Capital Income Taxation and Resource Allocation

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Chapter 5: Investment, Finance, and Taxation

Chapter 5

INVESTMENT, FINANCE, AND TAXATION

In terms of this book's aims, the most important aspect of the firm's decision making is its investment planning, and this aspect will now be considered. In this chapter, the analysis is entirely partial analytic. The question is how does taxation affect the optimal employment of capital given the time path of the market rate of interest. Later chapters will study the implications of the partial analytic results for a situation of market equilibrium where the market rate of interest is endogenous.

Investment planning and financial planning are tightly intertwined and are determined simultaneously by the firm. As shown in the previous chapter, the existing tax systems do not treat the alternative financial instruments uniformly. The profitability requirement an investment project has to satisfy therefore depends crucially on the way it is financed. The discussion of this aspect in the context of financial optimization and under alternative financial constraints is the theme of this chapter.

The chapter does not only apply the previous findings on the firm's financial decisions to an analysis of its real decisions. In addition, it provides a further extension of the analysis of financial decisions in that it offers a hypothesis on the determinants of the maximum marginal debt–asset ratio – or the minimum marginal equity–asset ratio – that so far has been treated as exogenous. This hypothesis is based on the fact that corporate tax laws do not provide an unlimited loss-offset. With a limited loss-offset, there is a trade-off between the deductibility of debt interest and that of accelerated depreciation and thus the firm's planned future path of capital accumulation reduces its present scope for debt financing. In the light of the wide-spread use of generous depreciation allowances in the Anglo-Saxon countries (see Chapter 3.1.3) this could be a point of practical importance.

In a broad sense, the discussion follows the tradition of the cost-of-capital literature starting with Jorgenson (1967). However, there are substantial deviations from this literature in that the role of personal capital income taxation for shareholder households is integrated and that the role of financial decisions in determining the cost of capital is emphasized. For example, the Jorgenson literature usually neglects completely the possibility that retained profits may serve as the marginal source of finance notwithstanding the fact that, empirically, a much larger proportion of new equity capital is formed through retentions than through new issues of shares. In this analysis, equity financing through retaining profits and through issuing new shares enjoy equal rights.

The discussion is more closely related to the contributions of King (1974a, 1974b, 1977) and Auerbach (1979a, 1983) than to the Jorgenson literature in the narrow sense. These contributions opened up a new direction of thought in the development of the theory of investment and taxation. They are extended here by endogenizing the financial decisions of the firm along the lines described and investigating the implications for the firm's real investment decision.

The chapter has four sections. The first derives a general formal condition for optimal investment planning and the second develops the hypothesis for the maximum marginal debt–asset ratio σ^* (or the minimum marginal equity–asset ratio ε^*) in the context of accelerated depreciation. Sections 5.3 and 5.4 are the heart of the chapter. They interpret the results that were formally derived in the first two sections with regard to their implications for optimal investment planning and compare these results with others that have previously been found in the literature.

5.1. The General Formal Condition of an Optimal Investment Policy

We first want to find out how taxation affects optimal investment planning for any given value of the maximum marginal debt–asset ratio σ^*. For this purpose, the intertemporal optimization problem of the firm described in Chapter 3.2 has to be considered and the Hamiltonian (3.35) has to be maximized with regard to the firm's investment, I. This cannot be done independently of the results concerning the optimal financial decisions of the firm found in the previous chapter for, when the level of investment varies, binding constraints on the net increase in debt (S_f) and/or on the volume of new share issues (Q) may change.

To capture the influence of the financial decisions it is useful to start with the general necessary condition for an optimum of the firm,

$$\frac{d\mathscr{H}^u}{dI} = \frac{\partial \mathscr{H}^u}{\partial I} + \frac{\partial \mathscr{H}^u}{\partial S_f}\frac{dS_f}{dI} + \frac{\partial \mathscr{H}^u}{\partial Q}\frac{dQ}{dI} = 0, \tag{5.1}$$

and to consider what form this condition takes under the tax systems classified in Figures 4.2 and 4.3. Note that non-negative real net investment was assumed in Chapter 4.1. for Types 1–3.

With financial preferences of Type 1 or 2, $S_f = \sigma^* I$ and either $dQ/dI = 0$ or $\partial\mathscr{H}^u/\partial Q = 0$. Hence (5.1) becomes

$$\frac{\partial\mathscr{H}^u}{\partial I} + \frac{\partial\mathscr{H}^u}{\partial S_f}\sigma^* = 0 \qquad (\theta_d^* \leq \theta_r^*). \tag{5.2}$$

With Types 4 and 7, $dQ/dI = \partial\mathscr{H}^u/\partial S_f = 0$, and with Types 5 and 8, $\partial\mathscr{H}^u/\partial Q = \partial\mathscr{H}^u/\partial S_f = 0$. Since (5.1) and (5.2) coincide for these four types, Condition (5.2) must again hold. Thus, in general, this condition has to be satisfied for all those types where the tax burden on distributed profits is at least as heavy as that on retained profits ($\theta_d^* \leq \theta_r^*$).

With Type 3, $S_f = \sigma^* I$ and $Q = I(1 - \alpha_1 \tau_r) - S_f$. Hence (5.1) becomes

$$\frac{\partial\mathscr{H}^u}{\partial I} + \frac{\partial\mathscr{H}^u}{\partial S_f}\sigma^* + \frac{\partial\mathscr{H}^u}{\partial Q}(1 - \sigma^* - \alpha_1 \tau_r) = 0 \qquad (\theta_d^* \geq \theta_r^*). \tag{5.3}$$

Since $\partial\mathscr{H}^u/\partial Q = 0$ holds with Type 2 and $\partial\mathscr{H}^u/\partial Q = \partial\mathscr{H}^u/\partial S_f = 0$ holds with Types 5 and 8, (5.3) has to be satisfied for these types, too. Moreover, this equation holds for Type 6, as, for this type, we have $\partial\mathscr{H}^u/\partial S_f = \partial\mathscr{H}^u/\partial Q$ and $dS_f/dI + dQ/dI = 1 - \tau_r\alpha_1$. Thus, Condition (5.3) has to be satisfied for all those types where distributed profits are not taxed more heavily than retained profits ($\theta_d^* \geq \theta_r^*$).

By calculating the expression

$$\frac{\partial\mathscr{H}^u}{\partial I} = \lambda_K - \frac{\theta_d^*}{\theta_r^*}(1 - \alpha_1 \tau_r) \tag{5.4}$$

from (3.28) and (3.35) and utilizing the values for $\partial\mathscr{H}^u/\partial Q$, $\partial\mathscr{H}^u/\partial S_f$, and λ_D that are given in (4.11), (4.12), and (4.13) the shadow price of capital can be obtained from (5.2) and (5.3) respectively for the two cases $\theta_d^* \leq \theta_r^*$ and $\theta_d^* \geq \theta_r^*$. After a number of simple algebraic transformations we get

$$\lambda_K = \theta_d^* \left[\frac{1 - \sigma^* - \alpha_1 \tau_r}{\max(\theta_d^*, \theta_r^*)} + \frac{\sigma^*}{\theta_p}\left(1 + \alpha_3 \frac{\tau_r}{\theta_r}\right)\right], \tag{5.5}$$

an expression that holds simultaneously for the two cases.

On the basis of (5.5) it is a straightforward matter to derive a general expression for an optimal employment of capital using the necessary optimization condition $\dot{\lambda}_K - \lambda_K r\theta_p/\theta_c = -\partial\mathscr{H}^u/\partial K$ and Equations (3.28) and (3.35). Since (5.5) implies $\dot{\lambda}_K = 0$, it follows after a few steps that

$$r = \frac{f_K - \delta - \tau_k}{\dfrac{\theta_p(1-\sigma^* - \alpha_1 \tau_r)}{\max(\theta_d^*, \theta_r^*)} + \dfrac{\sigma^*}{\theta_r}\left[1 - \tau_r\left(1 - \alpha_3 + \dfrac{\alpha_2}{\sigma^*}\right)\right]}. \tag{5.6}$$

This equation is a necessary condition for a solution of problem (3.29). It is the counterpart of the laissez-faire marginal condition (2.16) and indicates the influence of taxation on the firm's employment of capital. If Condition (5.6) is solved for f_K, the other side of the equation shows the *user cost of capital*[1] which in a certain sense can be considered as the firm's discount rate for evaluating the cash flow from an investment project. This transformation is not carried out, however, since r is the link between the planning problems of the firm and the household.

At first sight, Equation (5.6) might seem quite complicated. Note, however, that it holds for different tax systems that are characterized by different assumptions for the relative sizes of the capital income tax rates τ_d, τ_r, τ_c, and τ_p; for the deductibility of actual (α_3) and imputed interest costs (α_2); and for the generosity of tax depreciation rules (α_1). Moreover, it takes account of the full complexity of the financial problem by assuming an optimal financial decision, given the constraints the firm faces. A detailed interpretation of the equation that singles out a number of illuminating special cases and reduces the complexity significantly will be given in Sections 5.3 and 5.4. Prior to that, however, a further dovetailing of the firm's investment and financial decisions will be discussed.

5.2. Firm Growth, Accelerated Depreciation, and Equity Finance

5.2.1. The Rivalry between Accelerated Depreciation and Debt Interest

Without providing any economic justification, it has been assumed for the time being [cf. (4.7)–(4.9)] that no more than a certain proportion σ^* of net investment can be financed through loans. The value that this proportion – the maximum marginal debt–asset ratio – can have was left open. The determinants of σ^* will now be discussed, and the question is why a value $\sigma^* < 1 - \alpha_1 \tau_r$ can prevail; that is, why the firm wishes to finance a strictly positive proportion $\varepsilon^*(\varepsilon^* \equiv 1 - \alpha_1 \tau_r - \sigma^* > 0)$ of its real investment with

[1]This concept [albeit not Equation (5.6)] dates back to Jorgenson (1967, p. 143).

equity capital rather than with debt even under circumstances where, according to the rules derived in the previous chapter, debt appears to be the cheapest source of finance.

Many reasons have been given for the fact that, despite tax advantages, investment projects are not usually fully debt financed. The extensive literature on the Modigliani–Miller theorem has collected a list of reasons that extend from bankruptcy costs to moral-hazard problems.[2] Expression (5.6) is flexible enough to take account of these reasons through a suitable choice of σ^*, and indeed we will discuss in many places in this book the implications of exogenously limiting the scope for debt financing.

This section offers an endogenous explanation of σ^* in that it investigates the implications of a simple but important argument proposed by DeAngelo and Masulis (1980) in a different analytical framework. The argument is based on the fact that corporate income tax is endowed with a limited loss-offset. A limited loss-offset means that there is rivalry between debt interest and other deductible items such as additional depreciation allowances. The higher the latter, the lower the useful size of debt interest, and hence the lower the optimal debt–equity ratio of the firm. DeAngelo and Masulis study this rivalry with the aid of a one-period model with uncertainty. The following pages analyze the intertemporal dimension of their argument.[3]

It is particularly important to study the problem within an intertemporal framework because there are at least two aspects that cannot possibly be clarified in a static model. First, there is the question of where the additional depreciation allowances come from. The authors mentioned assume these to be exogenously given and seem to have in mind extraordinary depreciation allowances that are added to the normal allowances. An *accelerated* tax depreciation that does not increase the sum of depreciation allowances beyond 100% of the value of an asset but merely changes the time pattern of depreciation is not considered. With regard to empirical relevance, however, such accelerated depreciation seems to be particularly interesting. A priori the impact it has on the debt–equity ratio of the firm is by no means obvious. It is true that the rivalry argument holds for new assets where the tax depreciation exceeds economic depreciation. However, with old assets it is exactly the opposite. Here, tax depreciation falls short of economic depreciation and hence there is additional scope for deducting

[2]For an overview of the literature see Swoboda (1981) and Modigliani (1982).

[3]For other extensions compare, for example, Zechner and Swoboda (1983) and Bartholdy et al. (1985).

debt interest. The net effect depends on the relative sizes of new and old assets and hence on the firm's investment policy. These relationships will have to be clarified in detail.

The second aspect relates to the legal constraints on the firm's financing policy. Is it admissible to assume that these constraints allow a sufficiently large scope for debt financing so that additional depreciation allowances really will compete with debt interest? This question seems particularly justified since, as explained in Chapters 3.2.2 and 4.1, in the Anglo-Saxon countries the firm is forced by law to retain the tax advantage from accelerated depreciation in the form of deferred taxes ($\alpha_1 \tau_r I$). This requirement itself implies that the full amount of net investment cannot be financed through loans. At the very least it is not obvious whether the rivalry between depreciation and debt interest requires borrowing to be reduced beyond the legal constraint; that is, whether σ^* has to fall short of $1-\alpha_1\tau_r$ if a negative tax base is to be avoided.

5.2.2. *Equity Formation and Tax Exhaustion*

The starting point for an evaluation of the role of a limited loss-offset is the corporate tax base Z which follows from our model for the parameter constellation that characterizes the existing tax systems:

$$Z \equiv f(K,L) - \tau_k K - wL - \delta K - \alpha_1 I - rD_f. \tag{5.7}$$

This tax base can be derived from (3.17) if the gross dividends Π^d are omitted and T_k is substituted according to (3.2). It is assumed that debt interest is tax deductible ($\alpha_3 = 0$), and accelerated depreciation is allowed ($\alpha_1 > 0$). The possibility of non-deductible debt interest ($\alpha_3 = 1$) is not considered for the time being. Its implications are studied below in Section 5.2.3. In line with the limitation of parameter constellations given in Chapter 3.1.4, it is assumed that imputed interest costs are not deductible ($\alpha_2 = 0$) and that personal interest income is taxed at a rate which is strictly above the capital gains tax rate which may or may not be equal to zero ($\tau_p > \tau_c \geq 0$).

The existing legal possibilities for a loss-offset are typically concerned with an expression like (5.7) and limit the amount of tax rebate which the tax functions (3.5) and (3.17) would imply if this expression were negative. Consider the extreme case where there is no loss-offset at all. Here, the firm will try to choose a debt policy such that $Z \geq 0$ is permanently sustained, for, in the case $Z < 0$, marginal debt interest would not be tax deductible and, according to the analysis of Chapter 4.4, there would be an incentive for replacing the net increase in debt as far as possible with retaining profits and/or issuing new shares.

Existing tax laws, however, are not quite so harsh. The United States, for example, allows extensive loss transfers between firms and in most countries there is at least some degree of loss-offset, since there are more or less generous possibilities for carrying losses forward or backward. Thus the corporate tax base may temporarily become negative without depriving the firm of the advantage of tax deductibility of debt interest. On the other hand, no country has laws that are generous enough to allow its tax system to be perverted into a system where capital incomes are subsidized instead of taxed. For this reason, it is assumed that the legal regulations for a loss-offset implicitly determine the maximum marginal debt–asset ratio σ^* in such a way that, with an actual marginal debt–asset ratio equal to this maximum ratio ($S_f/I = \sigma^*$), the tax system will just be prevented from converting into a system where a non-vanishing proportion of capital incomes is subsidized. More formally, it is assumed that, within the admissible range $0 \leq \sigma^* \leq 1 - \alpha_1 \tau_r$, σ^* is determined such that in the long run the corporate tax base becomes as small as possible without, however, violating the condition

$$\lim_{t \to \infty} Z^*(t) \geq 0, \tag{5.8}$$

where

$$Z^*(t) \equiv Z(t)/[r(t)K(t)] \tag{5.9}$$

is the corporate tax base relative to some measure of aggregate capital income.[4] This assumption is an idealization that may not always appropriately reflect reality in the short run. For the long run, when the representative firm grows along a steady-state path,[5] it seems, however, that it does approximate the legal loss-offset rules found in practice.

Before beginning an analysis of the implications of (5.8), it is useful to stop a moment and think about the meaning of this assumption. If a country employs accelerated depreciation rules it does not usually require

[4]This measure merely has the function of a scale variable here. Any other variable that in the long run is strictly positive and proportional to rK, for example $\theta_p rK$ or K, would yield the same results.

[5]Anticipating the results of the intertemporal general equilibrium model developed below, it is assumed that, for $t \to \infty$, the growth rates of all endogenous variables of the model of the firm asymptotically approach a given non-negative constant. It will be shown that, as in the laissez-faire model, this growth rate has the size $n + g$ which is the sum of the population growth rate and the growth rate of labor-augmenting technological progress.

the use of these rules under all circumstances; rather, it imposes them merely as upper bounds on the time patterns of tax depreciation that firms are allowed to choose. In principle, firms could stick to more conservative depreciation rules if they wished to. Thus, an implicit assumption underlying Condition (5.8) is that it is more attractive for firms to deduct accelerated depreciation than to deduct debt interest. Only in this case will they try to adjust their *debt* policy to satisfy (5.8). Appendix D legitimates this assumption. It is shown there, for the general case of arbitrary acceleration allowances, that the firm will indeed prefer a deduction of depreciation to a deduction of debt interest if, as was assumed, the personal capital income tax rate exceeds the capital gains tax rate ($\tau_p > \tau_c$).

To understand the intuition behind this result, consider first the tax systems of Types 1, 2, 4, and 5 in Figure 4.2 which characterize the classical and closely related systems of capital income taxation ($\theta_r^* \geq \theta_d^*$). In these systems, retained profits dominate new issues of shares at least in a weak sense and debt financing is equivalent or superior to retaining profits. Suppose the firm wishes to extend its debt financing up to the point where there is a rivalry between the deductibility of debt interest and accelerated depreciation. Then the policy of using accelerated depreciation methods can be seen as substituting in some initial phase retained profits for debt financing – or, equivalently, capital gains for interest income – and carrying out the reverse substitution thereafter. As the time path of the corporate tax liability is unaffected by these substitutions and there is a positive discount rate, the shareholder gains if, as assumed, his personal tax rate on interest income exceeds his capital gains tax rate. Thus he prefers the use of accelerated depreciation rules even though this deprives his firm of some of the advantages of debt financing.

The shareholder's gain can even be greater under the full imputation or closely related systems ($\theta_d^* > \theta_r^*$; Types 3 and 6 in Figure 4.2), for in these systems it is possible that new issues are a better source of equity finance than retentions and that *they* are substituted for debt financing and vice versa.[6] With these systems, it is a fortiori true that the firm wishes to make use of the accelerated depreciation allowances.[7] Thus it does make sense to

[6]Note that, in the partial and full imputation systems, the fact that distributed profits are subject to a reduced corporate tax burden does not mean that the tax saved through the deduction of accelerated depreciation is being reduced. The imputation methods that are in operation provide a corporate tax rebate to the shareholder household regardless of the taxes his company actually paid.

[7]It would even be possible in this case to assume that the capital gains tax rate equals the personal income tax rate ($\tau_p = \tau_c$). With the classical or closely related systems ($\theta_r^* \geq \theta_d^*$),

look for that value of σ^* which just satisfies (5.8), given a value of $\alpha_1 > 0$.

Some definitions are useful as a preparation for the analysis. Analogously to the respective marginal ratios defined in Chapter 4.1, we define σ and ε as the *average debt–asset ratio* and the *average equity–asset ratio*, respectively:

$$\sigma(t) \equiv D_f(t)/K(t), \tag{5.10}$$

$$\varepsilon(t) \equiv 1 - \alpha_1 \tau_r - \sigma(t). \tag{5.11}$$

It is implicitly assumed with these definitions that the stock of deferred taxes, $\alpha_1 \tau_r K$, does not count either as debt or equity, notwithstanding the fact that, in a legal sense, it is a liability to the government.[8] Moreover, the *effective price of capital*

$$P_K = \frac{\theta_p(1 - \sigma^* - \alpha_1 \tau_r)}{\max(\theta_d^*, \theta_r^*)} + \sigma^* \tag{5.12}$$

is introduced. The effective price of capital was used in primitive notation in (3.33) and it was assumed there that

$$P_K > \sigma(0). \tag{5.13}$$

Equation (6.4), that will be derived in the next chapter, proves that, when P_K is given by (5.12), this assumption is equivalent to assuming that the firm starts with a strictly positive market value of equity $[M(0) > 0]$.

In order to calculate σ^*, transform (5.7), using Euler's theorem and the marginal condition (3.38), into the expression

$$Z = (f_K - \delta - \tau_k)K - rD_f - \alpha_1 I \tag{5.14}$$

and replace the term $f_K - \delta - \tau_k$ according to Equation (5.6) where $\alpha_2 = \alpha_3 = 0$. After dividing the expression thus achieved by rK,

$$Z^*(t) = P_K - \sigma(t) - \alpha_1 \frac{\hat{K}(t)}{r(t)} \tag{5.15}$$

is found. With the aid of this equation, and assuming that $rK > 0$ $\forall t$, it is possible to derive σ^*.

however, the preference for accelerated depreciation vanishes when $\tau_p = \tau_c$. Thus, when accrued capital gains are fully included in the personal income tax base and the classical system prevails, firms may deliberately choose true economic depreciation in order to gain additional scope for useful debt finance. These results follow immediately from the analysis of Appendix D. Note, however, that the case $\tau_p = \tau_c$ is purely theoretical. Without exception, all countries employ tax systems where $\tau_p > \tau_c$. Cf. Chapter 3.1.2. (This is true although the 1986 U.S. tax reform requires all realized capital gains to be included in the personal income tax base.)

[8] Cf. Chapter 3.2.2..

Consider first the cases $\alpha_1 = 0$ and/or $\lim_{t\to\infty}\hat{K}(t) = 0$. For these cases, it can be shown that $\sigma^* = 1 - \alpha_1\tau_r$ (which is the highest admissible value). To see this, note that, according to (5.12), $\sigma^* = 1 - \alpha_1\tau_r$ when $P_K = \sigma^*$. Because of the assumption $\sigma(0) < P_K$ and the fact that $\sigma(t) < \sigma^* \forall t$ when $\sigma(0) < \sigma^*$ and $\dot{D}_f/\dot{K} = \sigma^*$ it obviously follows that $\lim_{t\to\infty}\sigma(t) \leq P_K$. Taking the limit of (5.15) we thus have

$$\lim_{t\to\infty} Z^*(t) = P_K - \lim_{t\to\infty} \sigma(t) \geq 0, \tag{5.16}$$

as was required with Condition (5.8).

Next, consider the case $\alpha_1 > 0$ and $\lim_{t\to\infty}\hat{K}(t) > 0$. Here $\sigma^* = 1 - \alpha_1\tau_r$ is impossible since, when $P_K = \sigma^*$ and $\lim_{t\to\infty}\sigma(t) = \sigma^* = P_K$, it holds that

$$\lim_{t\to\infty} Z^*(t) = -\alpha_1 \lim_{t\to\infty} [\hat{K}(t)/r(t)] < 0, \tag{5.17}$$

which is a violation of (5.8). Instead there must be a lower value of σ^* such that, when σ^* is binding, it holds that $\lim_{t\to\infty}\sigma(t) = \sigma^* < P_K$ to compensate for the term $-\alpha_1 \lim_{t\to\infty}[\hat{K}(t)/r(t)]$. According to (5.15), the highest conceivable value of σ^* that just satisfies (5.8) is obviously defined through the condition[9]

$$P_K - \lim_{t\to\infty} \sigma(t) - \alpha_1 \lim_{t\to\infty} [\hat{K}(t)/r(t)] = 0. \tag{5.18}$$

If it is noted that $\lim_{t\to\infty}\sigma(t) = \sigma^*$ when σ^* is binding and (5.12) is used, then it follows from this equation that the maximum marginal debt–asset ratio is given by

$$\sigma^* = 1 - \alpha_1\tau_r - \alpha_1 W \max(\theta_d^*, \theta_r^*) \tag{5.19}$$

or, equivalently, that the minimum marginal equity–asset ratio, $\varepsilon^* = 1 - \alpha_1\tau_r - \sigma^*$, is given by[10]

$$\varepsilon^* = \alpha_1 W \max(\theta_d^*, \theta_r^*). \tag{5.20}$$

[9]Note that the value of σ^* that satisfies this condition ensures that $\lim_{t\to\infty}Z^* = 0$ but not necessarily that $\lim_{t\to\infty}Z = 0$. Thus it would be wrong to say that the present approach requires a vanishing tax revenue for $t\to\infty$. Rather, it can be shown that, with a suitable choice of the initial stock of debt, temporarily or permanently strictly positive tax payments can arbitrarily be "produced". The value of σ^* that just satisfies the steady-state property $\lim_{t\to\infty}Z^* \geq 0$ is independent of the initial stock of debt. When $\lim_{t\to\infty}\hat{K} > 0$, any lower value of σ^* would imply $\lim_{t\to\infty}Z^* = \text{constant} > 0$ and any higher one would imply $\lim_{t\to\infty}Z^* = \text{constant} < 0$. Cf. in this context the discussion of Equation (5.58) in Section 5.4.3.4 below.

[10]Cf. Equation (4.8).

Here,

$$W \equiv \lim_{t\to\infty} \{\hat{K}(t)/[\theta_p r(t)]\}, \quad 0 \leq W < 1/\theta_c, \tag{5.21}$$

is a magnitude which we call the *growth factor* of the firm.[11] Because of the transversality condition (3.36) and because of the fact that (5.5) implies $\hat{\lambda}_K = 0$, W falls short of $1/\theta_c$ if a solution of the planning problem of the firm exists.

Equations (5.19) and (5.20) were derived for the case $W > 0$ and they reveal for this case that $\sigma^* < 1 - \alpha_1 \tau_r$ and $\varepsilon^* > 0$. Note, however, that these two equations correctly reproduce the result $\sigma^* = 1 - \alpha_1 \tau_r$ and $\varepsilon^* = 0$ that was derived above for the cases $\alpha_1 = 0$ and/or $W = 0$. Thus Equations (5.19) and (5.20) can be taken to hold for all cases considered.

The interpretation of Conditions (5.19) and (5.20) in the light of the two initial questions, which could not be answered by the one-period model of DeAngelo and Masulis, is straightforward. On the one hand, it turns out that the phenomenon of accelerated depreciation per se cannot be made responsible for the fact that part of a firm's net investment is financed through equity capital. Instead the size of the growth rate of the firm also plays a very important role. Only with a growing firm ($W > 0$) does accelerated tax depreciation reduce the scope of debt financing, for only here is the volume of tax depreciation permanently above the volume of true economic depreciation. On the other hand, it follows from the result $\sigma^* < 1 - \alpha_1 \tau_r$, which (5.19) yields for the case $\alpha_1 W > 0$, that in general it is not sufficient to reduce the net increase in debt merely by the amount of deferred taxes, that is by the current tax savings from accelerated depreciation. A growing firm has to reduce the debt-financed proportion of its net investment even further than this if it wants to ensure that it will permanently be able to enjoy the advantage of debt interest deductibility. At least the proportion $\alpha_1 W \max(\theta_d^*, \theta_r^*)$ must be financed with new equity capital!

While it seems plausible that (5.19) shows σ^* as a falling function of $\alpha_1 \tau_r$ and $\alpha_1 W$, the role of the term $\max(\theta_d^*, \theta_r^*)$ that entered via the marginal condition (5.6) is still unclear. In order to understand this role, it must be considered that, in the case of partial equity financing, with a given market rate of interest the marginal product of capital is higher and with a given marginal product of capital the market rate of interest is lower the heavier

[11] It will be shown in Chapter 8 that, in an intertemporal general equilibrium, $\hat{K}$ and r will develop such that the growth factor obtains the value $W = (n+g)/(\rho + \eta g) < 1$ *independently of the tax law.*

the tax burden on the return from equity capital. Given the subsidy effect $\alpha_1 \tau_r$ from accelerated depreciation, an increased tax burden on equity capital means a reduction of the expression $\max(\theta_d^*, \theta_r^*)$ which follows from the assumption that the firm chooses optimally among retentions and new issues of shares as its marginal source of equity finance, and an increased marginal product of capital and/or a reduced market rate of interest imply, as can be confirmed by looking at (5.14), that, relative to the stock of capital, a higher stock of debt is possible without endangering the deductibility of debt interest. This explains why, according to (5.19), a reduction of the term $\max(\theta_d^*, \theta_r^*)$, given $\alpha_1 \tau_r$, results in a rise of σ^*.

To interpret the result it is also useful to consider the average equity–asset ratio as defined in (5.11). Clearly the above considerations imply that, in a growing economy, the steady-state value of this ratio satisfies

$$\varepsilon^{\infty} \geq \alpha_1 W \max(\theta_d^*, \theta_r^*). \tag{5.22}$$

This expression will hold with equality if debt financing dominates new issues of shares and retentions strictly; that is, if the tax systems are those of Types 1 through 3 from Figure 4.2. Thus far (5.22) provides a simple, empirically testable, hypothesis for the development of the equity–asset ratio.

It goes without saying that this explanation for a strictly positive equity–asset ratio is just one of the possible explanations. Clearly other reasons for the necessity of forming and maintaining an equity capital stock may be present in addition to the one considered here. The Miller equilibrium that was discussed at some length in the previous chapter is one of the possibilities, and others are conceivable. While these other possibilities will not be explicitly discussed in this book, some scope will be provided for them in that, in addition to the endogenous explanation of the firm's financial constraints through (5.19) and (5.20), the implications of an exogenous determination of σ^* or ε^*, respectively, will be studied. In general it will therefore be assumed that $\sigma^* \leq 1 - \alpha_1 \tau_r - \alpha_1 W \max(\theta_d^*, \theta_r^*)$ or, equivalently, that $\varepsilon^* \geq \alpha_1 W \max(\theta_d^*, \theta_r^*)$.

On the other hand, there is evidence that the hypothesis studied in this section is concerned with more than a negligible effect. In countries that introduced accelerated depreciation schemes, firms seem frequently seriously concerned with the loss-offset constraints they are facing. Extensive use is made of the possibilities of carrying losses forward and backward, and hectic activity aimed at extending and exploiting the scope for inter-firm loss transfers can be observed. A good example is Canada where very generous depreciation rules have been introduced in the period following the major tax reform of 1972. According to a recent report of the Canadian Department of Finance[12], in the 6-year period from 1977 to 1982 about 45% of Canadian investment was undertaken by firms that "rarely" paid taxes and 15% by firms that "sometimes" paid taxes. Only 30% of investment took place in firms that "usually" paid corporate income tax.[13] Different, but related effects were observable in the United States after the introduction of the Accelerated Cost Recovery System in 1981. A fire sale on corporate tax losses, called "safe-harbor leasing", took place shortly after the reform, and the U.S. corporate tax revenue declined dramatically.[14] The safe-harbor leasing arrangements were soon disallowed and corporate tax revenue recovered a little, but, as a consequence, a corporate loss overhang similar to the Canadian situation developed. This is one of the reasons why the United States returned to more conservative depreciation allowances with the tax reform of 1986.

In other countries, the situation may have looked less dramatic than in North America. Note, however, that this does not necessarily preclude the forces described from being operative. According to the hypothesis developed it is the threat of becoming tax-exhausted in the future that determines the choice of financial instruments in the present. This threat may be large even for companies that pay high taxes in the present. With accelerated tax depreciation, large future investment makes it wise to plan large future profits, and large future profits require much equity finance, but little debt finance today.

In Chapter 4.3.3 it was reported that the equity–asset ratios of large corporations in Western industrial countries drastically declined during the sixties and seventies,[15] and it was argued that this development contradicts the frequently contended lock-in effect of a double taxation of dividends. The previous considerations support this argument. On the one hand they provide the possibility of interpreting the described development as an adjustment process towards a given steady-state level. On the other hand, and this is certainly the more interesting implication, they suggest that, at least in some countries, the observed reduction of the equity–asset ratio was the consequence of a reduction in its steady-state level that was induced by

[12]See DFC (1985, pp. 17 n. and Table 9). (I am grateful to David Sewell for a useful discussion of the issue.)

[13]The ministry defined "rarely" as "fewer than three of the six years", "sometimes" as "three or four years", and "usually" as "at least five years".

[14]Cf. Chapter 3.1.3, expecially Footnote 2.2.

[15]Cf. Table 4.1.

a change in the growth factor. This suggestion is supported by the fact that during the sixties and seventies economic growth rates in most countries declined. If this means that entrepreneurial expectations with regard to the long-run growth chances of the firms were being revised downward, then the decline in the equity–asset ratios seems fully compatible with the requirements of a rational financing policy. This explanation of a frequently mentioned empirical fact contrasts sharply with the contention, which is typically put forward by industry representatives, that private investment and economic growth were being reduced *because* of the erosion of the equity base. Precisely the opposite effect is suggested by the model.

5.2.3. Non-deductible Debt Interest

As announced, the case of non-deductible debt interest ($\alpha_3 = 1$) is now briefly considered. If the interest the firm pays on its outstanding debt cannot be deducted from its profit tax base then there is no rivalry between debt interest and additional depreciation allowances. Hence a limited loss-offset has no implications for the size of the maximum marginal debt–asset ratio σ^*. In the analysis of the firm's investment choice it can therefore be assumed that only the legal constraint (4.3) has to be satisfied:

$$\sigma^* = 1 - \alpha_1 \tau_r. \tag{5.23}$$

5.3. Investment-neutral Taxes

Using the results of the previous section concerning the value of the maximum marginal debt–asset ratio σ^*, we now want to begin with an economic discussion of the marginal condition (5.6). For convenience this condition is repeated here together with Equations (5.19) and (5.23):

$$r = \frac{f_K - \delta - \tau_k}{\dfrac{\theta_p(1 - \sigma^* - \alpha_1 \tau_r)}{\max(\theta_d^*, \theta_r^*)} + \dfrac{\sigma^*}{\theta_r}\left[1 - \tau_r\left(1 - \alpha_3 + \dfrac{\alpha_2}{\sigma^*}\right)\right]}, \tag{5.6}$$

$$\sigma^* = 1 - \alpha_1 \tau_r - \alpha_1 W \max(\theta_d^*, \theta_r^*) \qquad \text{for } \alpha_2 = \alpha_3 = 0, \tag{5.19}$$

$$\sigma^* = 1 - \alpha_1 \tau_r \qquad \text{for } \alpha_3 = 1. \tag{5.23}$$

For the time being the analysis is limited to a search for those conditions under which Equation (5.6) can be reduced to the laissez-faire equation

$r = f_K - \delta$. Taxes or tax system with this property are called *investment neutral* since, given the time paths of the market rate of interest $\{r\}$ and the employment of efficient labor $\{L\}$, they do not affect the firm's investment decision. The choice of this concept is made in full awareness of the fact that, in a partial analysis, reliable results for the impacts of taxation on the process of accumulation of capital cannot yet be achieved. To get such results, it would be necessary, among other things, to know how taxation affects the time path of the market rate of interest, a problem that will only be treated in later chapters.

Despite the limited predictive power of the concept of investment neutrality, there are various reasons why this is a useful concept. One of these reasons is the bench-mark function of investment-neutral tax systems. It is easier to understand the effects of realistic tax systems if the effects of simpler idealized tax systems are known and can serve as a yardstick. Another reason is that the problem of partial analytic investment neutrality has straightforward consequences for international and intersectoral distortions in the capital structure brought about by the tax systems. This problem will be extensively treated in Chapters 6 and 7. Here it is sufficient to note that investment-neutral tax systems avoid such distortions.

Critics of formal analyses of taxation effects frequently tend to concentrate on neutrality results and attempt to demonstrate their irrelevance or even falsity by listing circumstances under which these results break down. Though legitimate, this attempt should not be taken too seriously. It is the very nature of a neutrality result that it is quite vulnerable to a modification of its assumptions. Economic models often predict that a certain variable will rise or fall as a result of an exogenous parameter change. Such a prediction is weak and therefore robust with regard to a change in assumptions. In comparison, a neutrality result is extremely precise and the a priori probability of ever observing it in reality is virtually zero. Only one point in the continuous spectrum of possibilities is identified, not 50% of them. Thus it is quite obvious that neutrality results cannot be taken as unconditional predictions. More than for other theoretical results, we must bear in mind that they are reported as *if–then* clauses where no claim is made that the *if* component is satisfied precisely in reality.

In the literature there has been extensive discussion of the problem of investment neutrality. The pertinent results will be summarized here in a systematic fashion in the light of Condition (5.6). In addition, further neutrality results that generalize the famous Johansson–Samuelson theorem to broader classes of systems of capital income taxation will be presented. These neutrality results are essential for understanding the remaining chapters of this book.

5.3.1. A Tax on Corporate Profits as the only Tax in the Economy

It is well known from the static theory of taxation[16] that a tax on pure profits does not bring about any changes in the behavior of the firm taxed. For if the firm maximizes its net-of-tax profit then it also maximizes its gross profit and hence chooses the same values of its control parameters as it would in the absence of taxation. It can easily be shown by use of Equation (5.6) that this result holds also for an intertemporally optimizing firm.

Assume for this purpose that there is a uniform tax on retained and distributed profits ($\tau_d = \tau_r > 0$), that there is no interest income tax, no capital gains tax, no tax on the capital stock, and no value-added tax ($\tau_p = \tau_c = \tau_k = \tau_v = 0$), and that the firm has to follow true economic depreciation for calculating its tax base ($\alpha_1 = 0$). With regard to the interest cost of the firm it is alternatively assumed that either the actual or the total, including imputed, interest cost can be deducted from the tax base. In the former case "profit" is defined in the legal or accounting sense and includes the return on equity capital. In the latter, "profit" is "pure economic profit".

In the first case, where only the actual interest cost is deductible, we have $\alpha_2 = \alpha_3 = 0$ and, taking note of (5.19), (5.6) becomes

$$r = \frac{f_K - \delta}{\dfrac{1-\sigma^*}{\theta_r} + \sigma^*} = f_K - \delta, \tag{5.24}$$

as contended. The tax system is of Type 2 from Figure 4.2. Because of the assumption of true economic depreciation, it holds that $\sigma^* = 1$, and the firm's net investment is fully debt-financed. Since the last unit of capital invested brings about a marginal profit of zero and hence does not bear any tax, taxation is irrelevant for the firm's investment decision.

If both actual and imputed interest cost can be deducted from the corporate tax base then $\alpha_2 = \alpha_3 = 1$. Ignoring (5.23), an investment neutrality follows immediately from (5.6):

$$r = \frac{f_K - \delta}{\dfrac{1-\sigma^*}{\theta_r} + \dfrac{\sigma^*}{\theta_r}\left(1 - \dfrac{\tau_r}{\sigma^*}\right)} = f_K - \delta. \tag{5.25}$$

Contrary to (5.24), this neutrality result holds independently of the size of

[16] Cf. e.g. Mill (1865, pp. 496–498) or Häuser (1959/60).

the maximum marginal debt-asset ratio, σ^*. It is irrelevant whether we set $\sigma^* = 1$, as required by (5.23) when $\alpha_1 = 0$, or set $\sigma^* < 1$ as there are other, exogenous reasons for a limitation of debt financing. Since the assumptions $\tau_d = \tau_r$, $\tau_c = \tau_p = 0$, and $\alpha_3 = 1$ describe the tax system of Type 8 from Figure 4.3 where all financial instruments are equivalent, the financial constraint cannot be binding and hence cannot enter the marginal investment condition.[17]

The neutrality result (5.25) seems first to have been derived in a precise form by Sandmo (1974, p. 291) who demonstrated the equivalence of a tax on pure economic profits and a tax on the real cash flow of a firm. More on this will be said below in Section 5.3.5. The neutrality result (5.24) can be attributed to Oberhauser (1963, pp. 67 n.) and Stiglitz (1973, pp. 25 n.) who showed that, when a corporate tax is the only tax imposed, it has no impact on the investment decision if the firm chooses debt financing at the margin and debt interest is deductible. As will be explained below in Section 5.3.4, however, Stiglitz addresses a more complex problem in giving his proof and justifies the assumption of debt financing of the marginal investment project in a way different from that implicitly assumed with the derivation of (5.24).

5.3.2. The Johansson–Samuelson Theorem

A tax that approximates the reality of existing systems of capital income taxation more closely than taxes on pure profits has been considered by Johansson (1961, pp. 106, 135, 148 n. and 211–216; 1969) and Samuelson (1964).[18] Both authors assume that all kinds of capital income are subject to a uniform marginal tax rate and they explore the influence of this rate on the present value of the returns an investment project yields. They find that this value is not affected by the tax and that hence investment neutrality prevails if tax depreciation is equal to economic depreciation and debt interest is deductible. The name "Johansson–Samuelson theorem" seems appropriate to characterize this important result.

Economic depreciation and deductible debt interest are crucial ingredients of the famous Schanz–Haig–Simons concept of capital income taxation.[19] In fact, the Johansson–Samuelson theorem can be seen as the theoretical basis of this concept. The tax to which the theorem refers is a

[17] See Chapter 11.3.5 for the revenue implications of the profit taxes.
[18] Cf. also D. Schneider (1969), p. 303; 1974, pp. 311–319) and Strobel (1970).
[19] Cf. Chapter 3.1.1.

Schanz–Haig–Simons tax in its purest form. Not all contributions in the literature concerning the influence of Schanz–Haig–Simons taxes on the firm's investment choice take sufficient note of this fact. Without exaggeration it can be said, however, that the Johansson–Samuelson theorem is the pivotal point of every microeconomic theory of the influence of taxation on the formation of capital. Being aware of this theorem is indispensible for understanding basic results of this book.

To capture the essence of Johansson and Samuelson's reasoning, suppose the present value of earnings generated by an asset is given by

$$\Omega(t) = \int_t^T \{\omega(u) - \tau[\omega(u) - \chi(u)]\} \exp\left[-\int_t^u (1-\tau) r(s)\, \mathrm{d}s\right] \mathrm{d}u, \tag{5.26}$$

where T, $T \leq \infty$, indicates the time horizon, ω the current real cash flow without the initial investment outlay, τ the uniform tax rate, and χ the flow of tax depreciation. After differentiating this expression with regard to t, for each point in time $t \leq T$,

$$\dot{\Omega}(t) = -(1-\tau)\omega(t) - \tau\chi(t) + (1-\tau) r(t)\Omega(t) \tag{5.27}$$

follows. Given the time paths $\{\omega\}_t^T$, $\{\chi\}_t^T$ and $\{r\}_t^T$, this differential equation describes possible paths in a (ω, t) diagram by determining precisely one slope for each point of this diagram. Figure 5.1 shows some examples. Which of these paths describes the true development of the present value of the remaining cash flow of an asset depends on the size of the final value of this asset at point in time T. If this final value, as implicitly assumed with (5.26), equals zero then the time path that leads through point $(0, T)$ is singled out, and the present value at each point in time $t \leq T$ is well defined.

In general the size of the present value depends on the tax rate τ. Assume however, with

$$\chi = -\dot{\Omega}, \tag{5.28}$$

that *true economic depreciation* is allowed. Then dividing (5.27) by $(1-\tau)$ yields

$$\dot{\Omega} = -\omega + r\Omega, \tag{5.29}$$

or the well known arbitrage condition

$$(\omega/\Omega) + \hat{\Omega} = r, \tag{5.30}$$

which says that the sum of the current rate of return ω/Ω and the relative capital gains $\hat{\Omega}$ equals the market rate of interest. Since the differential equations (5.29) and (5.30) are independent of the tax rate and since, at the

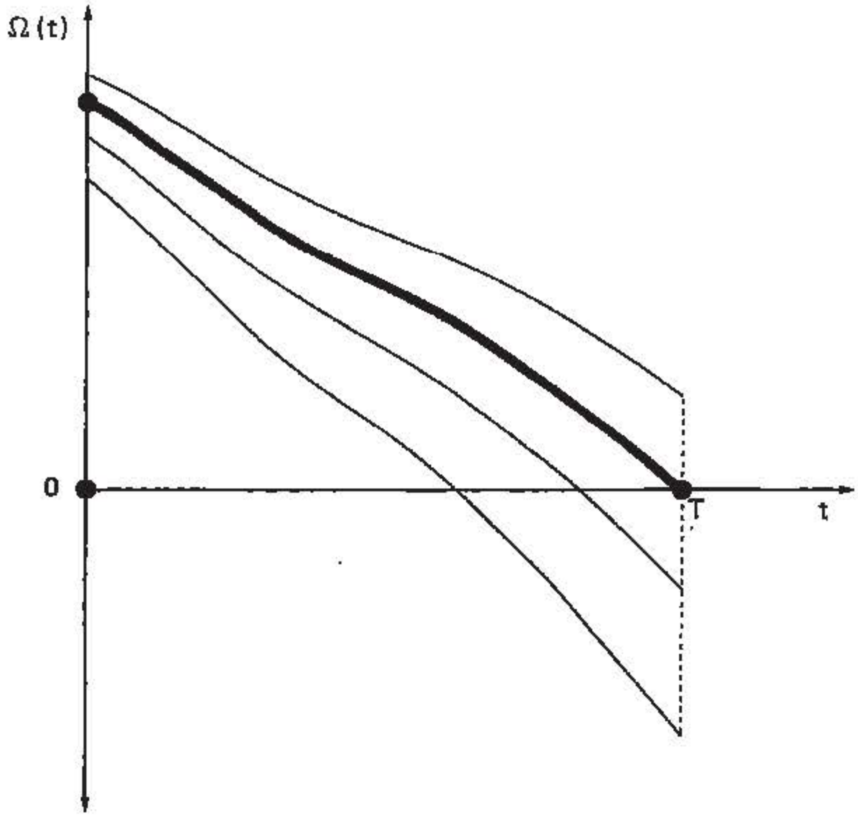

Figure 5.1. The present value of returns under the influence of a uniform tax on all kinds of capital income.

terminal point T, the asset has a given value independent of the tax rate, the total path $\{\Omega\}_0^T$ must be unaffected by the tax. This means, in particular, that the present value of the cash flow calculated at the initial point in time $t=0$ is not changed by the imposition of the tax. Thus, given the initial purchasing price of the asset, the firm's investment decision is not affected: a pure Schanz–Haig–Simons tax is investment neutral.

At first glance the result is surprising. A priori, one is tempted to believe that a tax which leads to a reduction of the net returns from a real investment project will imply a reduction of this project's market value and perhaps render it unattractive for the investor. The reason this belief is wrong is the assumption that *all* kinds of capital income, including the potential earnings from an alternative investment in the capital market, are taxed at the same rate. This assumption ensures that a market value, that before taxation made the effective rate of return on a real asset equal to the rate of return on a financial asset, will retain this property after taxation.

An elementary and problematic assumption underlying the result is true economic depreciation. In reality, true economic depreciation is usually not allowed if only because tax laws require the depreciation to be based on the purchase price of an asset. Since the present value of the returns generated by an asset that the firm buys is never below but often above this price[20] this aspect implies that too little depreciation is allowed for tax purposes. Contrary to initial appearances, this fact by itself does not yet mean that taxation discriminates against the employment of capital. It is true that the present value of the returns of an asset will fall under the influence of taxation if this present value was strictly positive. However, when only the level, and not the relative time pattern, of depreciation is false, the present value will not fall below the purchase price and the asset is still worth having. What matters for the question of whether the employment of capital is discriminated against is solely the treatment of marginal assets. With a marginal asset, the present value of future returns just equals the purchase price of this asset and hence the basis of tax depreciation coincides with the basis of economic depreciation. The preferability of a marginal asset can only be affected by the tax if the *speed* at which the value at purchase is depreciated does not coincide with the true diminution of the present value of returns. It was shown in Chapter 3.1.3 that in some countries the legal depreciation speed does not even approximate true economic depreciation, and this was the reason for allowing for a depreciation parameter $\alpha_1 \geq 0$. In Section 5.4.3 the question of how the size of this parameter can affect the investment decision will be discussed.

If the depreciation parameter α_1 has a value of zero, then the assumptions underlying the present model can easily be interpreted in the light of the Johansson–Samuelson theorem. Assume the firm's investment planning is optimized and, at time zero, a further unit of capital is invested. Then the cash flow at time t from this investment is given by

$$\omega(t) = f_K[K(t), L(t)]e^{-\delta t}, \tag{5.31}$$

the economic value of the capital remaining at time t is

$$\Omega(t) = e^{-\delta t}, \tag{5.32}$$

and true economic depreciation is

$$\chi(t) = \delta e^{-\delta t}. \tag{5.33}$$

If Equation (5.30) is calculated for these values then the familiar laissez-faire condition

$$r = f_K - \delta \tag{5.34}$$

for an optimal employment of capital is obtained.

[20]The point is less important under the narrow assumptions underlying the model employed in this book. Because of perfect competition and constant returns to scale, the present value of the returns of an asset must always equal its purchase price.

Just for confirmation, we now want to take a brief look at our general condition (5.6). If we set $\alpha_1 = \alpha_2 = \alpha_3 = \tau_k = \tau_v = 0$ and $\theta_r = \theta_d^* = \theta_p < 1$, $\theta_c = 1$, then Condition (5.34) is indeed obtained. A general uniform taxation of all kinds of capital income is therefore investment neutral if true economic depreciation is allowed for marginal assets.

As with the tax on pure economic profits analyzed by Sandmo, the result is again independent of the size of the maximum marginal debt–asset ratio σ^*. Once again there is, as can be seen from Type 5 from Figure 4.2, an equivalence between all three financial instruments, and once again the upper horizontal borderline of the solution space depicted in this figure is not a binding constraint for the firm's financial choice. This aspect shows that, under certain conditions, it is possible to study the impact of taxation on the firm's investment decision without explicitly mentioning its financial choice. Johansson and Samuelson made use of this possibility. Contrary to a suspicion the casual reader might have, this is not a weakness of their approach.

This section concludes with a generalization of the neutrality result just described. In their considerations, Johansson and Samuelson assume the investment is in a real asset. The mathematical structure of their proof, however, is general enough to be applied to the present value of the returns of a whole corporation and hence to the market value of its shares. Identify capital gains of these shares with negative economic depreciation. Then it follows immediately from (5.26)–(5.30) that a general uniform taxation of the sum of personal interest income, distributed profits, and *capital gains* will not affect the market value of the firm given the time paths of its control variables. Hence it is obvious that this form of taxation, too, cannot have any impact on the firm's optimal behavior.

In connection with the analysis of the firm's financial decisions from Figure 4.2, Equation (5.6) generalizes and confirms this interpretation. Indeed, both the financial neutrality and the neutrality result $f_K - \delta = r$ stay valid if, other things being equal, the assumptions $\theta_d^* = \theta_r = \theta_p < 1$ and $\theta_c = 1$ are replaced with $\theta_d^* = \theta_r^* = \theta_p < 1$ and $\theta_r^* \equiv \theta_c \theta_r$. Thus it does not matter to what degree retained profits are directly taxed, and to what degree they are taxed indirectly via the capital gains on company shares. It is only necessary that the combined tax factor θ_r^* that summarizes the effects of the corporate tax on retentions and the capital gains tax equals the combined tax factor for distributed profits θ_d^* and the tax factor for interest income θ_p. Every combination of direct and indirect taxation of retained profits that satisfies this condition is neutral with regard to the firm's investment and financing decisions.

5.3.3. The General Investment Neutrality of Capital Income Taxation under Full Financial Flexibility

Up till now three kinds of taxes that are investment neutral under an optimal financial planning have been analyzed: a profit tax with deductibility of actual and imputed interest cost, a profit tax with deductibility of actual interest cost, and a pure Schanz–Haig–Simons tax. It can be shown, however, that more general neutrality results are possible. One such result is the following. Remove the assumption of a uniform marginal tax rate, but suppose, still in line with the Schanz–Haig–Simons concept, that

(1) tax depreciation equals true economic depreciation ($\alpha_1 = 0$),
(2) only the actual interest cost is tax deductible ($\alpha_2 = \alpha_3 = 0$),
(3) there is no tax on the value of the capital stock ($\tau_k = 0$), and
(4) the firm enjoys full financial flexibility in the sense that $\varepsilon^* = \alpha_1 W \max(\theta_d^*, \theta_r^*)$.

Then all those systems of capital income taxation, which are described by the structure of the corporate tax rate for distributed profits (τ_d), the corporate tax rate for retained profits (τ_r), the personal income tax rate of the shareholder household (τ_p), and the personal capital gains tax rate (τ_c) and for which a solution of the optimization problem of the firm described in Chapter 3.2 exists, are investment neutral.[21]

The proof of this result can easily be given with the aid of the marginal condition (5.6). Set $\alpha_1 = \alpha_2 = \alpha_3 = \tau_k = 0$ and note that $\alpha_1 = 0$ implies that the maximum marginal debt–asset ratio is one ($\sigma^* = 1$) or, equivalently, that the minimum marginal equity–asset ratio is zero ($\varepsilon^* = 0$). Obviously Condition (5.6) reduces to the laissez-faire marginal condition for an optimal employment of capital:

$$r = \frac{f_K - \delta}{[\theta_p(1 - \sigma^*)/\max(\theta_d^*, \theta_r^*)] + \sigma^*} = f_K - \delta. \qquad (5.35)$$

This result includes all six of the combinations of tax factors θ_d^*, θ_r^*, and θ_p illustrated in Figure 4.2 and, except for the depreciation problem, should therefore be able to capture most of the tax systems that exist in the OECD countries. To the extent that these systems satisfy the rules of Schanz, Haig, and Simons and to the extent the Modigliani–Miller framework is applicable, taxation can be expected to be investment neutral.

Compared to Johansson and Samuelson's result, the most important

[21] This neutrality result even holds independently of size of the value-added tax rate. See Section 5.3.7.

aspect of this neutrality result is the *irrelevance of the degree of integration between corporate and personal taxation.* For example, the result is fully compatible with double taxation of dividends and does not require an equality between the personal income tax rate and the corporate tax rate for retained profits. Neither the levels nor the ordinal magnitude structure of the four capital income tax rates considered in this book $(\tau_p, \tau_c, \tau_r, \tau_d)$ impinge on the firm's investment decision.

The most objectionable assumption underlying the result concerns the depreciation rules. True economic depreciation is a purely theoretical concept that cannot easily be implemented in practice. It seems hardly possible to design a control system which would enable the authorities to monitor the gradual decline in the present value of returns generated by an asset. Moreover, it does not even seem that governments have *tried* to implement true economic depreciation. The situation described in Chapter 3.1.3 suggests the opposite: in a number, if not most, countries there has been a strong bias towards accelerated tax depreciation rules. It is clear therefore that, for these countries, the result cannot claim to depict reality, even as a first approximation–notwithstanding its theoretical significance.

The most debatable assumption underlying the result concerns the firm's financial decisions. In theoretical analyses of tax distortions, it is usually assumed that the firm does not attempt to optimize its financial decisions but chooses the financial instruments in fixed proportions regardless of whether the tax system discriminates against certain financial instruments or not.[22] Under such an assumption, investment neutrality cannot be expected if the tax factors deviate from the Johansson–Samuelson assumption $\theta_r^* = \theta_d^* = \theta_p$; the discrimination against a certain financial instrument would then necessarily carry over to a discrimination against the real investment project that is financed (at least in part) with this instrument. However, this assumption is not made in this section. On the contrary, it is assumed that the firm simultaneously optimizes its investment decision *and* its financial decision and chooses a policy that maximizes the market value of its equity capital. Within the legal constraints to its financial policy, it chooses the most favorable instrument among debt financing, financing through retained profits, and issuing new shares. This assumption is the reason that real investment choice is protected from discrimination against particular financial instruments. Admittedly this is an extreme view whose empirical relevance may be called into question. But it is a good reference

[22]Ironically, the most frequent assumption is that the firm finances its net investment only with that instrument which the tax system discriminates most heavily against. Cf. Section 5.4.2.

point for an analysis of tax distortions that may well compete with the other extreme that the firm makes no attempt whatsoever to react with its financial policy to the discrimination against particular financial instruments. It seems useful, at least for didactic purposes, to include the case of full financial flexibility, together with other cases, in the analysis of this book.

In order to understand the financial protection mechanism intuitively it is useful to begin with the Johansson–Samuelson result and the corresponding assumption $\theta_r^* = \theta_d^* = \theta_p$ and then to consider how the firm reacts to an increased burden on retained profits ($\theta_r^* < \theta_p$) and/or distributed profits ($\theta_d^* < \theta_p$).

If the direct or indirect (via capital gains taxation) marginal tax burden on retained profits exceeds the overall marginal tax burden on distributed profits and also exceeds the marginal tax burden on a potential interest income of the representative shareholder household, then the firm decides against retentions and chooses new share issues or debt as marginal sources of finance. The profitability requirement the marginal investment project has to satisfy is unchanged and there is no reason to choose another investment policy.[23]

If distributed profits are taxed more heavily than retained profits and also more heavily than the interest income of the representative shareholder household, then the firm has only limited possibilities for avoiding the additional tax burden. It is true that the firm can reduce its new share issues and can thus prevent further engagement of its shareholders. However, to pay back equity capital that has already been invested in the firm through repurchasing its own shares is excluded by Assumption (4.1). The best available reaction is to refrain from issuing new shares ($Q = 0$). Under these circumstances, an increased tax burden on distributed profits inevitably implies a reduction in the market value of equity. The reduction in the market value is a clear deviation from the Johansson–Samuelson result but it does not affect investment neutrality. The reason is that, whatever the time paths of the control variables, the firm's net dividends are reduced by a given percentage for each point in time and hence the market value declines by this same percentage. Obviously, this fact ensures that an increased tax burden on distributed profits does not affect the firm's preference ordering over alternatively possible time paths of control variables. As long as

[23] In the discussion that preceded the German tax reform of 1977, this substitution possibility was mentioned as a possible justification for the high tax rate on retained profits. See Hirsch (1966, pp. 426 n.).

confiscatory taxation is avoided ($\theta_d^* > 0$) investment neutrality is perfectly maintained.[24]

If retained *and* distributed profits are taxed more heavily than interest income of the representative shareholder then the firm has an incentive for avoiding both retentions and new issues of shares and prefers to finance its net investment with credit. Even in this case, therefore, there is no way a deviation from the Johansson–Samuelson assumption $\theta_r^* = \theta_d^* = \theta_p$ can affect the marginal condition for an optimal investment policy.

A common element of the three constellations considered and perhaps the simplest clue for understanding the neutrality result is that debt always belongs to the set of attractive financial instruments. Either debt is definitely the preferred marginal source of finance or, when another source is chosen, this source is equivalent to debt. Thus, wherever the funds needed for financing an investment project come from, *the marginal cost of finance is the cost of debt financing.* It is then quite obvious, in the light of Oberhauser and Stiglitz's neutrality result cited in Section 5.3.1, that, with a deductibility of debt interest, there is no way taxation could interfere with the marginal condition of the firm's investment policy.

Although debt financing is a clue for understanding the neutrality result it must be emphasized that it was not assumed that debt *is* the marginal source of finance. In three of the six types of system of capital income taxation considered in Figure 4.2 debt finance is equivalent to equity finance, and none of these can be accused of being empirically irrelevant. In fact, one of them, Type 4, characterized by the constellation $\theta_r^* = \theta_p > \theta_d^*$, might well approximate to a first degree the situation prevailing in some countries, including the United States before the 1986 reform.[25] Thus, the role of debt financing should be interpreted in a putative rather than a literal sense. The firm decides on its real investment projects *as if* these projects were debt financed.

This interpretation merits contrast with a position that seems to enjoy some popularity in the economic discipline. Obviously, it is argued, double taxation of dividends that characterizes most of the existing tax systems will favor debt over equity financing, but the empirical fact is that firms are nevertheless endowed with equity capital. Thus, the view that firms optimize their financial decisions in a Modigliani–Miller fashion is dismissed as unrealistic, so too is the view that debt financing could be relevant for the

[24]See Section 5.3.6 and, in particular, Chapter 6.2 for further discussions of the neutrality of dividend taxation.

[25]Cf. the discussion of the Miller equilibrium in Section 5.4.1.

firm's real investment decision. Apart from the fact that the existence of an equity does not contradict debt financing at the margin on logical grounds, there are two reasons for this position not being acceptable.

First, it overlooks the financial neutrality property of double taxation of dividends that was discussed in Chapter 4 (Sections 4.2.2 and 4.3.3). As it is not true that double taxation discriminates against equity finance as such, the mere observation that firms choose equity finance – even the observation that they choose equity finance at the margin – is no evidence whatsoever against the view that the marginal cost of finance is the cost of debt financing.

A second counterargument refers to the role of accelerated depreciation. Suppose, in addition to the double taxation of dividends ($\theta_p > \theta_d^*$), retained profits are taxed more heavily than interest income ($\theta_p > \theta_r^*$) such that the tax system really discriminates against (both kinds of) equity capital. Would the observation that, for example in the United States, equity is chosen at the margin now contradict the hypothesis that firms optimize their financial decisions in a Modigliani–Miller fashion? It would not. As shown in Section 5.2, the phenomenon of accelerated depreciation, quite obvious in the United States, could, in principle, explain this behavior. The fact that, despite $\theta_p > \max(\theta_d^*, \theta_r^*)$, the firm uses equity finance at the margin in order not to be deprived of the possibility of deducting debt interest does not contradict the Modigliani–Miller view that, in the absence of taxes, firms would be indifferent between debt and equity. Neither does it contradict the neutrality result that, when the tax laws require true economic depreciation, investment neutrality would prevail.

These remarks make it clear that the question of empirical evidence is far from being trivial or obvious. They do not, of course, preclude the possibility that firms nevertheless enjoy less financial flexibility than assumed in this section. As the empirical situation is unclear, this book does not take a particular stand either. In Section 5.4 and elsewhere the case of limited financial flexibility, both for exogenous and endogenous reasons, will be extensively discussed.

5.3.4. Criticism of Stiglitz's Neutrality Result

For the special case of the classical system of capital income taxation, Stiglitz (1973) studied the interaction of investment and finance and he also derived a neutrality result. Stiglitz assumed that the interest income of the shareholder household is taxed at least as heavily as retained profits but less than distributed profits:[26] $\theta_r \geq \theta_p > \theta_d^*$. He assumed true economic depreciation with a deductibility of debt interest and, as financial constraints, he considered a non-negativity of dividends [cf. (4.4)] and new issues of shares [cf. (4.1)].[27] Stiglitz did not consider a capital gains tax:[28] $\theta_c = 1$.

Among the two constellations $\theta_r = \theta_p > \theta_d^*$, $\theta_c = 1$, and $\theta_r > \theta_p > \theta_d^*, \theta_c = 1$, only the first is compatible with the tax systems considered in this book. This constellation describes a subcase of the tax system of Type 4 from Figure 4.2 which is characterized by $\theta_r^* = \theta_p > \theta_d^*$ and is one out of six possible types. The second constellation is not admissible in the framework of this model since a solution to the optimization problem of the firm would not exist. Stiglitz concentrates entirely on this constellation, however.[29]

According to the general rule for assessing the firm's financial preferences that was reported in Chapter 4.3.1, the second constellation implies that retentions dominate debt strictly. As illustrated in the following Figure 5.2, it is therefore "optimal" to choose the lower left corner of the solution space of the firm. This point is characterized by a retention of all profits and a complete absence of new share issues. The (possibly negative) net increase in debt is just high enough to cover that part of the firm's real net investment that cannot be financed through retentions.

The latter aspect is the clue to understanding Stiglitz's neutrality result. Obviously it implies that each additional unit of investment outlay has to be financed by funds raised in the capital market or by a reduction in the stock of financial assets the firm owns; that is, that the marginal investment is debt financed. For the reasons explained in Section 5.3.1 this implies that the tax system is investment neutral.

Despite a superficial similarity, the economic intuition behind this result has little in common with the mechanisms that were responsible for the

[26] Cf. ibid. pp. 17 n. in connection with the verbal considerations on p. 10. On p. 19 Stiglitz also mentions another constellation of tax rates, but because of a printing error and since this constellation is not discussed it is difficult to assess what he might have meant. Possibly he meant the constellation $\theta_d^* > \theta_p > \theta_r$. This case creates an unlimited incentive for the firm to lend in the capital market and, given that in reality there are no financial constraints to this activity, a solution to the firm's decision problem would not exist.

[27] In his verbal explanations Stiglitz assumed that the firm may repurchase its shares to a limited extent, but his formal analysis does not really take account of this assumption. The only role of the assumption seems to be to legitimate a lower marginal tax burden on dividends.

[28] It is true that in various places the paper alludes to a capital gains tax, but the only reason seems to be to justify the low level of the effective marginal tax burden on distributed profits that was mentioned in the previous footnote. (Cf. especially p. 10.)

[29] Cf. in particular p. 7 and pp. 17 n.

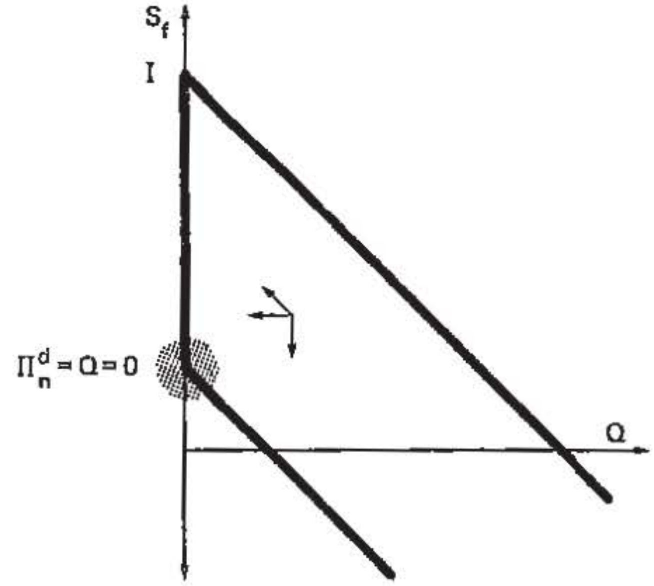

Figure 5.2. The case considered by Stiglitz.

neutrality result reported in the previous section. The essence of the latter was that debt *is never inferior* to new issues of shares or retentions and will therefore determine the cost of finance even if it is not chosen. In Stiglitz's case, debt is chosen although it *is inferior* to retentions.

Stiglitz's analysis certainly has intellectual appeal and would supplement the analysis of the previous section in an interesting way. There are, however, some problems connected with the fact that all profits are retained. First, as stated in the context of Chapter 4.3.3, a retention of profits is not compatible with the quite drastic empirical trend of a reduction in equity–asset ratios. Second, it follows from the discussion of Chapter 3.1.2 that the assumption $\theta_r > \theta_p$ favored by Stiglitz, which is crucial for the preference for retention, does not seem typical for the tax systems of Western industrial countries. Third, there will be a preference for distributing profits even in the case $\theta_r > \theta_p$ if, because a capital gains tax is levied, the constellation $\theta_r \theta_c \equiv \theta_r^* \leq \theta_p$ prevails. Fourth there is the existence problem mentioned above.

Since tax systems characterized by the conditions assumed by Stiglitz are conceivable, the non-existence of a solution seems to be the most severe of these problems. Stiglitz avoids the problem by assuming a finite time horizon where a distribution of profits must occur. However, this procedure has a certain ad hoc character and is not fully satisfactory. It would seem more plausible if the finite horizon could be derived from the optimization problem of the firm but, under the assumptions made by Stiglitz, this seems hardly possible as the following considerations will show.

A necessary condition for a termination of the firm at a finite point in time T is that the Hamiltonian becomes zero or negative:

$$\mathcal{H}^u(T) \leq 0. \quad (5.36)$$

Since there are neither new issues of shares nor dividends in Stiglitz's model ($R_n = Q = 0$) it follows from (3.35) that $\mathcal{H}^u = \lambda_K I + \lambda_D S_f$. If (4.13) and the above equations (5.5) and (5.19) are used and if account is taken of Stiglitz's assumptions through $\alpha_1 = \alpha_2 = \alpha_3 = \tau_k = 0$ then the expression $\mathcal{H}^u = \theta_d(I - S_f)$ emerges. Using the weak inequality (4.5) that now holds as an equality as well as Definition (4.6) one obtains

$$\mathcal{H}^u = \theta_d\theta_r[f(K,L) - \delta K - wL - rD_f], \quad (5.37)$$

where the term in squared brackets is the firm's gross profit. With a wage rate equal to the marginal product of labor and the neutrality result $f_K - \delta = r$ contended by Stiglitz, it follows because of Euler's theorem that

$$\mathcal{H}^u = \theta_d\theta_r r(K - D_f) > 0. \quad (5.38)$$

Under the assumptions made by Stiglitz this expression is permanently strictly greater than zero provided it has this property at any stage at all; that is, provided the firm ever engages in production. The reason is that, if at some point in time t^* it holds that $K(t^*) - D_f(t^*) > 0$, then it follows from the policy of retaining profits that $I(t) - S_f(t) > 0$ is feasible and will prevail for all $t \geq t^*$. Hence $K(t) - D_f(t) > 0$ for all $t > t^*$. Because of Condition (5.36), a termination of the firm can therefore be excluded.

Instead of an ultimate termination, the assumption of a finite time horizon might also be justified by the idea that the firm will be sold at a certain point in time without an actual termination of its economic activity. Stiglitz's assumption could more easily be defended with such an interpretation. The problem is, however, that in this case it would be necessary to assume a value function for the state variables at the time horizon, which itself would have to be derived from the policy planned for the time after the horizon. With a correct construction of this value function we would again have a problem with an infinite horizon, and the existence problem would show up again.

5.3.5. Investment Neutrality with Immediate Write-off and Non-deductible Debt Interest

All of the neutrality results described up to now assume true economic depreciation and a deductibility of debt interest ($\alpha_1 = \alpha_2 = \alpha_3 = 0$); that is, they require a strict application of the Schanz–Haig–Simons concept of capital income taxation. The deductibility of debt interest can easily be put into practice, but, as mentioned, the problems of implementing true economic depreciation for tax purposes are enormous. Because of the well known difficulties of correctly evaluating installed assets that are no longer traded in the market, no more than a very rough approximation of this depreciation rule can be expected in practice. Thus it is understandable that economists have tried hard to circumvent the depreciation problem and find other ways of achieving investment neutrality of capital income taxation.

An interesting possibility was considered by Brown (1948, pp. 309 n.), Musgrave (1959, pp. 343 n.), and Smith (1963), and Kay and King (1978, pp. 200–203) even took it as a basis of a reform proposal for the British tax system.[30] Its essence is to allow for an immediate write-off of real assets but, in exchange, to make debt interest non-deductible. The tax base is simply the real, non-financial cash flow from an investment project.

Why this possibility is investment neutral can easily be understood. In general, the net present value of an investment project is the integral over its cash flow discounted at some given rate r^*. Suppose now this cash flow is subject to proportional taxation without at the same time interest income being taxed such that the discount rate stays the same. Then the net present value is being reduced relatively at the same rate as the cash flow, and obviously it does not change its sign. After the introduction of the tax it therefore pays to carry out a certain investment project if and only if this project would have been carried out without taxation.

Formula (5.6) reproduces this result. In order to exclude an interest income tax and a tax on the value of the capital stock, we assume that $\tau_k = 0$ and $\theta_p = 1$. The profit tax is captured through the assumption $\theta_d^* = \theta_r^* < 1$, $\theta_c = 1$, and because of the non-deductibility of debt interest it holds that $\alpha_2 = 0$ and $\alpha_3 = 1$. The possibility of an immediate write-off is described by $\alpha_1 = 1$. As contended, (5.6) will then reduce to the laissez-faire optimality condition

$$r = \frac{f_K - \delta}{\frac{1 - \sigma^* - \tau_r}{\theta_r} + \frac{\sigma^*}{\theta_r}} = f_K - \delta. \tag{5.39}$$

[30]Cf. also the more sceptical considerations of the Meade Committee (1978, pp. 230–233, 239–243) to which Kay and King belonged. The Meade Committee uses the name *R-Base tax* instead of the name Brown tax which is used here.

While it is easy to see that a taxation of the real cash flow is investment neutral the relationship between this neutrality result and the results derived above is less obvious. To clarify this relationship consider the following problem. One dollar of capital is invested. For tax purposes deduction of depreciation at the proportional rate $\delta^*(t)$ at point in time t from the remaining book value of the asset is allowed. In the calculation of the taxable profit, interest cost of size $r^*(t)$ times this remaining book value is deductible. It is assumed that $\delta^*(t)+r^*(t)>$ constant >0 for all t, but otherwise the time paths $\{\delta^*\}$ and $\{r^*\}$ can be arbitrary. The problem is to calculate the present value of the tax savings arising from the deductibility of depreciation and interest cost when the tax rate has the constant value τ. It can be solved in only a few steps.

At point in time t, the remaining book value of the investment project is

$$\exp\left(-\int_0^t \delta^*(s)\,\mathrm{d}s\right)$$

and hence the current flow of tax savings from depreciation and interest deductions is

$$\tau[\delta^*(t)+r^*(t)]\exp\left(-\int_0^t \delta^*(s)\,\mathrm{d}s\right).$$

If the present value of this flow is calculated and the resulting expression is integrated over time,

$$\begin{aligned}
&\tau\int_0^\infty [\delta^*(t)+r^*(t)]\exp\left(-\int_0^t [\delta^*(s)+r^*(s)]\,\mathrm{d}s\right)\mathrm{d}t \\
&=\tau\left[-\exp\left(-\int_0^t [\delta^*(s)+r^*(s)]\,\mathrm{d}s\right)\right]_{t=0}^{t=\infty} \\
&=\tau[-\mathrm{e}^{-\infty}-(-\mathrm{e}^{-0})]=\tau \qquad (5.40)
\end{aligned}$$

is obtained.[31] This result shows that the present value of the tax savings from a deductibility of depreciation and interest cost just equals the tax savings that could be achieved with an immediate write-off and no interest deductibility for, with an investment volume of \$1, these tax savings are just \$$\tau$.

[31] Cf. D. Schneider (1969, p. 308 n.) who considered the two extremes of an immediate write-off and a write-off at the termination of the asset life and who already described the general result (5.40) verbally. Another formal proof of (5.40) where arbitrary depreciation, but a constant interest rate, was assumed was first communicated to me by Hans-Heinrich Nachtkamp in 1975.

It should be stressed that the result is completely independent of the time path of tax depreciation and of the particular definition of the discount rate r^*. It is only important that the deductible interest cost is calculated using an interest rate which is the same as the discount rate and that only the interest cost on the remaining book value of the asset is deductible. Under these circumstances, the equivalence between an immediate write-off and a gradual depreciation with interest deductibility will always exist. Consider, however, the special case $r^*(t)=r(t)$, $\delta^*(t)=\delta$, $\tau=\tau_d=\tau_r$ so as to depict the model structure of Section 5.3.1. Then the result says that a profit tax with immediate write-off but without deductibility of any interest cost is equivalent to a tax on pure economic profits. This is Sandmo's (1974) result cited above.[32]

Compared to a tax on pure profits, taxation of the current cash flow has the advantage of greater practical simplicity. This type of taxation saves the financial authorities having to distinguish between "something to which the name 'profit' is given and something which is labelled 'depreciation'" as Smith (1963, p. 90) remarks sarcastically. It almost seems that this form of taxation is something like the "Columbus's egg".

Unfortunately, a closer scrutiny shows considerable problems. As the only tax, the Brown tax will certainly provide the expected results. Its decisive weakness is, however, that it is not compatible with taxation of the interest income of the shareholder household. If interest income of households is taxed while firms are not allowed to deduct debt interest then, according to the analysis of Chapter 4.2.3, no solution of the planning problem of the firm exists since there is an unlimited incentive to substitute new issues of shares for debt financing. It is true that the existence problem could be removed by the introduction of further financial constraints but nothing justifies the expectation that the investment neutrality of taxation could then be maintained.

Thus the possibilities for achieving a partial analytic investment neutrality with the aid of an immediate write-off do not seem very promising at this stage. Fortunately, however, the partial analysis does not say the last word on the impact of taxation on the formation of capital. It will be shown that an immediate write-off, contrary to Brown, Musgrave, and Smith's proposal in connection with a complete deductibility of debt interest, deserves a more prominent role in the construction of a neutral tax system than it may have seemed just now.

[32]Cf. also Stiglitz (1976, p. 304), Boadway (1980, p. 285), and Boadway and Bruce (1979, 1984) who discuss various generalizations.

5.3.6. *The S-Base Tax and the Dividend Tax*

The essence of Brown's proposal was to tax the net cash flow from the real transactions of the firm. Since this means that financial institutions like insurance companies and banks would not be taxed,[33] the so-called Meade Committee (1978, pp. 239–245) proposed to include the net cash flow from financial transactions in the tax base, but not the transactions between the firm and its shareholders.[34] The Committee called this proposal *S-Base tax*. Since the real cash flow of the firm is $f(K, L)$-δK-I-wL and the financial cash flow is $S_{\mathrm{f}} - rD_{\mathrm{f}}$, the S-Base tax equals a tax on dividends net of the inflow of funds from new issues of shares:

$$R - Q = f(K, L) - \delta K - I - wL + S_{\mathrm{f}} - rD_{\mathrm{f}}. \tag{5.41}$$

Our model can be used straightforwardly to assess the influence of this tax on the firm's investment behavior.

It is only necessary to set all α-parameters in (5.6) and the rate of the tax on the value of the capital stock equal to zero; to assume that $\theta_{\mathrm{r}}^* = \theta_{\mathrm{p}} = 1$; to leave out the non-negativity constraint (4.4) for dividends; and to assume a strictly positive tax on dividends $\theta_{\mathrm{d}} = \theta_{\mathrm{d}}^* < 1$. The tax system is of Type 4 in Figure 4.2 in this case where debt and retentions are equivalent, and (5.6) shows with

$$r = \frac{f_K - \delta}{\dfrac{1 - \sigma^*}{\max(\theta_{\mathrm{d}}^*, 1)} + \sigma^*} = f_K - \delta \tag{5.42}$$

that, independently of the size of the maximum marginal debt–asset ratio σ^*, an investment neutrality of taxation can be expected.

It is obvious from the construction of the S-Base tax that it is very similar to a dividend tax whose neutrality properties have been investigated by King (1974b), Auerbach (1979a), and Bradford (1980, 1981) in various contexts. The only difference from such a tax is the deductibility of new issues of shares from the tax base. This deductibility has the advantage of avoiding tax discrimination against new issues of shares compared to debt and retentions. However, this advantage will only become operative if the firm wants to accumulate more equity capital than is possible by retaining all profits and not paying out any dividends. As long as dividends are paid,

[33] Cf. for a discussion of this problem Kay and King (1978, pp. 202 n.) and Head (1979, p. 214).

[34] The Committee also uses the name "(R + F) – Base" to characterize its proposal.

and, as shown in Chapter 4.3.3, a dividend tax as such does not punish this activity, the possibility of deducting new share issues from the tax base is quite meaningless. Clearly, (5.6) stays valid even when the non-negativity constraint (4.4) is taken into account. A need for a net flow of funds from the shareholders is to be expected only for new and rapidly growing firms that do not have sufficient internal funds and face narrow constraints on debt financing; only for these firms therefore can the difference between a pure dividend tax and the S-Base tax be expected to matter.[35] Note, however, that this requires that the S-Base tax has no limited loss-offset and that the government is prepared to pay money to the firm if necessary, a point on which the Meade Committee is not very explicit. If there is a limited loss-offset, then the S-Base tax is identical with a dividend tax.[36]

Brown (1948, p. 310) explained the neutrality of his tax on the real cash flow of the firm with the fact that the government acts as a fair partner: it contributes to the cost of a real investment project on the same terms as it participates in the returns and so does not give the private owners of the firm any incentive to change their investment decision. This is true a fortiori for the S-Base tax. Here the government participates not only in the real investment of the firm, but also in its financial investment. It waives its profit tax claim when the firm retains profits for real or financial investments in the same way that shareholders waive their claim on dividends; and it contributes to new issues of shares under the same conditions as private shareholders do (provided there is the unlimited loss-offset). Thus it is not surprising that the S-Base tax is neutral with regard to the firm's real and financial decisions.

In the light of these remarks, the suspicion may arise that the neutrality of the Brown tax and the S-Base tax implies that the government will not be able to collect any net tax revenue in present-value terms. The neutrality result would then seem quite self-evident, even trivial. In fact, however, the partnership of the government is not quite as fair as suggested. Only with regard to new investments is the government a fair partner of the shareholders. Existing assets are treated in a quite "unfair" manner in that the government taxes their returns without buying a partnership from the existing shareholders at the time when the tax law is introduced. The asymmetry between old and new assets is the reason for simultaneously

[35] Cf. Chapter 4.3.2.

[36] A formal proof of this contention can be found in Howitt and Sinn (1986). It is shown in this paper that the limited-loss offset of the dividend tax begins to play an important role when the tax rate is subject to change.

receiving revenue and avoiding tax distortions. The last chapter of this book discusses this and related problems in more detail. It is shown there that the revenues the cash flow taxes generate will not vanish either absolutely or relatively to other economic aggregates, but can even be expected to grow at the economy's natural rate in the long run.

Unfortunately, the proposal of the Meade Committee shares not only the advantages of the Brown tax but also the disadvantages. This proposal, too, suffers from the shortcoming that taxation of household interest income is not allowed. If $\theta_p < 1$, then $\theta_p < \theta_r^*$ and hence no solution of the optimization problem of the firm exists. Again therefore, as in the case considered by Stiglitz, there is an unlimited incentive for the shareholders to put their money into the firm which would then invest it in the capital market, and paying out the accumulated funds as dividends would never be an attractive proposition.[37]

This is not necessarily a criticism of the Meade Committee. The Committee clearly sees these difficulties for, as a supplement of the S-Base tax, it proposes to remove the income tax at the household level and to replace it with an expenditure tax. With this tax, the problem described above would not arise since interest income in itself is tax exempt. However, given the amount of attention the Committee pays to the problem of taxing the returns of financial institutions, it might also have been useful to discuss in more detail the problems arising from taxation of interest income in connection with the S-Base tax. This would have avoided the danger of the reader overlooking the radicalism of the proposal and underestimating the difficulties of its political implementation.[38]

5.3.7. The Value-added Tax

Although this aspect has not been stressed, all neutrality results derived above hold even when there is a value-added tax or, what amounts to the same thing, an expenditure tax of the Mill–Elster–Mombert type ($\tau_v > 0$): since τ_v does not appear in (5.6) the value-added tax is investment neutral regardless of whether it occurs as the only tax in the economy or in connection with other taxes.

It could be suspected that the reason for the disappearance of the tax rate

[37] Cf. Section 5.3.4 and Chapter 4.2.3.

[38] Cf., however, Meade Committee (1978, pp. 253 n.).

τ_v in the formula for the firm's marginal investment condition is the choice of the numeraire.[39] However, that is not the case. The only variable in (5.6) that could potentially be affected by a change in the numeraire is f_K, the marginal value product of capital. If, unlike before, we assume that the commodity price including the value-added tax equals unity, then the net-of-tax marginal value product of capital is $f_K/(1 + \tau_v)$. However, since the price of the capital good which always has to equal the net-of-tax price of the consumption good also falls to the value $1/(1 + \tau_v)$, the marginal value product of one value unit of capital is f_K in any case.

The economic reason for investment neutrality of the value-added tax will instead become apparent by comparing it with the Brown tax. While the base of the Brown tax is

$$\Pi_{\text{Brown}} = f(K, L) - \delta K - I - wL, \tag{5.43}$$

the base of the value-added tax is

$$C = f(K, L) - \delta K - I, \tag{5.44}$$

and obviously both equations together imply that

$$\Pi_{\text{Brown}} + wL = C. \tag{5.45}$$

This equation shows that a value-added tax equals a Brown tax plus a wage tax. This combination of taxes discriminates against the employment of labor.[40] However, if, for example because of a corresponding reduction in the wage rate, employment nevertheless stays constant, it has no influence on the firm's investment decision. The only way in which the firm's investment decision can be influenced by the value-added tax is via a reduction in the employment of labor and the subsequent change in the marginal product of capital.

5.4. Towards a Realistic Theory of Taxation and Investment

The previous sections concentrated on those special cases under which (5.6) implies investment neutrality, and no particular attempt was made to maximize "realism". This section interprets (5.6) from the viewpoint of existing tax systems and relaxes some of the assumptions. The aim is to get

[39] Cf. Chapter 3.3.
[40] Cf. Equation (3.39).

some first hints on the real distortions the existing systems might cause and to prepare for the chapters to follow.

The starting point for the analysis is the neutrality result of Section 5.3.3 which was derived for a large class of systems of capital income taxation that, to a first order of approximation, seemed to include the systems existing in the OECD countries. Crucial assumptions underlying this result were a high degree of financial flexibility on the part of firms, true economic depreciation for tax purposes, and an absence of a tax on the capital stock. These assumptions will be critically examined. As agreed in Section 5.2, it is assumed that $\sigma^* = 1 - \alpha_1\tau_r - \varepsilon^*$, $\varepsilon^* \geq \alpha_1 W \max(\theta_d^*, \theta_r^*)$, in order to allow for both an endogenous constraint on debt financing resulting from the firm's attempt not to be deprived of its loss-offset possibilities and even narrower exogenous constraints resulting from other causes. In principle, debt interest is assumed to be deductible ($\alpha_2 = \alpha_3 = 0$).

5.4.1. Taxation and Investment in a Miller Equilibrium

The financial flexibility assumed for the neutrality result of Section 5.3.3 included the case where, because of a strict dominance of debt over both retentions and new issues of shares [$\theta_p > \max(\theta_d^*, \theta_r^*)$], firms finance their net investment exclusively with debt. In a growing economy this means that, for $t \to \infty$, the equity–asset ratio approaches zero and the proportion of capital income that appears as interest earnings on bonds approaches unity, a strange implication.

A realistic theory of taxation and investment needs ingredients that help avoid this implication. One possibility is the interaction between growth, accelerated depreciation, and a limited loss-offset. But there are certainly others that are worth studying. The Miller equilibrium is a good example.

As explained above in Chapter 4.4, the increase in the proportion of interest income will raise the personal tax rate until a situation with $\theta_p = \theta_r^*$ is reached and the reason for the rise in the personal tax rate has vanished. This situation is a Miller equilibrium. As the shareholders diversify their wealth, the single firm has no influence on the marginal personal tax rate and is indifferent between debt and retained profits as sources of investment finance. However, in the aggregate, there is a well-determined interior solution for the financial decisions made by the industry. Too much debt financing creates a preference for retentions ($\theta_p < \theta_r^*$) and too much internal financing through retentions creates a preference for debt financing ($\theta_p > \theta_r^*$); this stabilizes the personal tax factor at the level of the combined tax factor of retentions ($\theta_p = \theta_r^*$) and implies that the Miller equilibrium will persist throughout.

With true economic depreciation ($\alpha_1 = 0$) and an equivalence between debt and retentions ($\theta_p = \theta_r^*$), the basic equation (5.6) reduces to the laissez-faire equation

$$r = f_K - \delta \qquad \text{(Miller equilibrium)}, \tag{5.46}$$

indicating a perfect investment neutrality.

Equation (5.46) proves a remarkable robustness of the neutrality result derived in Section 5.3.3. Again the degree of integration between corporate and personal taxation is irrelevant for the firm's investment decision and again the cost of finance is just the cost of debt financing as in a world without taxes. However, unlike before, the result now rests on the assumption of an interior debt–equity choice in the aggregate.

Note that the size of the maximum marginal debt–asset ratio σ^* that determines the upper boundary of the solution space (see Figure 4.1) has no direct influence on this result. Since this boundary cannot be a binding constraint for the single firm when $\theta_p = \theta_r^*$, its position cannot determine the investment decision. It would be perfectly compatible with a Miller equilibrium and the resulting marginal condition (5.46) if some firms were limited to using only equity finance. On the other hand, of course, the existence of narrow constraints on debt financing for many firms reduces the scope for variations in the personal tax base and hence the chances that a Miller equilibrium will exist. If there are reasons for a limitation of debt finance that determine an *aggregate* value of σ^* below that level of the *aggregate* value of the marginal debt–asset ratio (S_f/I) which is implied by a Miller equilibrium, then $\theta_p > \theta_r^*$ is possible, and (5.6) will not reduce to (5.46). The next two sections study this possibility in detail.

5.4.2. Less Flexibility in Financial Decisions: A Criticism of a Popular Formula

Consider now the case where there is no Miller equilibrium and where, because of some exogenous reason, the firm wishes to finance no more than a proportion $\sigma^* < 1$ of its net investment with debt or, equivalently, no less than a proportion $\varepsilon^* = 1 - \sigma^* > 0$ with equity even though true economic depreciation ($\alpha_1 = 0$) is required.

In this case, the general condition (5.6) becomes

$$r = \frac{f_K - \delta}{\dfrac{\theta_p(1 - \sigma^*)}{\max(\theta_d^*, \theta_r^*)} + \sigma^*}. \tag{5.47}$$

This expression reveals that in the case where the maximum marginal debt–asset ratio is less than unity ($\sigma^* < 1$), investment neutrality is no longer assured. Instead, the tax system will drive a wedge between the net-of-depreciation marginal product of capital, $f_K - \delta$, and the market rate of interest, r, when the marginal tax burden on interest income of the shareholder household falls short of the marginal tax burdens on both dividends and retentions[41] $[\theta_p > \max(\theta_d^*, \theta_r^*)]$. The lower σ^* the larger this wedge will be. Tax discrimination against equity capital carries over to discrimination against the firm's real investment.

It is useful to confront (5.47) with another result that has been achieved in the literature on taxation and investment. A popular formula that originates from the works of Harberger, Jorgenson, and others and that can be found in numerous textbooks and articles is

$$r = (1 - \tau)(f_K - \delta) \qquad \text{(traditional result)}, \tag{5.48}$$

where τ denotes "the" corporate tax rate. This formula is usually derived from considering an investor who can choose between investing his funds in the capital market or in his firm. If this investor chooses the capital market investment, the rate of return net of his personal income tax is $\theta_p r$, if he invests in his firm and distributes the profits the net-of-tax rate of return under a classical system of capital income taxation is $\theta_p(1 - \tau)(f_K - \delta)$. Thus he allocates his funds to the two alternative uses so that at the margin the two rates are equal: $\theta_p r = \theta_p(1 - \tau)(f_K - \delta)$. After dividing by θ_p this yields Equation (5.48).

Equation (5.48) confirms the result that the tax system drives a wedge between the marginal product of capital and the market rate of interest. Thus far there is some similarity with (5.47). A closer scrutiny, however, reveals substantial differences. Suppose, in order to attempt an approximation of (5.48), the firm chooses 100% equity finance at the margin, an unrealistic case that contradicts the dramatic empirical tendency towards debt financing reported in Chapter 4.3.3. Assume moreover that the classical system or a closely related system prevails where the corporate tax rate on dividends has a value similar to that on retentions ($\theta_d \approx \theta_r$) and capital gains are taxed at a lower rate than personal interest income ($\theta_c > \theta_p$) such

[41] Cf. Equations (3.14) and (3.15).

that $\theta_d^* \equiv \theta_d\theta_p \leq \theta_r^* \equiv \theta_r\theta_c$. Then Equation (5.47) becomes[42]

$$\theta_p r = \theta_c\theta_r(f_K - \delta) \qquad (\sigma^* = 0,\ \theta_r^* \geq \theta_d^*). \tag{5.49}$$

As $\theta_p < \theta_c$, this equation obviously reveals a smaller distortion in the firm's investment planning than (5.48). In countries where the maximum marginal personal tax rate exceeds the corporate tax rate, it may, even in the absence of a Miller equilibrium, be realistic for many firms that $\theta_p \approx \theta_c\theta_r$ so that (5.49) approximates the case of investment neutrality. Here, the traditional equation, (5.48), would be completely misleading, dramatically overestimating the tax distortions.

The reason for this strange divergence is that the traditional argument not only assumes away debt financing but also financing through retentions: implicit in (5.48) is the assumption that the firm exclusively uses new issues of shares for funding its investment. As is known from the previous chapter, in the classical system this is the least attractive source of finance. It is thus not surprising that the choice of this source creates the largest conceivable tax distortion.

Instead of new issues, retained profits are the marginal source of finance underlying (5.49). This is an implication of the firm's optimization, not an assumption. If the firm must choose equity as the marginal source of finance, then it chooses the cheaper of two alternative sources of equity finance, and under the classical system this is retentions.[43]

The failure to distinguish carefully between retentions and new share issues as alternative ways of equity formation has led many economists to believe that the double taxation of dividends that characterizes the classical system creates a serious disincentive for private investment. While Sections 5.3.3 and 5.4.1 showed that this belief might be wrong if a sufficient number of firms have access to debt financing, Equation (5.49) reveals that it would even be fallacious if each single firm were forced to use equity finance. *At the margin*, a tax reform that reduces the corporate tax burden on dividends will not succeed in stimulating private investment.

The reason for the irrelevance of dividend taxation is not that, when retentions are the preferred source of equity finance, the firm does not distribute dividends and hence pays no dividend taxes. Contrary to this

[42]A similar formula was first derived by King (1977, Table 8.1, p. 244) for the case of a predetermined choice of retained profits as the marginal source of finance.

[43]See Appendix B for the proof that, in the neighborhood of the steady-state growth path of the economy, profits will stay sufficiently large to completely finance the investment project and to allow the firm to pay dividends, Cf. also the discussion in Chapters 4.3.2 and 6.2.5.

supposition, the general assumption $\theta_p \geq \theta_p \theta_r$ implies that the firm does not mind distributing all profits in excess of the amount needed for investment finance, and when $\theta_p > \theta_p \theta_r$ it will definitely prefer doing this! Even when the firm is persistently subjected to the high burden of dividend taxes these taxes will not affect its marginal investment decision.

Seen from the perspective of the political debate on corporate tax reform which focusses primarily on the problem of double taxation, this is worth noting. It proves that a basic aspect of the investment neutrality result derived above stays valid even under extremely unfavorable conditions.

Note, however, that the neutrality of dividend taxation prevails only at the margin. When a tax reform goes far enough to reduce the overall marginal tax burden on dividends to or below that on retentions, $\theta_d^* \geq \theta_r^*$, then new issues will dominate retentions and *further* marginal reductions of the dividend taxes will stimulate investment to the extent σ^* is below unity; i.e., to the extent the firm forgoes the use of debt as the marginal source of finance.

In the extreme case of full equity finance and a dominance of new issues of shares over retentions, (5.47) becomes

$$r = \theta_d(f_K - \delta) \quad (\theta_d^* \geq \theta_r^*, \sigma^* = 0). \tag{5.50}$$

This equation sheds more favorable light on the traditional equation, (5.48), and indeed it can be justified in a similar way. It must be stressed, however, that (5.50) only fits partial or full imputation systems where the marginal corporate tax burden on dividends is sufficiently far below that on retained profits. For the classical system of capital income taxation that is typically considered in the pertinent literature, there is no way of reproducing the traditional formula (5.48) from (5.47).

5.4.3. The Investment Decision with Accelerated Depreciation

5.4.3.1. Introduction

As a further step in reducing the degree of abstraction in the analysis of tax effects, this section considers the possibility of accelerated tax depreciation ($\alpha_1 > 0$). Accelerated depreciation affects the firm's investment decision since it increases the present value of depreciation allowances and hence reduces the profitability requirement an investment project has to satisfy. But, as shown in Section 5.2, it also has strong implications for the firm's financial decisions in that it determines the maximum marginal debt–asset ratio σ^* endogenously. The repercussions of the firm's financial decision on its investment decision will be a particular theme of the following sections.

In Chapter 3.1.3 it was noted that the tax systems of some countries deviate so far from true economic depreciation that this method cannot be considered even as a very rough approximation of reality. The Anglo-Saxon countries, in particular, offered very generous depreciation allowances and they continue to do so despite some countervailing tendencies in the recent British and American tax reforms.

An extensive theoretical literature has dealt with the problem of depreciation and has illuminated substantial aspects of the way alternative depreciation rules affect the firm's investment decision.[44] However, typically only a single tax is considered, and even then it is sometimes not clear whether what is meant is a corporate tax as the only tax in the economy or a general tax on all kinds of capital income. Little effort has been made to study the role of accelerated depreciation in the context of alternative systems of capital income taxation that are characterized by different degrees of integration between corporate taxes, personal income taxes, and capital gains taxes.[45] Moreover, it seems that the interaction between the firm's investment decision and its financial decision in the context of accelerated depreciation and a limited loss-offset has not been explored at all.

A number of authors, including King (1975), Alworth (1979), and Boadway (1979a), derive cost-of-capital formulas for the case of accelerated depreciation assuming that the firm uses only debt and no equity capital for financing its investment projects.[46] While this assumption is admissible for a stationary firm, it cannot be justified for a growing one. As shown in Section 5.2, a growing firm needs equity finance at the margin for otherwise it must waive part of the deductibility of its debt interest. For a growing firm, the cost-of-capital formulas derived imply that, at some stage, the corporate tax will be perverted into a permanent and growing subsidy for the firm. This problem will be avoided in the analysis that follows.

[44] Cf. e.g. H. Schneider (1964, pp. 64 n.), Hall and Jorgenson (1967, 1971), D. Schneider (1969), Sandmo (1974), Schneider and Nachtkamp (1977), Boadway (1979a, 1980), Boadway and Bruce (1979).

[45] See, however, King (1977, Chapter 8) who offers various cost-of-capital formulas for the case of immediate write-off under alternative assumptions about the firm's choice of finance.

[46] King and Alworth assume that the part of investment that cannot be financed with deferred taxes is debt financed $[S_f = (1-\alpha_1\tau_r)I]$; Boadway assumes even a 100% debt finance at the margin $[S_f = I]$.

The discussion starts with simple cases and proceeds to more complicated ones. Throughout it is based on Equation (5.6). When accelerated depreciation is allowed while debt interest is deductible ($\alpha_2 = \alpha_3 = 0$) and there is no tax on the capital stock ($\tau_k = 0$), this equation becomes

$$r = (f_K - \delta)/P_K, \tag{5.51}$$

where

$$P_K \equiv \frac{\theta_p(1 - \sigma^* - \alpha_1 \tau_r)}{\max(\theta_d^*, \theta_r^*)} + \sigma^* \tag{5.52}$$

is the effective price of capital introduced in (5.12).

5.4.3.2. The Taxation Paradox: Basic Formulation

Consider a non-corporate firm or a corporation that operates under a fully integrated system of capital income taxation such that there is a uniform tax rate on all kinds of capital income ($\theta_d^* = \theta_r^* = \theta_p, \theta_c = 1$) and σ^* cannot be binding. For this firm, $P_K = 1 - \alpha_1 \tau_r$ and (5.51) reduces to

$$r = \frac{f_K - \delta}{1 - \alpha_1 \tau_r} \qquad (\text{for } \theta_r^* = \theta_p). \tag{5.53}$$

As one would expect, this expression shows that accelerated depreciation stimulates private investment: the higher α_1, the lower $f_K - \delta$ and hence the higher the stock of capital that satisfies (5.53).

Note, however, that the equation also reveals that the marginal product of capital ($f_K - \delta$) is below the market rate of interest; the higher the tax rate the more below it is. This shows that the taxation of retained profits acts like a subsidy on marginal investment projects. If accompanied by an increase of the tax rate on withdrawn profits (τ_d^*) and of the tax rate on interest income earned in the capital market (τ_p), an increase in the tax rate on retained profits (τ_r) induces the firm to employ a higher stock of capital! This interesting phenomenon was first described by Schneider (1969; 1974, pp. 311–319), Schneider and Nachtkamp (1970), and Strobel (1970). Schneider called it "*taxation paradox*", a name that is clearly to the point.[47]

The taxation paradox follows straightforwardly from the Johansson–

[47] Compare also Hall and Jorgenson (1971, pp. 53 n.). As an example for a more recent study of the problem see Steiner (1980).

Samuelson theorem that was described in Section 5.3.2. According to this theorem true economic depreciation is the borderline case where a tax rate change is investment neutral; i.e., where $f_K - \delta = r$. Decelerated tax depreciation implies that $f_K - \delta$ exceeds r and accelerated tax depreciation implies that $f_K - \delta$ falls short of r.

Equation (5.53) not only holds for a non-corporate firm or for a corporate firm in a fully integrated system of capital income taxation, but also for a corporate firm that operates in a Miller equilibrium which, regardless of the system of capital income taxation in operation, is characterized by $\theta_p = \theta_r^* \geq \theta_d^*$. Thus, when the corporate tax rate on retained profits rises and induces changes in the capital gains tax rate and the personal income tax rate such that θ_p and θ_r^* fall pari passu, investment will again be stimulated. Here too, the taxation paradox is operative.

There are two differences though from the situation of the non-corporate firm. First, since all systems are characterized by $\theta_d^* \leq \theta_p$, it is irrelevant how θ_d^* is affected in detail by the change in τ_r. In the Miller equilibrium, the firm does not rely on new issues of shares as a source of finance and hence the tax treatment of this source as captured by θ_d^* (see Chapter 4) does not matter. Second, unlike before, the joint movement of θ_r^* and θ_p is not implied directly by the tax law but is brought about in a quite indirect way through a transition from one Miller equilibrium to another. It is very likely that this would be a time consuming process.

Schneider (p. 302) conjectured that an increase in the tax burden on interest income – that is, a reduction in the discount rate – together with an increase in the "profit tax rate" is a necessary condition for the taxation paradox. While this conjecture is right under the specific assumptions he made, Equation (5.53) reveals that it is not generally correct. Whether θ_r and θ_p change uniformly while $\theta_r \theta_c = \theta_p$ and $\theta_c = 1$ or whether θ_r and θ_c change in opposite directions while $\theta_r \theta_c = \theta_p =$ constant is quite irrelevant for the taxation paradox. What ultimately matters is the change of τ_r and not that of τ_p. If, for example, the government introduces a tax exemption for capital gains but compensates for this exemption with an increase in the tax rate on retained profits, then it can be expected that private investment will rise. When θ_p stays constant, this would happen even without a time consuming transition to another Miller equilibrium.

5.4.3.3. Isolated Variations in the Corporate Tax Rate: Two Extreme Financial Assumptions

The cases considered above had in common that the overall marginal tax burden on retained profits was the same as the tax rate on interest income.

Any change in the tax rate on retained profits (τ_r) was accompanied by a change in the personal tax rate on interest income or the capital gains tax rate (τ_c), and it was irrelevant whether or not the tax rate on distributed profits changed. This section returns to the general case $\theta_p \geq \max(\theta_d^*, \theta_r^*)$ and considers isolated variations in the corporate tax rate, given the personal tax rate and given the capital gains tax rate. In the light of Schneider's conjecture, this case seems particularly interesting.

Unlike with the discussion of the firm's marginal investment condition under true economic depreciation it will turn out that the corporate tax rate on retained profits (τ_r) and the corporate tax rate on distributed profits (τ_d) can simultaneously affect the firm's marginal investment condition when accelerated depreciation is allowed. Account must therefore be taken of the fact that, given the basic characteristics of the system of capital income taxation under consideration, τ_d usually changes together with τ_r. Typically it holds that

$$\partial\tau_d/\partial\tau_r = \tau_d/\tau_r = \text{constant}, \qquad (5.54)$$

and this is what will be assumed.[48] In the classical system $\partial\tau_d/\partial\tau_r = 1$, in the full imputation system $\partial\tau_d/\partial\tau_r = 0$, and in the partial imputation systems $0 < \partial\tau_d/\partial\tau_r < 1$. The assumption makes it possible to speak of variations in *the* corporate tax rate without specifying whether what is meant is τ_r or τ_d.

Before the firm's investment decision under these new assumptions is studied it is useful to distinguish two kinds of taxation paradox.

Taxation Paradox of Type A

The marginal product of capital falls short of the market rate of interest or, equivalently, the effective price of capital is below unity ($f_K - \delta < r, P_K < 1$).

Taxation Paradox of Type B

A rise in the tax rate on retained profits brings about a fall in the marginal product of capital or, equivalently, a fall in the effective price of capital [$\partial(f_K - \delta)/\partial\tau_r < 0$, $\partial P_K/\partial\tau_r < 0$].

Both of these paradoxes existed simultaneously in the cases considered above where various tax rates were changed at the same time. However, when an isolated increase in the corporate tax rate is considered, this need

[48] This assumption is not meant to exclude isolated variations in τ_d due to a change in the "degree of double taxation of dividends" or to a change in the "degree of integration between corporate and personal taxation".

not be the case any more. Instead, the choice of financial instruments is crucial for the question of whether or not a taxation paradox occurs and of which type it is. This and the next section will demonstrate this. The discussion begins in this section with the extremes of very narrow and very wide constraints on debt finance. The next section considers an intermediate case.

Consider the possibility of wide constraints first. If the firm merely takes into account the legal constraints on debt financing (see Chapter 4), then $\sigma^* = 1 - \alpha_1 \tau_r$ and $\varepsilon^* = 0$. Hence (5.51) reduces to

$$r = \frac{f_K - \delta}{1 - \alpha_1 \tau_r} \qquad (\text{for } \alpha^* = 1 - \alpha_1 \tau_r), \tag{5.55}$$

which is the same formula as (5.53). Trivially, (5.55) again implies both types of paradoxes: the marginal product of capital is below the market rate of interest, the higher τ_r the more below it is. No change in the discount rate, as Schneider claimed, and no change in the capital gains tax rate has to accompany the rise in τ_r in order to induce the firm to employ more capital.

The explanation for this result is related to the irrelevance of the degree of integration between the corporate tax and the personal income tax that in Section 5.3.3 was derived for the case of true economic depreciation. Once again the optimization of the firm's financial decisions implies that the current cost of capital for one value unit of financial funds is the market rate of interest r, independently of taxation. The difference from true economic depreciation is, however, that now it is not the total net investment that has to be financed but only that part of it $[(1 - \alpha_1 \tau_r)I]$ that cannot be financed through deferred taxes, the tax savings from accelerated depreciation.[49] One value unit of capital that, after deducting depreciation, brings about a current increase in revenue of size $f_K - \delta$, only has to bear a capital cost of the size $(1 - \alpha_1 \tau_r)r$. This explains the algebraic form of Equation (5.53) for the case where maximum debt finance is admissible.

Consider now the other extreme where $\sigma^* = 0$ and $\varepsilon^* = 1 - \alpha_1 \tau_r$; i.e., the case where the part of net investment that cannot be financed by deferred taxes is exclusively equity financed. In this case $P_K = \theta_p(1 - \alpha_1 \tau_r)/\max(\theta_d^*, \theta_r^*)$, and hence (5.51) becomes

$$r = \frac{\theta_r \theta_c}{(1 - \alpha_1 \tau_r)\theta_p}(f_K - \delta) \qquad (\text{for } \sigma^* = 0,\ \theta_r^* \geq \theta_d^*) \tag{5.56}$$

or

$$r = \frac{\theta_d}{1 - \alpha_1 \tau_r}(f_K - \delta) \qquad (\text{for } \sigma^* = 0,\ \theta_d^* \geq \theta_r^*), \tag{5.57}$$

[49] Cf. Chapter 3.2.2. as well as Constraint (4.3).

depending on whether retentions ($\theta_r^* \geq \theta_d^*$) or new issues of shares ($\theta_d^* \geq \theta_r^*$) are the second-best source of finance.

It was shown above that, in the case of true economic depreciation, capital income taxation drives a wedge between the marginal product of capital and the market rate of interest when debt strictly dominates the other sources of finance [$\theta_p > \max(\theta_d^*, \theta_r^*)$], but equity is nevertheless used at the margin. Thus it should be expected that the taxation paradox of Type A will not show up when accelerated depreciation is allowed but α_1 is sufficiently small. Both Equations (5.56) and (5.57) confirm this expectation. However, as $\theta_c > \theta_p$ and $\theta_d > \theta_r$ when $\theta_d^* \geq \theta_r^*$, it is also obvious that the paradox of Type A shows up again if α_1 is sufficiently close to unity.[50] Thus, with very generous depreciation schemes, it will still be the case that the marginal product of capital is below the market rate of interest.

Unlike before, however, this does not necessarily indicate that there is also a paradox of Type B. Equation (5.56) that characterizes the classical or closely related systems of capital income taxation shows that $\mathrm{d}(f_K - \delta)/\mathrm{d}\tau_r \left\{\begin{smallmatrix} > \\ = \end{smallmatrix}\right\} 0$ for $\alpha_1 \left\{\begin{smallmatrix} < \\ = \end{smallmatrix}\right\} 1$. Thus a rise in the corporate tax rate will reduce private investment for all values of α_1 less than unity, including those which generate the paradox of Type A. Even in the limiting case of an immediate write-off ($\alpha_1 = 1$) a tax-rate increase will not be able to stimulate private investment. Here, investment will only just remain unchanged.

On the other hand, when (5.54) is used, (5.57) implies that $\mathrm{d}(f_K - \delta)/\mathrm{d}\tau_r \left\{\begin{smallmatrix} > \\ = \\ < \end{smallmatrix}\right\} 0 \Leftrightarrow f_K - \delta \left\{\begin{smallmatrix} > \\ = \\ < \end{smallmatrix}\right\} r \Leftrightarrow \tau_d/\tau_r \left\{\begin{smallmatrix} > \\ = \\ < \end{smallmatrix}\right\} \alpha_1$. This ensures that with the full imputation and closely related partial imputation systems both types of paradox coincide. They both occur if, and only if, the degree of integration between corporate and personal taxation as measured by τ_d/τ_r is less than the value of the depreciation parameter α_1.

These results can best be understood if one distinguishes a *subsidy effect* and a *discrimination effect*. The subsidy effect represents the investment incentive that is implicit in the use of accelerated depreciation schemes. Its stength depends on the degree of acceleration as measured by α_1 and the size of the corporate tax rate on retained profits, τ_r. The discrimination effect indicates the tax discrimination of the chosen source of finance relative to debt financing which, according to the rules derived in the previous chapter, can be assessed through comparing the tax factors θ_d^*, θ_r^*, and θ_p. The subsidy effect was fully present in all cases considered, but the discrimination effect appeared with different strengths.

In the case where retentions and debt were equivalent sources of finance

[50]Recall that $\theta_c > \theta_p$ is a general assumption for the case of accelerated depreciation. Cf. Chapter 3.1.4. It is also a realistic assumption for all existing systems of capital income taxation as accrued capital gains are nowhere fully included in the personal income tax base.

and in the case where debt was the chosen source of finance, the discrimination effect was completely absent. In these cases therefore, the taxation paradox showed up most clearly.

In the case where new issues of shares were the chosen source of finance (because of $\theta_d^* > \theta_r^*$) the discrimination effect was determined through the overall corporate tax burden on dividends relative to that on interest income or, as dividends and interest income are subject to the same personal tax burden, through the corporate tax burden on dividends alone. The lower this tax burden was – that is the closer the tax system was to a full imputation system – the larger were the chances for the two types of taxation paradox showing up.

In the remaining case where retained profits were the chosen source of finance, because the classical or closely related systems with $\theta_d^* < \theta_r^*$ prevailed, the discrimination effect was given by the joint burden of the capital gains tax and the corporate tax on retained profits relative to the tax burden on personal interest income. Suppose the capital gains tax rate had been the same as the personal tax rate. Then the discrimination effect would have been measured solely by the corporate tax rate on retentions and, except for the case of an immediate write-off, this effect would always have been stronger than the subsidy effect. Both types of taxation paradox would have been evident. In fact, however, the capital gains tax rate was lower than the personal income tax rate. With regard to marginal variations in the corporate tax rate this fact was irrelevant: the *change* in the discrimination effect still overcompensated the *change* in the subsidy effect, and so there was no taxation paradox of Type B. However, the size of the discrimination effect as such was smaller, and with a sufficiently fast depreciation the subsidy effect dominated, producing a paradox of Type A.

5.4.3.4. Taxation Paradox despite Limited Loss-offset

With a sufficiently narrow constraint on debt financing neither of the two types of taxation paradox is assured. With debt financing up to the legal constraint ($\sigma^* = 1 - \alpha_1 \tau_r$) they are both assured, but problems with a limited loss-offset must be reckoned with. For a growing firm it follows from (5.17) that, with a binding constraint on debt financing, Equation (5.55) could only hold if there were an unlimited loss-offset and the corporate tax base were allowed to go towards minus infinity. An interesting question is whether there is a taxation paradox for a growing firm even when the corporate tax has a loss-offset constraint and the firm builds up sufficient amounts of equity capital to avoid conflicting with it.

Suppose the minimum marginal equity–asset ratio the firm wants to satisfy belongs to the set of behavioral hypotheses defined by

$$\varepsilon^* = (1+\lambda)\alpha_1 W \max(\theta_d^*, \theta_r^*). \tag{5.58}$$

Here λ is a constant large enough to ensure that $\varepsilon^* \geq 0$, and $\alpha_1 W \max(\theta_d^*, \theta_r^*)$ is the value of ε^* calculated with (5.20). By construction, the hypothesis that was derived in Section 5.2 from the interaction between accelerated depreciation and a limited loss-offset is contained in (5.58) for the case $\lambda = 0$. This value of λ ensures that the limited loss-offset constraint (5.8) is only just satisfied. Any higher value of λ implies a fortiori that the constraint is satisfied, and any lower value implies that it is violated.

Let ε_A^* and ε_B^* denote those values of ε^* which just fail to produce the taxation paradoxes of Type A and Type B, respectively; that is the values which imply that $P_K = 1$ and $\partial P_K/\partial \tau_r = 0$. Inserting (5.58) into (5.52) while noting that $\varepsilon^* \equiv 1 - \sigma^* - \alpha_1 \tau_r$ one calculates

$$\varepsilon_A^* = \begin{cases} \alpha_1 \tau_r / [(\theta_p/\theta_r^*) - 1] & \text{for } \theta_r^* \geq \theta_d^*, \\ \alpha_1 \theta_d \tau_r / \tau_d & \text{for } \theta_d^* \geq \theta_r^*, \end{cases} \tag{5.59}$$

and[51]

$$\varepsilon_B^* = \begin{cases} \alpha_1 \theta_r & \text{for } \theta_r^* > \theta_d^*, \\ \alpha_1 \theta_d \tau_r / \tau_d & \text{for } \theta_d^* > \theta_r^*, \end{cases} \tag{5.60}$$

where the upper case in (5.60) also holds for the left-hand, and the lower case also for the right-hand, derivative of P_K for τ_r. Any value of ε^* that falls short of these critical values produces the respective taxation paradox, any value that exceeds them produces tax effects on the firm's investment decision similar to those with true economic depreciation.

To interpret (5.59) and (5.60) note first that

$$\varepsilon_A^* \begin{Bmatrix} > \\ = \end{Bmatrix} \varepsilon_B^* \quad \text{for} \quad \theta_r^* \begin{Bmatrix} > \\ < \end{Bmatrix} \theta_d^*, \tag{5.61}$$

where the inequality sign in the upper case follows from $\theta_r^* \equiv \theta_r \theta_c$ and $\theta_c > \theta_p$. This result confirms the findings of the last section. When the full imputation system or a closely related system with $\theta_r^* < \theta_d^*$ prevails, the conditions for both types of taxation paradox coincide, but not when a classical or a closely related system prevails where $\theta_r^* > \theta_d^*$. Here, the

[51] Note that the derivative of $\sigma^* = 1 - \alpha_1 \tau_r - (1+\lambda)\alpha_1 W \max(\theta_d^*, \theta_r^*)$ for τ_r happens to be zero when $\varepsilon^* = \alpha_1 \theta_d \tau_r / \tau_d$ and $\theta_d^* > \theta_r^*$. For this reason, the same value for ε_B^* as in (5.60) would result if the condition $\partial P_K/\partial \tau_r = 0$ were not calculated under the hypothesis (5.58) but under the hypothesis $\varepsilon^* = 1 - \alpha_1 \tau_r - \sigma^*$, $\sigma^* = $ constant.

condition for a paradox of Type A is weaker than that for a paradox of Type B, and again the fact that capital gains are taxed at a lower rate than personal interest income is the reason.

Note that, because of $\varepsilon^* = 1 - \alpha_1 \tau_r - \sigma^*$, the result $\varepsilon_B^* = \alpha_1 \theta_r$ for $\theta_r^* > \theta_d^*$ defines a critical value of the maximum marginal debt–asset ratio given by $\sigma_B^* = 1 - \alpha_1$. Provided old and new assets are subject to the same depreciation system, $(1 - \alpha_1)I$ is the net increase of tax-written-down assets. The critical value therefore implies that, under the classical and closely related systems, the Type-B paradox will occur if the net increase in the firm's stock of debt exceeds the net increase in its stock of tax-written-down assets. In the case of an immediate write-off ($\alpha_1 = 1$), a possibility that for a number of years applied to a considerable part of British investment,[52] this means that, given the rate of interest, a rise in the corporate tax rate reduces private investment demand provided that debt is used *at all* to finance marginal investment projects. This is certainly a weak condition.

Let us now check whether a paradox will occur under the hypothesis on the firm's financial choice modelled in Section 5.2. Compare the value $\varepsilon^* = \alpha_1 W \max(\theta_d^*, \theta_r^*)$ from (5.20) with ε_B^* as given by (5.60). It follows that

$$\varepsilon^* < \varepsilon_B^* \Leftrightarrow \begin{cases} W\theta_c < 1 & \text{for } \theta_r^* > \theta_d^*, \\ W\theta_c \dfrac{\tau_d}{\tau_r} \dfrac{\theta_p}{\theta_c} < 1 & \text{for } \theta_d^* > \theta_r^*. \end{cases} \tag{5.62}$$

As $\tau_d < \tau_r$ when $\theta_d^* > \theta_r^*$ and as $\theta_p < \theta_c$, this condition is satisfied for all systems of capital income taxation provided that $W\theta_c < 1$. That this inequality holds was shown with (5.21) to result from the transversality condition (3.36) of the firm's optimization problem.[53] Thus $\varepsilon^* < \varepsilon_B^*$, and because of (5.61), $\varepsilon^* < \varepsilon_A^*$. It follows that both types of taxation paradox prevail when the equity–asset ratio is endogenously explained through the interaction between a limited loss-offset and accelerated depreciation:

$$P_K < 1, \quad \frac{\mathrm{d}P_K}{\mathrm{d}\tau_r} < 0 \quad \text{if} \quad \alpha_1 > 0, \quad \varepsilon^* = \alpha_1 W \max(\theta_d^*, \theta_r^*), \quad \text{and}$$

$$\frac{\partial \tau_d}{\partial \tau_r} = \frac{\tau_d}{\tau_r} = \text{constant.} \tag{5.63}$$

The fact that $\varepsilon^* < \varepsilon_B^* \le \varepsilon_A^*$ when $\varepsilon^* = \alpha_1 W \max(\theta_d^*, \theta_r^*)$ implies that there exists some non-degenerate range of strictly positive values of the parameter

[52] Cf. Chapter 3.1.3.

[53] In fact, in an intertemporal general equilibrium, it even holds that $W < 1$. Cf. Footnote 11.

λ where (5.58) would still produce the two types of paradox. For a growing firm [i.e., for a firm with $\lim_{t\to\infty}\hat{K}(t) = \text{constant} > 0$], such values would imply that the corporate tax base relative to the imputed capital income generated by the firm's capital stock $[Z^* \equiv Z/(rK)]$ approaches a strictly positive constant as time goes to infinity. The value of this constant is $\lambda\alpha_1 W\theta_p$.[54] This can easily be seen if Condition (5.8) is replaced with $\lim_{t\to\infty}Z^*(t) \geq \lambda\alpha_1 W\theta_p$. Instead of (5.18), this assumption yields

$$P_K - \lim_{t\to\infty}\sigma(t) - \alpha_1 \lim_{t\to\infty}[\hat{K}(t)/r(t)] - \lambda\alpha_1 W\theta_p = 0$$

and, if use is made of Definition (5.21), (5.58) results.

Summarizing, we can conclude that, provided a solution to the firm's decision problem exists, both types of taxation paradox can occur under all systems of capital income taxation even when the firm builds up sufficient equity capital to avoid exhausting its loss-offset possibilities. In fact, there is some scope for accumulating so much equity capital that, despite accelerated depreciation, the corporate tax base will permanently stay strictly positive and grow in strict proportion to the total imputed capital income the firm is generating (rK).

This shows that the taxation paradox is not the remote theoretical possibility which it might appear at first glance. Instead, this paradox emerges as a quite plausible case when the firm optimizes its financial decisions in line with the Modigliani–Miller framework. Under this assumption, it cannot be ruled out on any trivial grounds that, with accelerated depreciation, capital income taxation as such presses the marginal product of capital below the market rate of interest and that a rise of the corporate tax rate, even one that is not accompanied by a change in other tax rates, will stimulate private investment.

5.4.4. *Taxing the Capital Stock*

As a further step towards a more realistic theory of taxation and investment consider finally a periodic taxation of the value of the stock of capital employed by the firm ($\tau_k > 0$) that occurs in some countries. Here, the result is obvious. A tax on the stock of capital is a burden on all units of this stock

[54]Likewise, corporate income, as defined in national income accounting, relative to the imputed capital income, $\tilde{Z}^* \equiv [f(K,L) - wL - \delta K - rD_f]/(rK)$, approaches the value $\tilde{Z}^* = (1+\lambda)\alpha_1 W\theta_p$ or, because of (5.58), $\tilde{Z}^* = \varepsilon^*\theta_p/\theta_r^*$ when $\theta_r^* > \theta_d^*$ and $\tilde{Z}^* = \varepsilon^*/\theta_d$ when $\theta_d^* > \theta_r^*$. Because of (5.60) it follows that both types of taxation paradox prevail if $\tilde{Z}^* < \alpha_1\theta_p/\theta_c$ (for $\theta_r^* > \theta_d^*$) or $\tilde{Z}^* < \alpha_1\tau_r/\tau_d$ (for $\theta_d^* > 0_r^*$), respectively.

including the marginal ones. Thus the employment of capital is discriminated against by this tax. Set $\alpha_2 = \alpha_3 = 0$ to assume deductible debt interest. Then (5.6) becomes

$$r = \frac{f_K - \delta - \tau_k}{P_K}, \tag{5.65}$$

where P_K is the effective price of capital defined in (5.52). The formula shows that the tax on the capital stock by itself drives a wedge between the marginal product of capital and the market rate of interest. Thus, given the employment of labor and capital, the market rate of interest has to fall; and given the market rate of interest and the employment of labor, the employment of capital has to fall.

The wedge counteracts the taxation paradox of Type A -- the fact that accelerated depreciation may result in a marginal product of capital below the market rate of interest. However, it does not affect the Type B paradox which was derived exclusively by considering the change in P_K that resulted from a change in τ_r. The existence of a tax on the stock of capital discriminates against private investment, but it does not eliminate the possibility that a corporate tax increase would favor it.