributions into the future.

A similar remark applies to the familiar argument that the dividend tax provides an incentive to distribute profits through share repurchases.[13] This argument is empirically insignificant if a company's own shares are con-

[13]Cf. Feldstein and Green (1983, in particular p. 19).

cerned[14] but, as will be discussed in Chapter 6, it may have some relevance for a purchase of other companies' shares. Nevertheless, here too, it is not obvious that a lock-in effect is produced. If, for example, share repurchases are a constant fraction of ordinary dividends they will act like a lower effective dividend tax rate, leaving the optimal time path of corporate distributions completely unaffected.

It seems that in order to derive a lock-in effect produced by a dividend tax, it would be necessary to assume that the effective tax rate on dividends is falling over time.[15] Announced *future* tax cuts, *improving* possibilities of share repurchases, possibilities of avoiding a taxation of accumulated earnings through liquidation and the like are among the candidates for a possible explanation. The following analysis abstracts from such effects. It is assumed that there are no loopholes in the tax system and that the dividend tax rate stays constant over time. Under these circumstances there is no lock-in effect caused by dividend taxation.

This result is perfectly compatible with the empirical facts. On the one hand, corporations usually do pay out large fractions of their profits as dividends.[16] On the other hand, there is clear evidence that the equity base of corporations did not improve in the past, as the lock-in effect would imply, but rather worsened dramatically. This evidence is reported in Table 4.1.[17] During the sixties and seventies, the equity–asset ratios of large industrial companies of a number of important countries have been significantly reduced, in some countries to less than $\frac{1}{2}$ of the initial value in less than two decades.[18] No lock-in effect, rather, to coin a similar phrase, a (limited) *lock-out* effect is suggested by the data.

The lock-out effect is compatible with any of the six tax systems shown in Figure 4.2, but clearly the first three of these seem particularly plausible in

[14]Auerbach (1979a) reports figures according to which, even in the United States where the legal constraints seem to be particularly loose, share repurchases were significantly less than one tenth of dividends. In recent years, share repurchases may have occurred somewhat more frequently in the United States. From a world perspective, they remain definitely negligeable though.

[15]See Howitt and Sinn (1986) for the analysis of a non-constant dividend tax rate.

[16]Cf. Footnote 9.

[17]The table gives the ratio of equity to total accounting value of assets. Some caution is appropriate for cross-country comparisons, since equity capital is not always defined the same way. The development of the equity–asset ratios over time indicated in the table is not affected by this limitation.

[18]Cf. Albach (1975, Table 1) who, using German data, calculates, for the time period of 1953–1973, a reduction of the equity–asset ratio from 57% down to 34%. The direction of this result is also confirmed by von Torklus (1969).

Table 4.1
The development of equity–asset ratios

	Large industrial companies[a]		Total industry[b]	
	1962	1975	1973	1980
USA	65.3	49.6	52.1	49.8
United Kingdom	64.1	38.6	–	–
Netherlands	67.0	35.2	–	–
France	57.0	23.1	–	–
West Germany	37.6	22.9	24.5	22.0
Sweden	37.5	20.0	–	–
Japan	30.0	16.6	17.7	18.7
Italy	34.0	14.4	–	–
Austria	–	–	26.6	18.5

[a]Source: Gruhler (1976, p. 43, Table 23)
[b]Source: Richter and Petrusch (1983, p. 138).

the light of the evidence reported in Table 4.1. They have in common that debt financing dominates both retentions and new issues of shares, and hence a solution where the maximum marginal debt–asset ratio σ^* is binding is chosen. If this maximum marginal debt–asset ratio was above the average debt-asset ratio ($\sigma^* > D_f/K$) – say because the maturing stock of post-war corporations consolidated sufficiently to cope with a relatively lower equity base – then the results in the figure could easily be reproduced.[19]

4.3.4. The Miller Equilibrium

While the above analysis suggested that six different types of solution must be distinguished in order to capture the tax rate constellations that are possible in the OECD countries, there is an interesting argument that reduces the number of types to just two. This argument follows from a hypothesis established by Miller (1977) and is based on the progressivity of marginal personal tax rates.[20] Applied to the present model, Miller's

[19]Another hypothesis, based on an endogenous explanation of σ^*, is suggested in Chapter 5.2.

[20]Miller discussed the gains from leverage without formally distinguishing between new issues of shares and retentions as alternative sources of equity finance. The following discussion is inspired by his paper and remarks of Auerbach (1983), but no claim is made to be authentic.

hypothesis says that the policy of distributing dividends and retaining as little as possible which pays in the case $\theta_r^* < \theta_p$ (Types 1–3 and 6) results in a rise in the personal tax base. This rise reduces θ_p relative to θ_r^* until these two tax factors become equal and hence an indifference between debt and retentions as marginal sources of finance is reached. In the light of this argument, the decline in the equity–asset ratios reported in Table 4.1 can be seen as an adjustment phenomenon that disappears as soon as a situation with $\theta_p = \theta_r^*$ is achieved.

An exact representation of the "*Miller equilibrium*" in a dynamic model of the firm would not be an easy task since, in a transition phase, the tax rates are no longer constants and the *change* in the marginal personal tax rate on dividends would affect the firm's choice between debt and retentions.[21] However, the steady-state properties of this model can rather easily be determined. Consider a growing economy where a positive real net investment provides for a natural scope of profit retentions and assume there is a progressive personal tax system, where the marginal tax rate is a function of personal income relative to some aggregate scale variable but not a function of time. Suppose there are many identical firms and many identical households.[22] Since the households hold well-diversified portfolios of all shares, a single firm cannot, through its own actions, alter the shareholders' marginal personal tax rate, but all firms together can. Firms face only the legal financial constraints [there is no requirement to finance part of net investment with equity ($\varepsilon^* = 0$)] and either the classical or the partial imputation system applies ($\theta_d^* < \theta_p$); therefore only debt and retentions need to be considered as potential sources of finance. Given the size of pre-tax aggregate capital income, the personal tax base is a rising function of interest income and a falling function of equity income as capital gains and dividends enter the personal tax base after the corporate tax has been deducted from retained and distributed profits respectively and as accrued capital gains are not fully included in the personal tax base.[23]

For this economy, alternative steady states are conceivable that differ with regard to the proportions in which net investment is financed with retained profits and new credit and hence differ with regard to the proportions in which capital income appears as capital gains and dividends on the one hand and interest earnings on the other. As the personal tax base of the shareholders rises when interest income is substituted for equity income

[21] Cf. Howitt and Sinn (1986).

[22] Miller presented his argument for the case of households that have different marginal tax rates. In the present context this aspect does not seem essential.

[23] Cf. Chapter 3.1.1 and 3.1.2.

and since such a substitution pays when $\theta_p > \theta_r^*$, a steady state with this constellation of tax factors would obviously not be stable. Instead it seems likely that a transition to another steady state is induced on which a sufficiently low fraction of capital income appears as equity income and where thus the marginal personal tax rate is sufficiently high to ensure that $\theta_p = \theta_r^*$. Similarly, a steady state with $\theta_p < \theta_r^*$ could not prevail since, with this constellation of tax factors, there is an incentive to substitute equity income for interest income, hence a force that increases θ_p relative to θ_r^*. Only a steady state with $\theta_p = \theta_r^*$ – that is, a steady state on which firms are indifferent between debt and retentions – would be stable. Each single firm would now face the constellation $\theta_r^* = \theta_p > \theta_d^*$ which characterizes the solution of Type 4 in Figure 4.2. Its dividend policy and its planned time path of the debt-equity ratio would be irrelevant for its market value. Nevertheless, the aggregate flow of dividends and the aggregate debt–equity ratio in the economy would be well-defined.

Consider now the remaining possibility $\theta_d^* = \theta_p$ that characterizes the full imputation system. Here, the firm is indifferent between debt and new issues of shares, and a substitution of interest income for dividends neither affects the personal income tax base nor the personal tax rate. Nevertheless, for reasons similar to those explained above, the personal tax rate would be a falling function of the fraction of capital income that appears in the form of capital gains, and a Miller equilibrium, where this tax rate equals the combined tax rate on retained profits, would emerge. The tax factors of all three kinds of capital income would now be equal ($\theta_r^* = \theta_p = \theta_d^*$) and the firm would be indifferent between all three elementary sources of finance as illustrated with Type 5 in Figure 4.2. Note that, unlike the previous case, this equilibrium merely determines the share of capital gains in the total capital income. It will neither determine the proportions of interest income and dividends nor the aggregate debt–equity ratio.

There are many potential objections to the view that the Miller equilibrium could be a good description of reality. In particular it seems doubtful whether the forces Miller described are strong enough to bring about the required adjustment in the marginal personal tax rate. At least three reasons for an insufficient adjustment can be given. First, there are countries like Denmark, Canada, West Germany, and, since the 1986 tax reform, the United States where the *maximum* marginal personal tax rate is at or below the critical level of the corporate tax rate where an indifference between debt and retentions would occur.[24] For these countries, a Miller equilibrium must obviously be dismissed as impossible or at least extremely

[24]Cf. Chapter 3.1.2.

implausible. Second, there may be other reasons for an interior debt–equity choice such as the hypothesis that will be formulated in Chapter 5.2. Such reasons would imply that the firm's financial behavior, and hence the marginal personal tax rate, do not react as elastically as assumed above. Third, the scope for variations in the fraction of capital income that appears as capital gains might be small since the economy does not grow and hence has no need for profit retentions. It is true, in principle, that it is possible to retain profits even when there is no real net investment. However, under such circumstances, the government debt or the debt of the household sector would grow relative to the other aggregates in the economy. This may create existence problems and will most likely result in a violation of liquidity constraints.

Whatever the significance of these objections: it must be admitted that the forces underlying the Miller equilibrium may well be operative in reality. They may not be strong enough to bring about an equality of θ_r^* and θ_p, but they tend to reduce the gap between these two tax factors. In this light, the solutions of Type 4 and 5 in Figure 4.2 appear as important special cases that, at least for didactic purposes, deserve particular attention in the further analysis of this book.

4.4. Existence Problems with Non-deductible Debt Interest

The analysis of the firm's decisions will now be concluded with a brief look at the case of non-deductible debt interest ($\alpha_3 = 1$). It is true that this case is practically irrelevant. However, it merits interest in view of various reform proposals that have been made in the literature.

There are only two constellations of tax factors that, given the basic assumptions $\theta_d^* \leq \theta_p \leq \theta_c$ and $\theta_r^* \leq \theta_p$ from (3.14) and (3.15), are compatible with the existence requirements of a solution to the optimization problem of the firm. As it is known from (4.14) that the case $\theta_p < \theta_c$ can be excluded since retentions would dominate debt financing and the firm would never pay out any dividends only the possibility $\theta_p = \theta_c$ which implies an equivalence of retentions and debt as sources of finance remains. Knowing this fact and using the definition $\theta_r^* \equiv \theta_c \theta_r$ it is immediately obvious that (4.17) allows only for the cases $\theta_d^* < \theta_r^*$ and $\theta_d^* = \theta_r^*$. In general, it therefore turns out that

$$\theta_d^* \leq \theta_r^* \leq \theta_c = \theta_p \qquad (\text{for } \alpha_3 = 1) \tag{4.25}$$

is required. It is assumed throughout this book that this condition holds whenever the case of non-deductible debt interest is treated.

The two possible constellations of the tax factors are classified as Types 7 and 8. The preference patterns that follow from (4.12), (4.14), and (4.17) for these types are illustrated in Figure 4.3. Obviously the preference patterns are the same as those of Types 4 and 5 from Figure 4.2. There is an indifference between debt financing and retentions, and new issues of shares are either dominated by these two financial instruments or they are equivalent to them.[25]

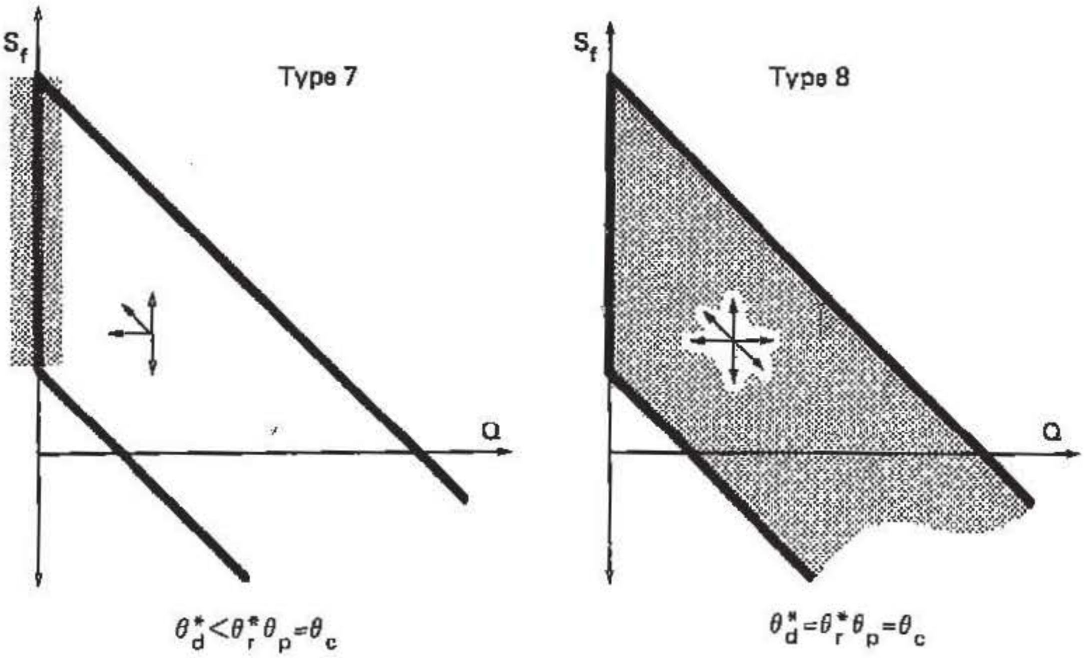

Figure 4.3. The optimal financial structure with alternative tax systems in the case of non-deductible debt interest.

Quite plausibly, the result shows that the incentives for forming equity capital are reinforced if interest on debt is not tax deductible. The existence of a solution requires, however, that the non-deductibility of debt interest for the firm is combined with a uniform tax rate on interest income and capital gains for the shareholder household (cf. also Section 4.2.2) and that the overall tax rate on retained profits does not exceed the overall tax rate on distributed profits. If the first of these conditions is not satisfied ($\theta_p < \theta_c$), the firm never has any incentive to pay out dividends, and if the first is satisfied but not the second, then there is an unlimited incentive to issue new shares and to invest the funds thus received in the capital market.

[25] Anticipating a result derived in Chapter 5.2.3 it is assumed that the upper horizontal borderline does not effectively reduce the solution space ($\sigma^* = 1 - \alpha_1 \tau_r$). Because of the indifference between debt and at least one source of equity, this borderline would not be binding in any rate.