
STUDIES
IN MATHEMATICAL AND
MANAGERIAL ECONOMICS

Editors

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VOLUME 32



NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

ECONOMIC DECISIONS UNDER UNCERTAINTY

HANS-WERNER SINN

UNIVERSITY OF MANNHEIM



1983

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

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ISBN 0 444 86387 7

First published as: *Ökonomische Entscheidungen bei Ungewißheit* in: Die Einheit der Gesellschaftswissenschaften, ed. by E. Böttcher, J.C.B. Mohr (Paul Siebeck), Tübingen 1980.

PUBLISHERS:

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM – NEW YORK – OXFORD

DISTRIBUTORS FOR THE U.S.A. AND CANADA:

ELSEVIER SCIENCE PUBLISHING COMPANY, INC.
52 VANDERBILT AVENUE
NEW YORK, N.Y. 10017

PRINTED IN THE NETHERLANDS

INTRODUCTION TO THE SERIES

This is a series of books concerned with the quantitative approach to problems in the social and administrative sciences. The studies are in particular in the overlapping areas of mathematical economics, econometrics, operational research, and management science. Also, the mathematical and statistical techniques which belong to the apparatus of modern social and administrative sciences have their place in this series. A well-balanced mixture of pure theory and practical applications is envisaged, which ought to be useful for universities and for research workers in business and government.

The Editors hope that the volumes of this series, all of which relate to such a young and vigorous field of research activity, will contribute to the exchange of scientific information at a truly international level.

THE EDITORS

to Annette, Philipp and Rüdiger

Preface

This book was first published in German as *Ökonomische Entscheidungen bei Ungewißheit*, Tübingen 1980. Thanks to the initiative of the editors of the present series, it is now available in English. Except for minor amendments, the English version closely follows the German original.

The translation would not have been accomplished without the help of Juli Irving-Leßmann, a charming professional economist, whom a benevolent fate sent from Australia to Mannheim. With great patience and care she read the manuscript in various stages of the translation process, corrected my mistakes and polished my style. I gratefully acknowledge her assistance.

Acknowledgement must also be given to Reiner Bernhardt, Martin Kriechbaum, Bernd Gutting, Brunhild Schindhelm, Martin Sikora, and Walter Winkelmann for providing various kinds of technical assistance and to Christa Kininger who typed the manuscript promptly and with great accuracy.

I have to record my intellectual debt to all those who gave their comments on the German version and on related papers. I should like to mention Elmar Helten, Peter Howitt, Heinz König, Wilhelm Krelle, Hans Schneeweiß, Ann Schwarz-Miller and, in particular, Hans Heinrich Nachtkamp, who has been a permanent source of encouragement and inspiration.

Finally, I owe much gratitude to my wife Gerlinde who not only bore more than her fair share of our family commitments while the book was being written, but also, as a fellow economist, gave unsparingly of her professional help. The English edition is dedicated to our children, who, albeit unwittingly, also bore some of the burden imposed.

Mannheim, November 1982

Hans-Werner Sinn

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Introduction

The Fundamental Issues Involved

Why do we need a theory of uncertainty? It is a fact that almost all man's economic decisions are made under conditions of uncertainty, but this fact alone does not provide a strong enough argument for making the effort necessary to generalize ordinary preference theory designed for a world of perfect certainty. In accordance with *Occam's Razor*, the mathematician may well welcome a generalization of assumptions even if it does not promise more than a restatement of known results. The economist, however, will only be well disposed towards making the effort if he can expect to achieve new insights and interesting results, for he is interested in the techniques necessary for the generalization only as means to an end, not as ends in themselves. A stronger reason for developing a theory of uncertainty, therefore, seems to be the fact that there are kinds of economic activities to which the non-stochastic preference theory has no access or has access only through highly artificial constructions. Such activities include portfolio decisions of wealth holders, speculation, and insurance. These will be considered in detail in the last chapter of the book.

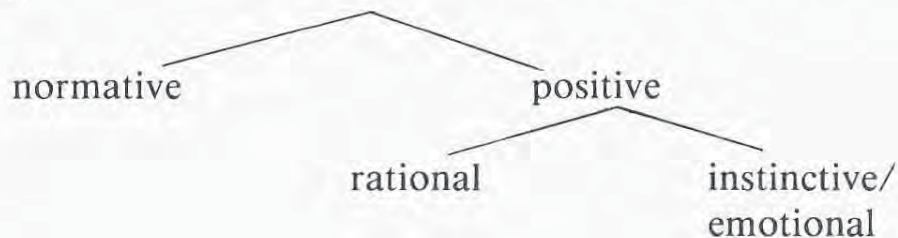
The main purpose of this book, however, is not to apply a theory of uncertainty to concrete economic problems, the purpose rather is to formulate such a theory. This may sound pretentious, especially in the light of the effort that has been involved in constructing the current theoretical edifice, starting from Cramer and Bernoulli and proceeding by way of von Neumann and Morgenstern. However, the aim of this book is comparatively modest. It is not to replace the current theoretical edifice by a new one but to examine it critically and to carry out some modifications, renovations, and extensions, which may improve its efficacy. Examples of such modifications are

- a rehabilitation of the Principle of Insufficient Reason (ch. I B 3),
- an improved argument for approximating nearly arbitrary von

Neumann-Morgenstern utility functions by indifference curves in a (μ, σ) diagram (ch. II D 2.2),

- the development of a preference hypothesis based on the psychophysical law (ch. III A),
- the introduction of the 'blood out of a stone' rule (ch. III B),
- the use of a specific risk preference function in multiperiod planning problems (ch. IV B 2).

Although the theory of uncertainty presented here did not necessarily have to be limited to a specific range of economic activities, it was necessary to make a fundamental decision about its methodological direction. The following figure shows the alternatives that are available in principle.



The first decision to be made is whether we want to explain how people behave (positive analysis), or to give them advice about 'right' behavior (normative analysis). If a decision is made in favor of positive analysis, the next question concerns the model of man to be used. Does man, after careful consideration, choose the best from a set of alternatives available to him (rational behavior), or is his behavior simply an automatic response to external stimuli (instinctive or emotional behavior)? The choice between these two models of man that MARCHAL (1949, p. 129) once appropriately characterized as *homme de Descartes* and *homme de Pavlov* remains a problem, even if attention is limited to the range of economic activities. The truth most probably lies between these extremes: man is, as ALBERT (1978, p. 65) says, a 'fehlbares theoretisches Tier' (fallible theoretical animal). By far the majority of the founders of economic theories solved the two-part methodological decision in a way that was designed to kill two birds with one stone. They assumed rationality, however defined, and saw their theories as tools which could be used both to identify wise decisions and to describe the actual behavior of man. In the present study the author has also adopted this solution.

However, doubts about this decision remain. They are concerned not so much with the usefulness of the theory presented here for normative analysis but with its usefulness for positive analysis.

Basically, the procedure of the economic theory of uncertainty is to

derive concrete rules of behavior from a few plausible axioms by way of a rather complicated process of logical deduction. Suppose everyone accepts these axioms as devices for rational behavior. Does it necessarily follow that their implications will also be accepted? The fact that this question cannot be self-evidently answered in the affirmative is the reason for the above-mentioned doubts. Certainly KNIGHT (1921, p. 236) was right when he stated that the evolution of man is a development towards more rationality; rationality in fact, has turned out to be his evolutionary advantage compared to other species. But has man yet reached that high standard necessary for an unhesitatingly affirmative answer to be given? Probably not. The absurd implications of assuming that he has reached such a standard are graphically illustrated by SAVAGE (1954, p. 20): ‘... if anyone who believed the axioms of mathematics also believed all that they imply and nothing that they contradict, mathematical study would be superfluous for him; such a person would ... be able to state the ten-thousandth or any other term in the decimal expansion of π on demand.’ Thus, from the point of view of positive theory, the usefulness of the axiomatic method in preference theory definitely has limits originating in the imperfect analytical ability, intelligence, or rationality of the economic actors considered.

This recognition, however, should not be interpreted as leaving no space whatsoever in an explanatory theory for the axiomatic method. The fact that economic decision makers *want* to behave rationally, gives rise to the hope that the axiomatic approach, although imperfect, will result in an approximately correct description of behavior, much like MARSCHAK’S (1950, p. 111) saying that men ‘cannot be “all fools all the time”’.

Contrary to the way it may seem at first glance, the decision for the *homme de Descartes* (alias *homo oeconomicus*) and against the *homme de Pavlov* is not a decision against psychology. This cannot be the case since, *ex definitione*, a preference theory is a psychological theory. But even where professional psychology is concerned, nothing would be further from the author’s thoughts than an attempt to dissociate himself from its findings. The time when ROBBINS (1935, pp. 83–90) could banish psychology to the outskirts of economics as being ‘the happy hunting-ground of minds averse to the effort of exact thought’ and when he could state, with sympathetic indulgence, that even great economists like Gossen, Jevons, and Edgeworth were led astray by psychology are long since past. Instead, an attempt should be made to integrate the results of professional psychology into the model of the rationally calculating decision maker. Doing this should make it possible to substantially reduce the large degree of tolerance necessarily left over by using rationality axioms and thus make it possible to achieve

more specific conclusions about human behavior, i.e., conclusions with a higher informational content. As mentioned above, this book makes an attempt in this direction by introducing the psychophysical law into the theory of economic decision making under uncertainty.

The structure of the book is as follows. The first chapter provides the basis for economic decision making under uncertainty. It is primarily concerned with the question of whether it makes sense to transform vague ideas about the possible outcomes of an economic decision into an objective probability distribution of such outcomes. Following that, the second chapter addresses the problem of a rational evaluation of objective probability distributions. Various decision criteria proposed in the literature are compared and assessed with respect to their usefulness. The framework for economic decisions produced in the first two chapters is rather general. Therefore, chapter three is devoted to the task of filling out this framework with two supplementary hypotheses about the structure of human preferences. The fourth chapter deals with special problems arising if, either simultaneously or sequentially, decisions have to be made about multiple risks. Finally, in chapter five, the general decision theory previously developed is applied to various economic problems.