Capital Income Taxation and Resource Allocation

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Chapter 8: Intertemporal General Equilibrium with Taxation

Chapter 8

INTERTEMPORAL GENERAL EQUILIBRIUM WITH TAXATION

The partial analysis showed how the firm reacts to government tax policy when the factor price paths are given, and it was possible to derive a number of immediate implications for tax effects on the intersectoral and international structure of capital from this. Now the partial analytic results will be integrated into the general intertemporal equilibrium model of Chapter 2 in order to study the intertemporal allocation effects of taxation. The analysis returns to a one-sector model and abstracts from taxation that differentiates with respect to political or legal criteria.

The present chapter provides both a technical background to the analysis and a discussion of the basic problems arising when government activity is considered in an intertemporal equilibrium framework. The first section studies the role of government activity in the decision problem of the household. The second tries to clarify whether, and if so in what sense, the coordination function of the capital market will be maintained in an economy with government intervention. The third derives the formal conditions of intertemporal general equilibrium. The three following *chapters* are devoted to interpreting these conditions.

8.1. The Optimization Problem of the Household under the Influence of Taxation

As with the presentation of the optimization problem of the firm under the influence of taxation, it is again useful to keep the laissez-faire model in mind (especially, Chapter 2.4) and use it as a basis for discussing the modifications resulting from taxation. All modifications apply to the wealth of the household. There is a new definition of wealth, a new equation of motion for wealth, a new initial condition, and a new liquidity constraint. Until the properties of the market equilibrium are analyzed, all variables endogenous to the decision problem of the household are characterized by the superscript "h" and all variables endogenous to the decision problem of the firm are characterized by the superscript "u".

As before, the human capital of the representative household is

$$A(t) = \int_t^\infty \left[\exp \int_t^u - \theta_p r(v)\,dv\right]\theta_w w(u) L(u)\,du \tag{8.1}$$

with the one difference that the tax factors for personal capital income, θ_p, and personal wage income, θ_w, are present. Again, the time path of L is explained through an exogenous growth of the household size N and the efficiency factor G: $L = NG$, $\hat{N} \equiv n = \text{constant} \geq 0$, $\hat{G} \equiv g = \text{constant} \geq 0$.

A new component of wealth that was not in the laissez-faire model is the present value $\tilde{F}^e$ of the expected flow $\{F^e\}$ of government transfers to the representative household:

$$\tilde{F}^e(t) = \int_t^\infty \left[\exp \int_t^u - \theta_p r(v)\,dv\right] F(u)\,du. \tag{8.2}$$

It is assumed that the representative household considers the path $\{F^e\}$ exogeneous to its planning problem; that is, it neglects any repercussions from its tax payments on the transfers it receives. This assumption is on the same level as the assumption of competitive behavior and results from the fact that the household whose behavior is to be modelled is too small to affect aggregate variables in a perceptible way.

Further wealth components are the stock of bonds, issued by private firms and the government,

$$D^h \equiv D_f^h + D_g^h, \tag{8.3}$$

and the value of corporate shares, M^u, defined in (3.24). The total wealth entering the household's decision problem is therefore

$$V^h \equiv A + \tilde{F}^e + M^u + D^h. \tag{8.4}$$

The time derivative of this expression is

$$\dot{V}^h = \dot{A} + \dot{\tilde{F}}^e + \dot{M}^u + \dot{D}^h, \tag{8.5}$$

where the differential equations

$$\dot{A} = r\theta_p A - w\theta_w L, \tag{8.6}$$

$$\dot{\tilde{F}}^e = r\theta_p \tilde{F}^e - F^e, \tag{8.7}$$

$$\dot{M}^{u} = r\frac{\theta_{p}}{\theta_{c}}M^{u} - \frac{\Pi_{n}^{du}}{\theta_{c}} + Q^{u}, \tag{8.8}$$

follow from (8.1), (8.2), and (3.22). As in the laissez-faire model, $S^{h} \equiv \dot{D}^{h}$ is the household's supply of savings in the credit market. But, unlike before [cf. (2.25)], this supply is now described by

$$S^{h} = w\theta_{w}L + \Pi_{n}^{du} + r\theta_{p}D^{h} + F^{e} - Q^{u} - \tau_{c}(\dot{M}^{u} - Q^{u}) - C^{h}(1 + \tau_{v}). \tag{8.9}$$

It is the sum of net labor income $w\theta_{w}L$, net corporate distributions Π_{n}^{du} [cf. (3.28)], net interest earnings on bonds $r\theta_{p}D^{h}$, and expected government transfers F^{e} minus the funds injected into the firm for newly issued shares Q^{u}, the capital gains tax $\tau_{c}(\dot{M}^{u} - Q^{u})$ [cf. (3.13)], and consumption expenditure $C^{h}(1 + \tau_{v})$ evaluated at the gross price including the value-added tax. Inserting Equations (8.6)–(8.9) into (8.5), analogously to (2.28), the following equation of motion for the household's wealth is obtained:

$$\dot{V}^{h} = r\theta_{p}V^{h} - C^{h}(1 + \tau_{v}) \tag{8.10}$$

Further characteristics of the opportunity set of decision alternatives available to the household are its historically given stock of wealth,

$$V^{h}(0) = A(0) + \tilde{F}^{e}(0) + M^{u}(0) + D^{h}(0) > 0, \quad D^{h}(0) = D_{0}, \tag{8.11}$$

and the liquidity constraint

$$V^{h} - \beta(A + \tilde{F}^{e}) \geq 0. \quad 0 < \beta \leq 1, \tag{8.12}$$

that, in the extreme case $\beta = 1$, excludes all borrowing against the household's human wealth and the present value of government transfers.

The implications of the optimization problem

$$\max_{\{C^{h}\}} \bar{U}(t) = \int_{t}^{\infty} e^{-\rho(v-t)} N(v)\, U[C^{h}(v)/N(v)]\, dv$$

$$\text{s.t. (8.10) and (8.11)} \tag{8.13}$$

can be straightforwardly calculated, given the paths $\{N\}$ and $\{G\}$. As in the laissez-faire model, it is assumed that Constraint (8.12) is not binding. The justification for this assumption will be examined with (8.54)–(8.55) after analyzing the general intertemporal equilibrium. The current-value Hamiltonian belonging to (8.13) is

$$\mathcal{H} = NU(C^{h}/N) + \lambda[r\theta_{p}V^{h} - C^{h}(1 + \tau_{v})]. \tag{8.14}$$

The conditions

$$U'(C^h/N) = \lambda(1 + \tau_v) \tag{8.15}$$

and

$$\dot{\lambda} - \rho\lambda = -\lambda r\theta_p \tag{8.16}$$

are the corresponding necessary conditions for an optimum.

Both conditions imply that the time path of consumption is no longer chosen such that the subjective rate of time preference,

$$\gamma \equiv \rho - \hat{\lambda} = \rho + \eta(\hat{C}^h - n), \tag{8.17}$$

equals the market rate of interest r, but such that it equals the net-of-tax market rate of interest:

$$\gamma = r\theta_p. \tag{8.18}$$

Thus the taxation of interest earnings drives a wedge between the market rate of interest and the subjective rate of time preference, and it can be expected that this will reduce the incentive to save.

That this expectation is correct can be seen from equating (8.17) and (8.18). Since

$$\hat{C}^h - n = (r\theta_p - \rho)/\eta \tag{8.19}$$

is obtained it follows that, given the path of the market rate of interest $\{r\}$, the household reacts to the introduction of taxation by reducing the growth rate of its per capita consumption; that is, by consuming more in the present and less in the future.

It should be stressed that the value-added tax does not induce substitution effects in intertemporal consumption planning for it raises the consumer price for all points in time by the same percentage. The only potential influence comes from the fact that this tax reduces the net real wage and hence may affect labor supply. This problem, however, has been put aside for the time being by assuming that labor supply is inelastic.[1]

A further requirement that the intertemporal decision of the household has to satisfy is the transversality condition. Completely analogously to (2.40), this condition can be reduced to the postulate

$$\lim_{t \to \infty} \left[\hat{V}^h(t) - \theta_p r(t) \right] < 0. \tag{8.20}$$

[1] It can easily be shown that (8.19) will be maintained with variable labor supply if the household is endowed with a utility function that is separable with regard to consumption and leisure.

8.2. The Role of Government in Coordinating Economic Plans

It has been shown how, *given* the time paths of the market rate of interest and the wage rate, the intertemporal plans of the firm and the household react to taxation. Now the case will be considered where the factor price paths $\{r\}$ and $\{w\}$ themselves have adjusted so that these plans are compatible with one another, in short: the general intertemporal equilibrium in an economy with government activity.

Government activity affects the process of intertemporal allocation via expenditure and revenue. Here the analysis is confined to a study of the role of government revenue. It would certainly be an interesting task to transfer the growth theoretic considerations on the role of government expenditure in the neoclassical growth model, as first done by Timm (1963), to the case of a general intertemporal equilibrium, but the problems this would incur are complex enough to justify a separate investigation. Timm's "e-Effekt" and other effects of government expenditure on intertemporal allocation are excluded in the present investigation by the analytical tool of lump sum transfers to private households.

While the assumption of lump sum transfers can be interpreted quite literally, there are two aspects under which it can also be interpreted as a special form of neutral government expenditure. On the one hand it is possible to imagine that the government buys goods from private firms that are perfect substitutes for private consumption goods and then distributes these without charge to private households. Provided what the households receive is less than what they would have bought anyway, this policy of the government is indistinguishable from a policy of paying out monetary lump sum transfers. The households will simply fill the gap between their consumption targets and the quantities received by the government. The time path of the sum of government and private expenditure will be identical with the path that would have been chosen in the case of monetary transfers.[2] If the assumption of perfect substitutability is to be avoided, then a second possibility is that government chooses the time profile of its expenditure through an appropriate debt policy such that, at each point in time, public consumption is proportional to private consumption. To ensure that, with such a policy, the consumption planning of private households stays autonomous and can, so to speak, take the allocative leadership, it has to be assumed, however, that the *relative* time profile of private consumption is independent of wealth, because preferences are

[2]This point was first made by Friedman (1962) and Bailey (1962).

homothetic,[3] and that the private utility function is separable with regard to public and private goods.

As in the laissez-faire situation, intertemporal general equilibrium is again defined so as to determine the factor price paths $\{r\}$ and $\{w\}$ in a way that clears all markets at all points in time. With the superscript "u" indicating variables endogenous to the firm's decision problem this means for the labor market that

$$L^{\mathrm{u}} = L, \tag{8.21}$$

for the commodity market that

$$f(K^{\mathrm{u}}, L^{\mathrm{u}}) = \delta K^{\mathrm{u}} + I^{\mathrm{u}} + C^{\mathrm{h}}, \tag{8.22}$$

and for the capital market that

$$D^{\mathrm{h}} = D_{\mathrm{f}}^{\mathrm{u}} + D_{\mathrm{g}}^{\mathrm{s}}, \tag{8.23}$$

where $D_{\mathrm{f}}^{\mathrm{u}}$ denotes the level of debt planned by firms and $D_{\mathrm{g}}^{\mathrm{s}}$ the level of debt planned by the government. However, because of government activity, a further condition for a general intertemporal equilibrium is the government's intertemporal budget constraint which, at each point in time, requires the present value of revenue to equal the sum of outstanding debt and the present value of expenditure:

$$\tilde{T}^{\mathrm{e}} = D_{\mathrm{g}}^{\mathrm{s}} + \tilde{F}^{\mathrm{s}}. \tag{8.24}$$

The flow of government expenditure is $\{F^{\mathrm{s}}\}$, and the present value of this flow is

$$\tilde{F}^{\mathrm{s}}(t) = \int_t^{\infty} F^{\mathrm{s}}(u)\left[\exp\int_t^{u} - r(v)\theta_{\mathrm{p}}\,\mathrm{d}v\right]\mathrm{d}u. \tag{8.25}$$

This expression is similar to (8.2); the only difference is that variables planned by the government rather than variables expected by the household are considered. The present value of tax revenue the government expects,[4] is

[3]Homothetic preference have already been introduced by assuming a constant elasticity of marginal utility. Compare, in this context, the remarks on tax incidence in Chapter 10.1.

[4]Since interest payments from the government to the household sector are subject to taxation, the intertemporal transformation possibilities available to the government without a violation of its intertemporal budget constraint are characterized by the net-of-tax market rate of interest $r\theta_{\mathrm{p}}$.

Note in this context that, according to Definition (3.7), the variable $T_{\mathrm{f}}^{\mathrm{e}}$ in (8.27) captures only those interest income taxes that result from transactions between private agents. For the sake of analytical simplicity it was assumed in Chapter 3.1, without any loss of generality, that the government services its debt at a rate of interest that equals the net-of-tax market rate of interest and, in exchange, exempts interest on government bonds from the personal income tax.

defined as

$$\tilde{T}^e(t) = \int_t^\infty T^e(u)\left[\exp\int_t^u - r(v)\theta_p \mathrm{d}v\right]\mathrm{d}u. \tag{8.26}$$

The flow of expected tax revenue is composed of the value-added tax T^e_v, the tax on the stock of capital T^e_k, the wage income tax T^e_w, the interest income tax T^e_i (excluding taxes on interest from government bonds), the personal and corporate income taxes on dividends T^{*e}_d, the corporate tax on retained profits T^e_r, and the capital gains tax T^e_c:

$$T^e \equiv T^e_v + T^e_k + T^e_w + T^e_i + T^{*e}_d + T^e_r + T^e_c. \tag{8.27}$$

The respective tax functions were described in Chapter 3.1.

If (8.24) is differentiated using (8.25) and (8.26), $r\theta_p \tilde{T}^e - T^e = \dot{D}^s_g + r\theta_p \tilde{F}^s - F^s$ or

$$\dot{S}^s_g = F^s - T^e + r\theta_p D^s_g \tag{8.28}$$

results where $S^s_g \equiv \dot{D}^s_g$ is the planned budget deficit. This equation is the flow counterpart of (8.24) and says that the excess of current government expenditure on transfers and debt interest above its current tax revenue must be financed by borrowing in the capital market.

In Chapter 2.5 it was shown that, in the case without government activity, one of the three market clearing conditions (8.21)–(8.23) is redundant so that, for example, commodity futures markets are dispensable if perfect capital and labor markets exist. Whether this result can be maintained if government activity is taken into account must now be checked.

For this purpose the stock equilibrium condition (8.23) is, analogously to (2.44), replaced by an equivalent flow-cum-stock condition:

$$S^h = S^u_f + S^s_g, \quad D^h(0) = D^u_f(0) + D^s_g(0). \tag{8.29}$$

According to (8.3), (8.9), (3.1), (3.6), (3.7), and (3.13) the households' supply of loans is

$$\begin{aligned} S^h = {} & wL - T_w + \Pi^{du}_n + rD^h_f - T^h_i + r\theta_p D^h_g \\ & + F^e - Q^u - T^u_c - C^h - T^h_v, \end{aligned} \tag{8.30}$$

where it follows from (3.25), (3.26), and (3.27) that

$$\begin{aligned} \Pi^{du}_n = {} & f(K^u, L^u) - \delta K^u - wL^u - rD^u_f - T^u_k \\ & + S^u_f + Q^u - I^u - T^u_r - T^{*u}_d. \end{aligned} \tag{8.31}$$

After inserting this expression into (8.30) and subtracting $S^u_f + S^s_g$ on either side,

$$S^{h} - S_{f}^{u} - S_{g}^{s} = w[L - L^{u}] + [f(K^{u}, L^{u}) - \delta K^{u} - I^{u} - C^{h}] + [S_{g}^{e} - S_{g}^{s}] + r[D_{f}^{h} - D_{f}^{u}] \tag{8.32}$$

results where

$$S_{g}^{e} = F^{e} - T^{h} + r\theta_{p}D_{g}^{h} \tag{8.33}$$

and

$$T^{h} = T_{v}^{h} + T_{k}^{u} + T_{w} + T_{i}^{h} + T_{d}^{*u} + T_{r}^{u} + T_{c}^{u}. \tag{8.34}$$

The variables S_{g}^{e} and T^{h} are defined in principle like S_{g}^{s} and T^{e} from (8.28) and (8.27) and denote the government budget deficit and the tax revenue. There are differences, however. The variable S_{g}^{e} is not the deficit planned by the government; i.e., the government's credit demand revealed in the capital market. Instead it is the deficit expected by households in the sense of an expected net income flow that results from their own and their firms' transactions with the government sector. Analogously, T^{h} is not the tax revenue expected (or forecast) by the government but the tax payments that are planned by the households where the planning is made either directly or through the firms the households own and control.

Assume now, according to (8.23) and (8.29), that there is a capital market equilibrium. Then (8.32) simplifies to

$$0 = w[L - L^{u}] + [f(K^{u}, L^{u}) - \delta K^{u} - I^{u} - C^{h}] + [S_{g}^{e} - S_{g}^{s}]. \tag{8.35}$$

Compared to (2.45) this expression shows that, when government activity is considered, the intertemporal allocation problem obtains a new dimension. Capital and labor market equilibria are no longer sufficient to ensure a commodity market equilibrium. In addition, the time paths of the budget deficits expected by the household and planned by the government have to coincide.

This condition of intertemporal compatibility of plans is comparatively strong, since no market mechanism is available to satisfy it. No uniform opinion has emerged among economists on the role of the government budget deficit in the economy. At least three different doctrines can be distinguished.

Monetarists usually take the stand that unexpected changes in the government budget deficit are the major cause of disturbances in the development path of the capitalist economy.[5] From the point of view of the

[5]A somewhat different but related position is held by Phelps (1965). Phelps examines the influence budget deficits have on the growth path of the economy in a world with lump sum

present approach, this stand means that the volatility of the government budget deficit produces persistent deviations between S_g^e and S_g^s that bring about disequilibria in the commodity market.

Keynesians also attribute a great significance to variations in the government budget deficit, but with another sign. They stress the imperfection of markets and mistrust the markets' ability to coordinate the plans of private agents. The Keynesian view is that fluctuations in the government budget deficit are necessary in order to support the coordination function of markets.

A third view, known under the name "Ricardian equivalence",[6] is held by representatives of the rational expectations school. This view is contrary to both Monetarist and Keynesian opinion on the role of the budget deficit. Its essence is that government budget deficits are neutral because private market agents are seen as having perfect foresight. Deficits neither disturb private allocation as Monetarists believe, nor stabilize private production as Keynesians think.

The discussion between the protagonists of these three doctrines continues and no end is in sight. For the sake of this analysis, an optimistic decision will be made that, at least superficially, approximates the theory of rational expectations most closely. It is assumed that $S_g^e = S_g^s$; that is, that the single household correctly anticipates the time path of the net flow of funds it receives from the government sector. There are three reasons for this decision.

First, although it is obvious that, in reality, households are *not* endowed with *perfect* foresight[7] it is completely unclear how incomplete their information is and in which direction it deviates from perfect foresight. It is true that the idealizing assumption of perfect foresight describes a special case but this case is not situated at the extreme margin of meaningful assumptions. Even for a possibly more realistic – and certainly also more

taxes if private agents are unable to anticipate these deficits. However, he does not see these deficits necessarily as disturbances of the allocation process but, on the contrary, interprets them as a means of compensating for the distortions that, because of fiscal illusion, are brought about by an existing government debt (see in particular pp. 32 n.).

[6]Ricardo himself believed in fiscal illusion rather than equivalence between debt and tax financing. For a formal analysis of Ricardian equivalence see Gandenberger (1971/72) and Barro (1974). Gandenberger also gives an overview of the intense discussion of the theme in the public finance literature.

[7]The perception, and even more so the description, of an observable phenomenon always involves abstraction. In a strict sense, abstractions are empirically false. If only for this reason, consciously made idealizations cannot be measured against an objective truth for this is impossible. This should, however, not be used to argue that all abstractions are equally admissible.

complicated – theory this case could be a useful reference point.

Second, there are cycles and disturbances in the growth process of capitalist economies that might to a large extent be the result of an imperfect coordination of economic plans. However, the mere existence of such disturbances does not imply that, in a long-run analysis of economic growth under the influence of taxation, they ought to be modelled explicitly. In any case, the author shares the wide-spread belief that these disturbances contribute little to an explanation and evaluation of the long term allocative effects of taxation. This is not meant to imply that disturbances in economic activity are unimportant and that no measures are necessary to remove them. Perhaps the contrary would be true. Perhaps the Monetarist proposal to even out the time paths of budget variables in order to reduce the disturbances should be followed. Perhaps a skilled Keynesian stabilization policy is the prerequisite for applying neoclassical allocation models like the one used here. These questions are left open.

Third, it is only possible to isolate the substitution effects of taxation as agreed in the introduction of this book when perfect foresight concerning government's plans is assumed. By doing this we avoid confusing two things: allocative disturbances that, via income effects, result from imperfect foresight of households and disturbances that result from the specific characteristics of the tax system.

8.3. The Conditions for Market Equilibrium

With a perfect capital market, a perfect labor market, and perfect foresight of the time path of the government budget deficit, all markets in the model, including the commodity market, clear for all points in time. This section studies the formal properties of the economy's growth path that emerges under such circumstances.

Independently of taxation, as in (2.47), (8.22) implies the technological equation of motion

$$\dot{k} = \varphi(k) - (\delta + n + g)k - c \tag{8.36}$$

for the capital intensity k, defined as capital per efficiency unit of labor, where $\varphi(k) = f(k, 1)$ denotes production and c consumption per efficiency unit of labor. However, unlike (2.48), the equation of motion for c is affected by the tax system. Because of $\dot{c}/c = \hat{C} - \hat{L}$ and $\hat{L} = n + g$ the general expression

$$\dot{c} = (c/\eta)(\gamma - \gamma^{\infty}) \tag{8.37}$$

results from (8.17) where γ is the current and

$$\gamma^{\infty} \equiv \rho + \eta g \tag{8.38}$$

is the steady-state rate of time preference.

In the laissez-faire case the rate of time preference γ was equal to the market rate of interest r which in turn equalled the marginal product of capital $\varphi'(k) - \delta$. Now the three types of "interest rates" no longer coincide.

On the one hand it is known from the basic condition (5.6), and from $\varphi' = f_K$, that

$$r = [\varphi'(k) - \delta - \tau_k]/(\theta_p \tilde{P}_K), \tag{8.39}$$

where

$$\tilde{P}_K \equiv \frac{1 - \sigma^* - \alpha_1 \tau_r}{\max(\theta_d^*, \theta_r^*)} + \frac{\sigma^*}{\theta_p \theta_r}\left[1 - \tau_r\left(1 - \alpha_3 + \frac{\alpha_2}{\sigma^*}\right)\right] \tag{8.40}$$

is a magnitude that we want to call *wedge parameter.*[8] The maximum marginal debt-asset ratio σ^* that appears in (8.40) is, according to (4.8), given by

$$\sigma^* \equiv 1 - \alpha_1 \tau_r - \varepsilon^*, \tag{8.41}$$

where ε^* is the minimum marginal equity–asset ratio. In order to allow for both an endogenous explanation of ε^* along the lines discussed in Chapter 5.2 and even higher values that are exogenously determined, it is assumed that

$$\varepsilon^* \geq \alpha_1 \, \mathrm{W} \max(\theta_d^*, \theta_r^*) \qquad \text{for } \alpha_2 = \alpha_3 = 0 \tag{8.42}$$

and

$$\varepsilon^* \geq 0 \qquad \text{for } \alpha_3 = 1. \tag{8.43}$$

On the other hand, the analysis of the household's decision problem has shown with Equation (8.18) that the rate of time preference equals the net-of-tax market rate of interest:

$$\gamma = \theta_p r. \tag{8.44}$$

[8] As can be seen by comparing (8.40) with (6.3), the wedge parameter is the effective price of capital P_K divided by the personal tax factor θ_p in the special case where debt interest is deductible ($\alpha_2 = \alpha_3 = 0$). In general, however, the relationship between the effective price of capital and the wedge parameter is of a more complicated nature.

If (8.39) is inserted into (8.44) and then (8.44) into (8.37),

$$\dot{c} = \frac{c}{\eta}\left[\frac{\varphi'(k) - \delta - \tau_k}{\tilde{P}_K} - (\rho + \eta g)\right] \tag{8.45}$$

results. This equation of motion is the vehicle by which the tax system affects the growth path of the economy.

Together with (8.36), (8.45) defines a unique path in a (c, k) diagram that leads to a steady-state point with a strictly positive value of c. An example is illustrated in Figure 8.1 that corresponds to Figure 2.1 and is repeated here for convenience.

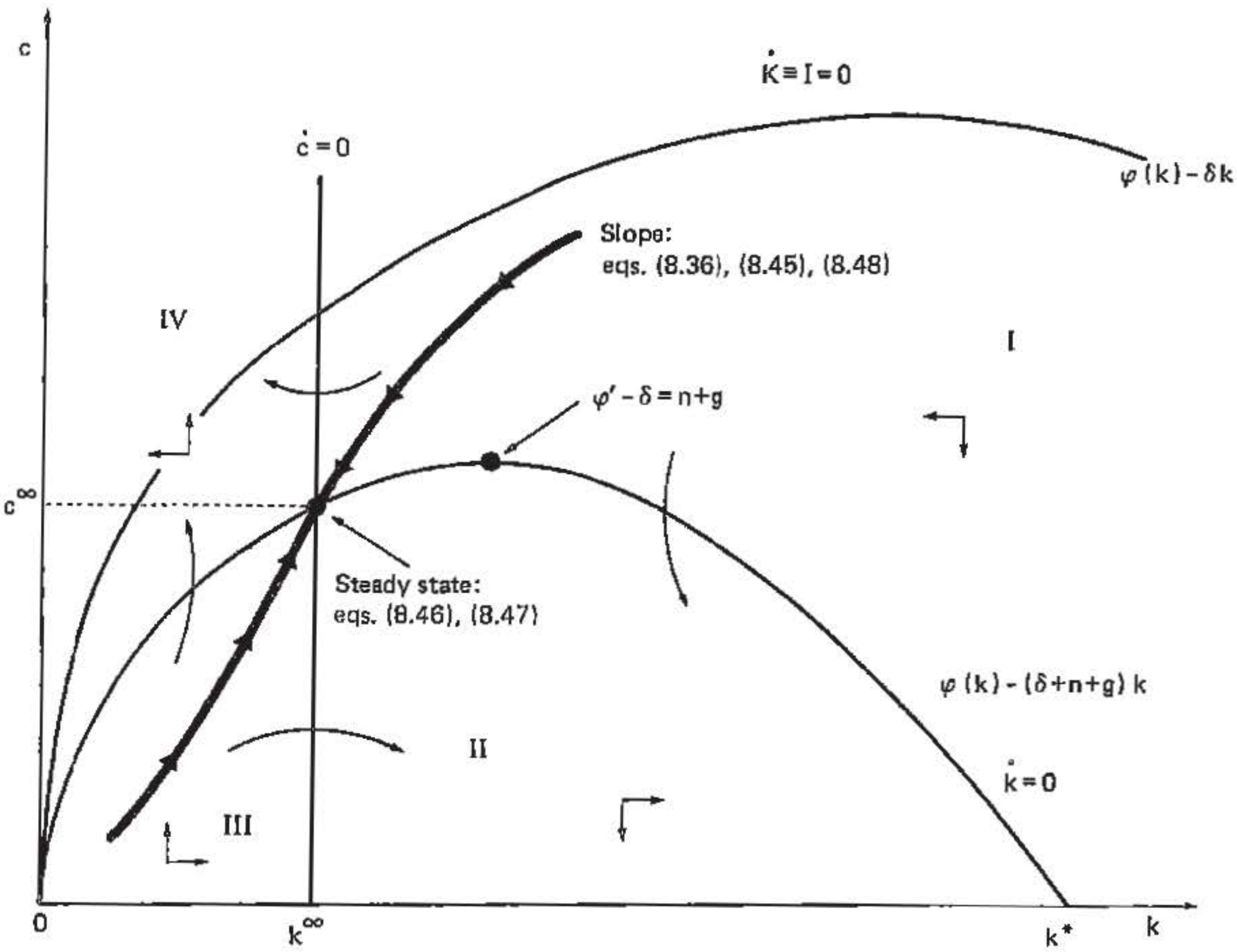

Figure 8.1. The general intertemporal equilibrium with taxation.

The steady-state point is defined by

$$\varphi'(k^\infty) - \delta = (\rho + \eta g)\tilde{P}_K + \tau_k \tag{8.46}$$

and

$$c^\infty = \varphi(k^\infty) - (\delta + n + g)k^\infty. \tag{8.47}$$

The slope of the path leading to this point is determined by

$$dc/dk = \dot{c}/\dot{k} \tag{8.48}$$

in connection with (8.36) and (8.45). The fact that both $\dot{k}$ and $\dot{c}$ are negative in Region I of the diagram, and positive in Region III, implies that the steady-state solution is stable and that the path leading to it has a strictly positive slope.[9]

The path leading to the steady-state point is the market equilibrium path. In Appendix C it is shown that paths that are compatible with (8.36) and (8.45), but do not lead to the steady-state point described with (8.46) and (8.47), cannot represent an intertemporal market equilibrium since they violate other conditions of a solution. Moreover it is shown that the transversality conditions (3.36), (3.37), and (8.20) of the planning problems of the model agents are satisfied on the path leading to the steady-state point. Two important assumptions have to be made, however.

First, as in (2.51), existence requires the steady-state rate of time preference, defined in (8.38), to exceed the natural rate of growth:

$$\gamma^{\infty} > n + g. \tag{8.49}$$

To interpret this condition, note that, because of (8.37), (8.38), (8.44), and $\lim_{t\to\infty} c(t) = \text{constant} > 0$,

$$\lim_{t\to\infty} r(t)\theta_{\mathrm{p}} = \gamma^{\infty} \equiv \rho + \eta g; \tag{8.50}$$

and that, because of $\lim_{t\to\infty} k(t) = \text{constant} > 0$,

$$\lim_{t\to\infty} \hat{K}(t) = n + g. \tag{8.51}$$

Together with (8.49), these two equations imply that the long-run net-of-tax market rate of interest, $\theta_{\mathrm{p}} r$, must exceed the long-run rate of growth of the capital stock, $\hat{K}$. Hence, the growth factor W defined in (5.21) must be strictly smaller than unity:

$$W = \frac{n + g}{\rho + \eta g} < 1. \tag{8.52}$$

If this condition is violated then, given the other aspects of the model, no market equilibrium exists.

Second, in order to ensure uniqueness it is necessary and sufficient that

$$(n + g)(\theta_{\mathrm{c}} \tilde{P}_K - 1) + \tau_{\mathrm{k}} \geq 0. \tag{8.53}$$

[9] The statements concerning the sign of $\dot{c}$ result since $\varphi'' < 0$ and since $\tilde{P}_K > 0$. The latter follows from (8.55) and (8.57) below.

Clearly this condition is satisfied for a stationary economy ($n + g = 0$). It is not obvious, however, under which circumstances it will hold for a growing economy ($n + g > 0$).

Consider first the case of deductible debt interest ($\alpha_2 = \alpha_3 = 0$). Here (8.40) becomes

$$\tilde{P}_K = \frac{1 - \sigma^* - \alpha_1 \tau_r}{\max(\theta_d^*, \theta_r^*)} + \frac{\sigma^*}{\theta_p} \qquad (\text{for } \alpha_2 = \alpha_3 = 0). \tag{8.54}$$

As it follows from $\theta_p \geq \max(\theta_d^*, \theta_r^*)$ that $d\tilde{P}_K/d\sigma^* \leq 0$, (8.54) implies in connection with (8.41) and (8.42) that

$$\tilde{P}_K > \frac{1 - \alpha_1 \tau_r}{\theta_p} > 0 \qquad (\text{for } \alpha_2 = \alpha_3 = 0). \tag{8.55}$$

Since $\tau_k \geq 0$, this inequality ensures that (8.53) is satisfied despite $n + g > 0$ if

$$(\theta_c/\theta_p)(1 - \alpha_1 \tau_r) \geq 1 \qquad (\text{for } \alpha_2 = \alpha_3 = 0). \tag{8.56}$$

Unlike (8.53), (8.56) is not a necessary, but still a sufficient, condition for a uniqueness of the intertemporal general equilibrium. It defines an admissible set of tax parameters $\alpha_1, \tau_c, \tau_p, \tau_r$, and τ_k for the case of a non-stationary economy with deductible debt interest. In the extreme case of an immediate write-off and an absence of a tax on the capital stock, (8.56) requires that the overall marginal tax burden on retained profits equals the marginal personal tax rate ($\theta_r^* = \theta_p$) as would be the case in a Miller equilibrium. In general, however, it is admissible that the marginal personal tax rate falls short of the marginal tax burden on retained profits. It is only necessary in this case that the acceleration of tax depreciation is sufficiently moderate. To illustrate this, consider the stylized facts $\tau_p = 0.28$ (0.4), $\tau_c = 0.6 \cdot \tau_p = 0.17$ ($= 0.25 \cdot \tau_p = 0.1$), and $\tau_r = 0.34$ (0.46) that according to Chapter 3.1.2 might characterize the situation of a typical U.S. corporation after (before) the 1986 tax reform. Assume $n + g > 0$ and $\tau_k = 0$. Then (8.56), and for an even stronger reason (8.53), will surely hold true if $\alpha_1 \leq 0.39$ (0.72), a condition that would be satisfied by the stylized fact $\alpha_1 = 0.3$ (0.5) reported in Chapter 3.1.3.

Next, consider the case of non-deductible debt interest ($\alpha_3 = 1$). For this case it follows from financial existence requirements discussed in Chapter 4.4 [see Equation (4.25)] that $\theta_d^* \leq \theta_r^* \leq \theta_p = \theta_c$. Moreover, the limitation of parameter constellations given in Chapter 3.1.4 says that either $\alpha_1 = 0$,

$\alpha_2 = 1$ or $\alpha_1 = 1$, $\alpha_2 = 0$. Thus (8.40) reduces to

$$\tilde{P}_K = 1/\theta_c \gtrsim 1 \qquad (\text{for } \alpha_3 = 1 \text{ and } \theta_d^* \leq \theta_r^* \leq \theta_p = \theta_c). \tag{8.57}$$

Obviously, this implies that (8.53) is satisfied.

To ensure existence and uniqueness of the market equilibrium path it is assumed in this and the following chapters that the necessary conditions (8.49) and (8.53) are met.

These conditions also have important repercussions on the position of the steady-state point. From (8.46) and (8.49) it follows that this point satisfies the condition

$$\varphi'(k^\infty) - \delta - (n + g) > (n + g)(\tilde{P}_K - 1) + \tau_k. \tag{8.58}$$

As the lefthand side of this inequality is the slope of the ($\dot{k} = 0$) curve, the steady-state point must be left of the maximum of this curve when the righthand side of (8.58) is non-negative. The latter is the case because of (8.53) and $\theta_c \leq 1$. Thus, necessary conditions for existence and uniqueness of an intertemporal general equilibrium imply that dynamically inefficient steady states right of the Golden-Rule point as well as this point itself can be excluded.

In addition to the ($\dot{k} = 0$) curve, which is the graph of the function $\varphi(k) - (\delta + n + g)k - c = 0$, Figure 8.1 contains a curve labeled $I = 0$. This curve is the graph of the function $\varphi(k) - \delta k$ and the geometrical locus of all those consumption points at which net investment is zero. Under certain conditions, it is also a boundary of the range for which the model is defined. The assumptions following (4.9) imply that there is a non-shrinking economy ($I \geq 0$) whenever there is a limited financial flexibility in the sense $\varepsilon^* > 0$ and a strict tax preference for debt financing [$\alpha_3 = 0$, $\theta_p > \max(\theta_d^*, \theta_r^*)$]. Under such circumstances, the market equilibrium path in the (c, k) diagram is therefore not defined in the range above the ($I = 0$) curve. Note that this limitation of scope does not exclude a discussion of declining growth paths in the (c, k) space if the natural rate of growth is strictly positive, that is, if $n + g > 0$.

As in the laissez-faire model, the liquidity constraint (8.12) that, at least in part, prohibits any borrowing against human capital and government transfers is again not binding under very mild conditions. In the extreme case $\beta = 1$, this constraint requires, because of (8.3) and (8.4), that the sum of the market value of company shares and the stocks of bonds, issued by private firms and the government, is permanently non-negative:

$$M + D_f + D_g \geq 0 \qquad \text{for all } t \geq 0. \tag{8.59}$$

It is shown in Appendix A [Equation (A.7)] that the basic assumption (4.10) implies $M > 0$.[11] Thus (8.59) is satisfied if, as in reality, households are net creditors of private firms and the government (D_f, $D_g \geq 0$).

For the theoretical analysis, negative values of D_f and D_g do not have to be excluded, though. It suffices to assume that, at each point in time, the stock of government debt is large enough to satisfy (8.59). Apart from its realism, this assumption can be justified by the theoretical argument, put forward by Gandenberger (1971/72, pp. 381 n.), that government debt can be considered as resulting from the households' ability to borrow against their future labor income collectively under more favorable conditions than individually. According to this argument, the reason for government debt is to ensure that Constraint (8.12) is never binding and hence cannot disturb the growth path of the economy.

[11]Using the explicit formula for the market value function, as given in (6.2), it is possible, but not illuminating, to reduce (8.59) to an expression with K, D_f, and D_g instead of M, D_f, and D_g.