

Contents lists available at [ScienceDirect](http://www.sciencedirect.com)

## Journal of Monetary Economics

journal homepage: [www.elsevier.com/locate/jme](http://www.elsevier.com/locate/jme)

## The fossil episode

John Hassler <sup>a,\*</sup>, Hans-Werner Sinn <sup>b</sup><sup>a</sup> IIES, Stockholm University, S-106 91 Stockholm, Sweden<sup>b</sup> Ifo Institute – Leibniz Institute for Economic Research at the University of Munich, Germany

## ARTICLE INFO

## Article history:

Received 14 February 2014

Received in revised form

17 August 2016

Accepted 18 August 2016

Available online 26 August 2016

## Keywords:

Fossil fuel

Biofuel

Food prices

Climate change

Green paradox

## ABSTRACT

Agriculture sector output (biocarbon) is a good substitute for oil in energy production but oil cannot be used as food. This one-way substitutability is analyzed in a dynamic general equilibrium model. It features three endogenous phases: a pure fossil, a mixed fossil and biocarbon and an absorbing biocarbon fuel only phase. In the latter two, the demand for biocarbon as fuel leads to increasing food prices. Depending on how easily capital and labor can reallocate, food prices increase by between 40% and 240%. The model is also used to analyze climate consequences of biocarbon fuel polices and of the shale revolution.

© 2016 Elsevier B.V. All rights reserved.

## 1. Introduction

The average price of a barrel of oil over the year 2013 was US\$109. Using an energy content of 1699 kWh/barrel, this implies an energy price of 6.4 cents/kWh. Over the same year, the average market price of corn was US\$259 per ton. Using an energy content of 4389 kWh per ton, the energy price of corn was quite close at 5.9 cents/kWh.<sup>1</sup> This closeness is not a unique coincidence. Over the period 2005–2015, the average energy prices of oil and corn were 4.9 and 4.5 cents/kWh, respectively. However, the prices of energy in these two forms have not always been close. Fig. 1 depicts yearly prices of oil and corn in a long historic perspective, both expressed per unit of energy content. The price of oil has converged slowly to the price of corn from below, reaching it in the last decade and coinciding with it rather closely since 2005.

After oil and corn prices first were equal, they have been tracking each other in a way that did not occur before. This is easier to see in Fig. 2, which depicts the same data as Fig. 1 but for the period of 1981–2015. The recent change in the dynamic relation between corn and oil prices has been more formally documented by Avolos (2014). He finds evidence of a structural break in 2006 making the corn price co-integrated with the oil price while no such relation existed before.

Fig. 2 also depicts the energy price of sugar, another agricultural staple good with a well-defined energy content. The figure shows that the sugar price per unit of energy is much higher than that of corn and oil for the whole period. Furthermore, there is no tendency that the oil price and the sugar price have become more correlated. A tentative interpretation of the increased correlation between oil and corn prices is that the degree of substitutability between oil and corn

\* Corresponding author.

E-mail address: [john@hassler.se](mailto:john@hassler.se) (J. Hassler).<sup>1</sup> Price data from the World Bank Commodity Markets Outlook, January 2014. The energy content of corn is taken from Penn State College of Agricultural Sciences (2012) as the energy content of shelled corn taking into account a water content of 15%. The energy content of oil is from EIA <http://www.eia.gov/energyexplained> Units and Calculators.

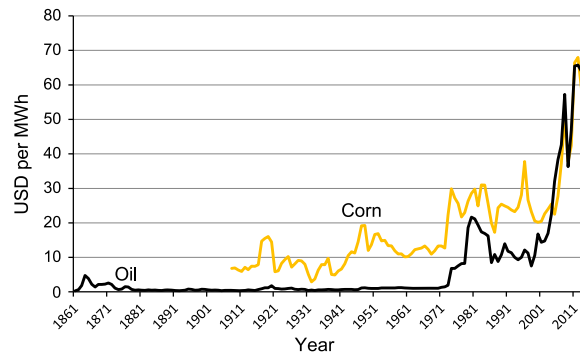


Fig. 1. Prices in current USD per unit of energy for corn and crude oil. Source: Ifo Institute calculations.

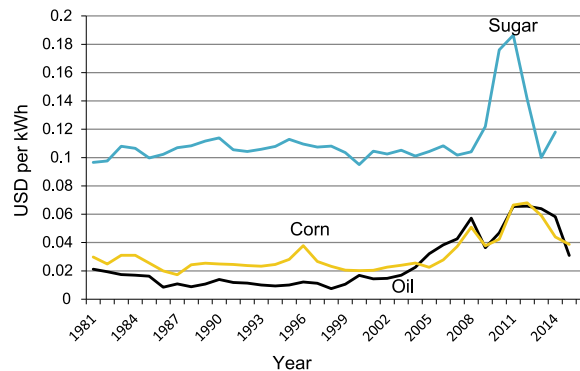


Fig. 2. Prices in current USD per unit of energy for corn, crude oil and sugar. Source: Ifo Institute calculations.

(but not oil and sugar) increased quite suddenly. With a higher degree of substitutability, any factor affecting the price of one good is more likely to also drive the prices of good substitutes in the same direction.

Over the last decades, the price of oil has been quite volatile. Our paper does not attempt to explain this volatility. Our focus is instead on providing an explanation for a sudden increase in the substitutability between biocarbon in the form of corn and fossil carbon in the form of oil. We then analyze the consequences of this for long-run global growth patterns. Our explanation will be based on a well documented feature of oil and corn – namely a unidirectional substitutability. For many uses, corn is an almost perfect substitute for fossil carbon. Corn can easily be used as fuel in heat production, both industrially and in owner-occupied housing. It can also be used in the transportation sector. In fact, in 2007, 8% of US agricultural land, or 30% of US total corn output, was used for the production of bioethanol (von Braun, 2008). However, fossil carbon is not a good substitute for most uses of biocarbon – we cannot eat fossil oil. All attempts to chemically convert the hydrocarbon in fossil fuel into edible substances have failed.

The unidirectional substitutability implies that the substitutability becomes operational only when the price of fossil carbon has reached the price of biocarbon, both measured per unit of energy. As argued above, this happened for corn in the middle of the previous decade but has not (yet) happened for sugar. When the price of fossil carbon is lower than that of biocarbon, the substitutability is non-operational and prices can move independently. When the price of fossil carbon has reached that of biocarbon two interesting mechanisms arise. On the one hand, increasing fuel prices can pull food prices with them. In particular, a trendwise increasing fuel price can cause a similar trend in food prices. On the other hand, the fact that biocarbon is a substitute for fossil carbon has an impact on fossil fuel prices. In particular, biocarbon may endogenously become a backstop technology for fossil fuel that can cap or limit future fuel prices.

To analyze the mechanisms described above, a dynamic general equilibrium model is required. A natural starting point is the seminal work by Dasgupta and Heal (1974). Like in their paper, our model has a production sector that uses labor, capital and energy to produce an output that can be consumed as well as used for capital investments. One source of energy is oil, and like in Dasgupta and Heal (1974), a secular positive trend in energy scarcity results in an upward trend in the relative price of energy. Our model also includes a second production sector, producing biocarbon from capital, labor and fixed factor called land. In line with the discussion above, biocarbon can be consumed directly as well as used as energy input in the first production sector.

Our model has a number of qualitative predictions. With just one unforeseen shock, the appearance of a technical device to exploit the fossil fuel reservoirs, namely steam and combustion engines (the Industrial Revolution), we can endogenously derive a development path with four stages: (1) A pre-industrial stage where food and fodder rival for land, and bioenergy is the only form of energy available. (2) A fossil phase where land is exclusively used for the production of food while energy is

taken exclusively from fossil sources. (3) A mixed phase where biocarbon and fossil carbon are both used as a source of energy and an increasing share of land is gradually used for biocarbon production. (4) A final biophase where the stock of fossil carbon is exhausted and, as in the pre-industrial phase, biocarbon serves both nutrition and energy needs.

These qualitative results hinge on the assumption that energy scarcity does not disappear. That some other technology than biocarbon production, e.g., cheap nuclear fusion, becomes the backstop for fossil fuel cannot be ruled out. Such a technology could break the trend in energy scarcity once-and-for-all and make energy cheaper than food forever. Another possibility is that such a technology is not developed, in which case we argue that the competition for the use of biocarbon will increase as fossil fuels become increasingly scarce. It is hard or impossible to quantify the likelihoods of these scenarios. A starting point for our analysis is that the second scenario is sufficiently likely to make the analysis worthwhile.

Our model is stylized but based on well-known and much-used assumptions about preferences and technology. Therefore, also the quantitative implications can be informative. The recent public attention to the consequences of increased competition between different uses of biocarbon motivates such an analysis. As an example, in January 2007, thousands of people protested against rising food prices in Mexico City, an event known as the Tortilla crisis.<sup>2</sup>

If capital and labor are free to move to the biocarbon producing sector, the increase in the food price caused by the transition to a fossil free economy need not be very large. Under a benchmark calibration, the price increases by about 40% during the transition. However, the price increase may be much larger if the output expansion of the biocarbon sector is hindered while the use of biocarbon as fuel is not restricted. In the extreme, if the distribution of capital and labor over the production sectors is fixed, the price increase during the transition is a whopping 250%.

A second motivation for the quantitative analysis is the increased interest in using biocarbon as fuel to curb climate change. Policies in order to stimulate the use of biocarbon as fuel have been introduced in many countries. In order to analyze the consequences for the climate of such policies, a simple climate module and a carbon circulation system is added to the model. This is done in order to examine the claim that public policies stimulating the use of biocarbon would mitigate the global warming problem. However, the reverse is true in our model, a ban or restrictions on biocarbon use for fuel use can slow down global warming. In addition, it would decouple food and fuel prices for an extended period of time.

Recent improvements in the technology for extracting fossil fuels, e.g., hydraulic fracturing (“fracking”), reduce scarcity and have had large impacts on the fossil price but does not change the fact that fossil fuel exists in limited supply. We also use our model to analyze the medium and long-run consequences of such a shift in fossil fuel reserves.

Our model is tractable and transparent and provides clear analytical results. Obviously, this comes at a cost in terms of realism of the underlying assumptions. In particular, various frictions like market power and adjustments costs are assumed away. Potentially important supply side details of alternative sources of energy are also ignored. Nevertheless, we think that the model provides useful results, also quantitatively. We are the first (to our knowledge) to analyze the intertemporal general equilibrium consequences of food-fuel competition in a setting with exhaustible resources. A transparent and tractable model, built on standard and well-known macroeconomic assumptions, may be particularly useful at such an early stage, leaving for future work the construction of models with less stylized assumptions solved using numerical techniques.

It is also important to note what issues are not addressed. In particular, our model is not suited for analyzing short-run movements in energy prices. Such movements were used above to document an increase in the co-movements between fossil carbon and biocarbon. However, the model will not feature such highly volatile high-frequency fluctuations. Even though shocks to demand or supply of fuel easily could be added, a frictionless model like ours is not likely to be useful in providing an understanding of these fluctuations.

The paper is organized follows. The next section describes formally our model. [Section 3](#) shows the existence of the distinct phases of fuel use. Expressions for balanced growth in the model are derived as well as a sufficient condition for the economy to pass through the four phases in order 1 through 4, with the last state being absorbing. [Section 4](#) analyzes how much the price of food changes due to the increased food-fuel competition and present some calibrated simulations. [Section 5](#) adds a carbon circulation model and a mechanism that determines the relation between atmospheric carbon concentration and the global mean temperature. The consequences of policies enacted in order to affect the use of biocarbon for fuel as well as a sudden increase in the amount of fossil reserves (the shale gas and oil revolution) are analyzed. [Section 6](#) concludes.

## 2. A model of multiple uses of biocarbon

Manufacturing output  $Y_t$  is given by

$$Y_t = A_{1,t} K_{1,t}^{\alpha_1} L_{1,t}^{1-\alpha_1-\nu_1} E_t^{\nu_1} \quad (1)$$

where  $A_{1,t}$  is an exogenous productivity trend,  $K_{1,t}$  and  $L_{1,t}$  are capital and labor and  $E_t$  is energy input. When prices later are defined, the manufacturing good is used as numeraire.

<sup>2</sup> Corn, imported from the US, had become nearly twice as expensive as during the previous winter, increasing the price of tortillas by 35%. A year later, wheat and rice prices had trebled, and the hunger riots had spread to 37 countries. See [Sinn \(2012, chapter 3\)](#).

The agricultural sector produces biocarbon  $F_t$  according to the production function

$$F_t = A_{2,t} K_{2,t}^{\alpha_2} L_{2,t}^{(1-\alpha_2-\nu_2)} G^{\nu_2} \quad (2)$$

where  $A_{2,t}$  is an exogenous productivity trend,  $K_{2,t}$  and  $L_{2,t}$  are capital and labor used in agriculture, and  $G$  is land, a fixed factor only used in agriculture. Sector 2 may be given a broader interpretation than agriculture, representing land-intensive production including e.g., wind and solar energy. In order to keep the analysis simple, energy use in agriculture is abstracted from. Including energy would produce a link between energy prices and food prices also when the price of food is above the energy price. Given a small income share of energy, this link would, however, be weak relative to the link due to the one-way substitutability between fossil carbon and biocarbon analyzed in this paper.<sup>3</sup>

Individual utility is given by

$$U = \sum_{t=0}^{\infty} \beta^t (\ln C_t + \theta \ln D_t) \quad (3)$$

where  $C_t$  is consumption of the manufactured good and  $D_t$  is consumption of biocarbon produced in the agricultural sector (food). The parameter  $\theta$  determines the relative taste for food.<sup>4</sup>

$B_t$  denotes the flow of biocarbon that is used as energy input in sector 1. Sector 1's energy input can also come from oil denoted by  $O_t$ , that is taken from a non-renewable and finite stock  $R_t$  without incurring extraction costs. In Section 5, we introduce fossil coal. Until then, we use the terms oil, fossil carbon and fossil fuel interchangeably.

$B_t$  and  $O_t$  are perfect substitutes. It is key for our analysis that there be a one-sided substitutability between biocarbon and oil. While biocarbon can be used for energy services, oil cannot be eaten. On the other hand, oil can be stored in ground at no cost while biocarbon must be consumed or used as energy immediately.

In addition to (1) and (2), the economy faces the following constraints:

$$\begin{aligned} Y_t &= C_t + K_{t+1}, F_t = D_t + B_t, E_t = O_t + B_t, \\ K_t &= K_{1,t} + K_{2,t}, L_t = L_{1,t} + L_{2,t}, \\ B_t &\geq 0, R_0 \geq \sum_{t=0}^{\infty} O_t, O_t \geq 0. \end{aligned} \quad (4)$$

The first constraint is the resource constraint for goods from sector 1. Manufacturing output is split between consumption and next period's capital stock. For analytical tractability, we assume that capital depreciates fully between periods. In our numerical examples, a period is 10 years, somewhat justifying the depreciation assumption.

The second equation states that output from agriculture (sector 2) is split between consumption and energy use. The third states that energy input stems from fossil carbon and the part of the biocarbon production that is not directly consumed as food. The fourth and fifth equations are, respectively, the aggregate resource constraints for capital and labor where the latter is exogenous. The sixth equation states that the use of biocarbon for energy has to be non-negative. This is a key restriction in the model. Allowing  $B_t < 0$  would imply that  $D_t > F_t$ , i.e., that consumers eat oil. Finally, there are non-negativity constraints on the stocks and flows of fossil carbon. The industrial technology for using fossil fuel is assumed to appear unexpectedly at time 0. We call the pre-industrial time phase 1, during which by assumption  $O_t = 0$ .

Markets are perfect so the planning allocation is identical to the competitive equilibrium.<sup>5</sup> The planning allocation is found by maximizing (3) subject to the production functions (1), (2) and the resource constraints (4).

The Kuhn–Tucker formulation of the planner's problem can be written as the maximization of

$$\sum_{t=0}^{\infty} \beta^t \left\{ \ln \left( A_{1,t} (K_t - K_{2,t})^{\alpha_1} (L_t - L_{2,t})^{\varepsilon_1} (O_t + B_t)^{\nu_1} - K_{t+1} \right) + \theta \ln (F_t - B_t) \right\} + \beta^t \left\{ \lambda_t^F \left( A_{2,t} K_{2,t}^{\alpha_2} L_{2,t}^{\varepsilon_2} G^{\nu_2} - F_t \right) + \lambda_t^B B_t + \lambda_t^O O_t \right\} + \lambda^R \left( R_0 - \sum_{t=0}^{\infty} O_t \right) \quad (5)$$

where  $\varepsilon_1 \equiv 1 - \alpha_1 - \nu_1$  and  $\varepsilon_2 \equiv 1 - \alpha_2 - \nu_2$ . The first-order conditions are listed in an appendix available on-line. The shadow value on the resource constraint for fossil fuel  $\lambda^R$  is strictly positive, since fossil fuel is always valuable for production and is assumed to be costless to extract. However, this is not the case for the current-value Kuhn–Tucker multipliers  $\lambda_t^B$  and  $\lambda_t^O$ . Key for the coming analysis will be whether they are strictly positive or not.

$\lambda_t^B > 0$  indicates that the constraint  $B_t \geq 0$  binds. The interpretation of such a situation is that in period  $t$ , the marginal value of biocarbon is higher than that of oil. Intuitively, if eating oil were possible, it would be optimal to do so. Since it is not possible, all biocarbon is used for direct consumption and nothing is burned, i.e.,  $B_t = 0$ .

<sup>3</sup> Based on Schnepf (2004), Desmet and Rossi-Hansberg (2015) calibrate the energy share in US agriculture to 5%. Stout (1990) provide evidence that the energy share in cereal output is substantially lower in developing countries than in developed.

<sup>4</sup> Preferences are homothetic and imply a constant income share of food given by  $\frac{\theta}{1+\theta}$ . Although somewhat unrealistic, this is a standard assumption. In the conclusion we comment on this, noting that in particular some interesting (and potentially worrisome) distributional aspects arise if we were to allow an income share of food that falls with income.

<sup>5</sup> The definition of a competitive equilibrium is standard and therefore omitted. Allowing market power in the oil market would not in itself change the analysis much since marginal revenues are proportional to the price in the current setting. If an oil monopolist also could affect the interest rate, this would lead to substantial complications. See Hassler et al. (2010) for a growth model with an oil producer with market power.

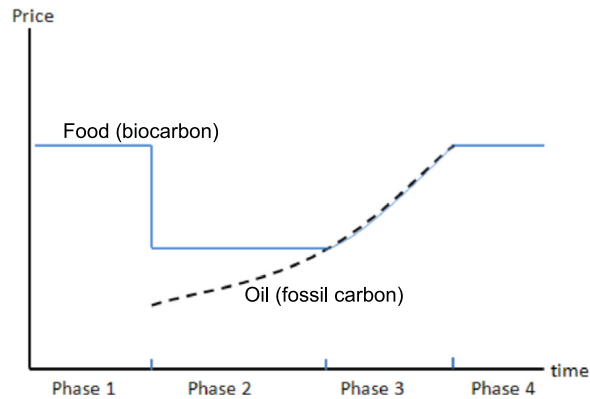


Fig. 3. Stylized paths of food price and oil price over the four model phases.

If  $\lambda_t^O > 0$ , the constraint  $O_t \geq 0$  binds. This indicates that shadow value of fossil carbon is higher than that of biocarbon. At first, this might seem to contradict the assumption that biocarbon is a perfect substitute for fossil carbon. But in contrast to biocarbon, fossil fuel can be stored.  $\lambda_t^O > 0$  implies that the non-storage constraint for biocarbon binds.

It is immediate to rule out that both constraints are binding, i.e., that both  $\lambda_t^B$  and  $\lambda_t^O$  are strictly positive, since then energy input in sector 1 is zero, implying that production and consumption of the manufactured good is zero. This cannot be optimal. Finally,  $\lambda_t^B = \lambda_t^O = 0$  is a possibility. Then, none of the constraint binds, implying that both fossil carbon and biocarbon are used as an energy input in sector 1.

### 3. Characterization of the allocation

Now turn to a characterization of the solution to the planner program. Since the planner solution will coincide with the allocation in a standard decentralized competitive equilibrium the notion of prices are used for relevant marginal products and marginal utilities.

After the technology for using fossil fuel has been discovered, the economy will move through a series of distinct phases, determined by which of the Kuhn–Tucker multipliers  $\lambda_t^B$  and  $\lambda_t^O$  is binding. In one of the phases, only fossil fuel will be used and the price of biocarbon will then be higher than the price of fossil carbon. Therefore, only the multiplier  $\lambda_t^B$  will be strictly positive.

In another phase both biocarbon and fossil carbon will be used as fuel. The share of biocarbon used as fuel will be determined such that the price of the two forms of fuel coincide. In this phase, both  $\lambda_t^B$  and  $\lambda_t^O$  will be zero.

Finally, there will be a phase when biocarbon will be the only source of fuel. Then  $\lambda_t^O > 0$ , while  $\lambda_t^B = 0$ . The interpretation of  $\lambda_t^O > 0$  is that at any time period in this phase, the price of fuel is strictly below the price that would have made it economically worthwhile to save oil for use in that period. This phase will resemble the initial phase before the discovery of the fossil fuel technology.

Under quite mild and realistic conditions described below, the economy will pass through the different phases in a particular order. Therefore, the phases are labelled by the numbers 1 through 4. The pre-industrial phase before the discovery of the fossil fuel technology is called phase 1. When the fossil technology is discovered, land, capital and labor used for the production of fuel can be released for other purposes. This reduces the scarcity of food, and its price therefore falls. Nevertheless, the fossil fuel price is below the food price during phase 2, provided the stock of fossil fuel is sufficiently large.<sup>6</sup> During phase 2, the fossil fuel price increases relative to the food price due to increased scarcity of the exhaustible stock of fossil carbon. Eventually, the fossil fuel price reaches the food price and phase 3 is entered. During this phase, the two prices rise jointly at the rate of interest. Under weak conditions this cannot continue forever. The existence of biocarbon as a backstop technology implies that eventually, all fossil reserves are exhausted and the economy enters the absorbing phase 4, when biocarbon is the single source of fuel. Abstracting from growth in technology and labor, the prices of food and oil develop schematically as shown in Fig. 3.

#### 3.1. Fuel phases

**Proposition 1.**<sup>7</sup> After discovery of the fossil fuel technology, the economy passes through a maximum of three distinct phases. If  $\lambda_t^B > 0$  and  $\lambda_t^O = 0$ , the economy is in phase 2 (Fossil) in period  $t$ . Fossil carbon is used ( $O_t > 0$ ) but no biocarbon is used as fuel ( $B_t = 0$ ). If  $\lambda_t^B = \lambda_t^O = 0$ , the economy is in phase 3 (Mixed) in period  $t$ . Fossil carbon is used ( $O_t > 0$ ) and some biocarbon is also

<sup>6</sup> If not, the economy enters into phase 3 immediately.

<sup>7</sup> Proofs that are not in the text are provided on-line at [http://hassler-j.iies.su.se/PAPERS/JME\\_2016App.pdf](http://hassler-j.iies.su.se/PAPERS/JME_2016App.pdf) or by request from the corresponding author.

used as fuel ( $B_t > 0$ ). If  $\lambda_t^B = 0$  and  $\lambda_t^O > 0$ , the economy is in phase 4 (Biocarbon) in period  $t$ . All fuel is in the form of biocarbon ( $O_t = 0, B_t > 0$ ). The economy has to be in phase 2 or 3 for at least one period.

To characterize the endogenous dynamic allocation of capital and labor as well as the use of biocarbon for its potential uses as food and fuel it is convenient to define the share of capital and labor used in biocarbon production as  $\kappa_t \equiv \frac{K_{2,t}}{K_t}$  and  $\Lambda_t \equiv \frac{L_{2,t}}{L_t}$ , respectively, the savings rate as  $s_t \equiv \frac{Y_t - C_t}{Y_t}$ , and the share of biocarbon used as fuel as  $\Phi_t \equiv \frac{B_t}{F_t}$ . Together with the use of fossil fuel, these four variables completely determine the endogenous dynamics of the model.

An optimal allocation requires that value of the marginal product of capital (output price times marginal product) is the same in both uses. Formally,  $\frac{1}{C_t} \frac{\alpha_1 Y_t}{K_{1,t}} = \frac{\theta}{D_t} \frac{\alpha_2 F_t}{K_{2,t}}$ , where the LHS is the product of the price (marginal utility) of the manufactured good times the marginal product of capital in sector 1. The RHS is the product of the price (marginal utility) of food times the marginal product of capital in sector 2. Quite intuitively, a high consumption rate ( $\frac{C_t}{Y_t} = 1 - s_t$ ) reduces the value of the marginal product of capital in sector 1. Similarly, a high consumption rate in sector 2 ( $\frac{D_t}{F_t} = 1 - \Phi_t$ ) reduces the same value in sector 2. Using the notation just introduced, the capital allocation condition is

$$\kappa_t = \frac{1}{1 + \frac{1 - \Phi_t}{1 - s_t} \frac{\alpha_1}{\theta \alpha_2}}. \quad (6)$$

The allocation of labor is determined by the condition that the value of the marginal product of labor should be equalized between the two sectors, yielding

$$\Lambda_t = \frac{1}{1 + \frac{1 - \Phi_t}{1 - s_t} \frac{\varepsilon_1}{\theta \varepsilon_2}}. \quad (7)$$

The savings rate is determined by the Euler condition,  $C_{t+1}/C_t = \beta \frac{\alpha_1 Y_{t+1}}{K_{1,t+1}}$ . Using the notation above, this becomes  $\frac{1 - s_{t+1}}{1 - s_t} = \beta \frac{\alpha}{s_t(1 - \kappa_{t+1})}$  which in the standard one-sector case when  $\kappa_{t+1} = 0$  yields the familiar solution  $s = \beta\alpha$ . Here,  $\kappa_{t+1} > 0$ , tends to raise savings since part of the savings is allocated to capital in sector 2. Furthermore, in contrast to the one-sector case, our model features non-trivial dynamics to the extent that  $\kappa_{t+1}$  varies over time. Using (6) shifted one period ahead, we can solve the Euler condition for  $s_t$ , yielding

$$s_t = \beta \frac{\alpha_1(1 - \Phi_{t+1}) + \theta \alpha_2(1 - s_{t+1})}{\beta \alpha_1(1 - \Phi_{t+1}) + (1 - \Phi_{t+1} + \beta \theta \alpha_2)(1 - s_{t+1})}. \quad (8)$$

Next, optimality when some biocarbon is used as energy input in sector 1 ( $\lambda_t^B = 0$ ), requires that the value of the marginal product of energy in sector 1 equals the alternative use of biocarbon as food:

$$\frac{1}{C_t} \nu_1 \frac{Y_t}{E_t} = \frac{\theta}{D_t}. \quad (9)$$

Finally, we have the Hotelling condition that conditional on oil being used in periods  $t$  and  $t + 1$ , ( $\lambda_{t+1}^O = \lambda_t^O = 0$ ), the gross increase in the value of the marginal product of energy should equal the gross interest rate.<sup>8</sup> Using the Euler condition to substitute for the latter, the Hotelling condition yields

$$\frac{O_{t+1} + B_{t+1}}{O_t + B_t} = \frac{\beta(1 - s_t)}{1 - s_{t+1}} \quad (10)$$

**Proposition 2.** Define the price of fuel as the marginal product of energy  $\frac{\nu_1 Y_t}{E_t}$  and the price of biocarbon as the ratio of the marginal utility of food to the marginal utility of the manufactured good  $\frac{\theta C_t}{D_t}$ . In phase 2, the price of fuel grows at a rate identical to the marginal product of capital (the interest rate). The price of energy is below the price of food in phase 2, and in phases 3 and 4, the two prices are equal.

That the price of fuel grows at the interest rate is an implication of the Hotelling result. When both capital and the exhaustible resource can be used as a store of value, an interior solution requires their respective returns to be identical.

<sup>8</sup> Recall that biocarbon cannot be stored. The Hotelling condition is therefore not required for optimality if oil is not used.



**Proposition 3.** In phases 2 and 4, there are balanced growth paths such that in phase 2,  $\Phi_t = 0$ ,  $s_t = s_F \equiv \beta \frac{\alpha_1 + \theta \alpha_2}{1 + \beta \theta \alpha_2}$ ,  $\Lambda_t = \Lambda_F \equiv \left(1 + \frac{\epsilon_1}{\theta \epsilon_2 (1 - s_F)}\right)^{-1}$ ,  $\kappa_t = \kappa_F \equiv \left(1 + \frac{\alpha_1}{\theta \alpha_2 (1 - s_F)}\right)^{-1}$ , and in phase 4,  $\Phi_t = \Phi_B \equiv \frac{\nu_1}{\nu_1 + \theta (1 - s_B)}$ ,  $s_t = s_B \equiv s_F + \frac{\beta \alpha_2 \nu_1}{1 + \beta \theta \alpha_2}$ ,  $\Lambda_t = \Lambda_B \equiv \left(1 + \frac{(1 - \Phi_B) \epsilon_1}{\theta \epsilon_2 (1 - s_B)}\right)^{-1}$  and  $\kappa_t = \kappa_B \equiv \left(1 + \frac{(1 - \Phi_B) \alpha_1}{\theta \alpha_2 (1 - s_B)}\right)^{-1}$ .

**Proof.** Follows directly from Eqs. (6)–(10).  $\square$

The savings rate is higher in a phase 4 balanced growth path. The increase is proportional to the product of  $\alpha_2 \nu_1$ . This is intuitive since a higher capital share in sector 2 and a higher energy share in sector one leads to a larger increase in the need for capital when phase 4 is entered. The difference in capital and labor allocations between the two phases is directly related to the share of biocarbon used as fuel in phase 4 ( $\Phi_B$ ).

**Proposition 4.** If one or both of the phases 2 and 4 are absorbing, their respective balanced growth paths are attained at the first period the economy is in the absorbing phase.

**Proof.** The law-of-motion for the savings rate is unstable at any steady state with a root  $\frac{ds_{t+1}}{ds_t} = \frac{1}{\alpha_1 \beta} > 1$ .  $\square$

An important finding is that the endogenous variables defined in Proposition 3 are independent of the growth rates of the technology trends  $A_1$  and  $A_2$ . This will be used when the climate module is appended to the model.

### 3.2. Ordering of the phases

**Proposition 5.** If technological growth rates are constant and imply that the growth rate of biocarbon output  $\gamma_F > \ln \beta$  in balanced growth, phase 2 is transitory while phase 4 is absorbing and always in balanced growth with a constant saving rate and constant allocation shares of capital, labor and biocarbon ( $s_t = s_B$ ,  $\kappa_t = \kappa_B$ ,  $\Lambda_t = \Lambda_B$  and  $\Phi_t = \Phi_B$ ).

Before discussing the intuition for the result, note that  $\ln \beta < 0$ , showing that the condition is weak.

To understand why phase 2 is transitory, assume the opposite. Then the price of fossil fuel has a growth rate given by the difference of the growth rate of manufacturing output and the growth rate of fossil fuel use, i.e.,  $\gamma_Y - \gamma_O$ .<sup>9</sup>  $\gamma_O$  is given by Eq. (10), which here implies that  $\gamma_O = \ln \beta$ , which is negative. The growth rate of the price of food, instead, is given by the difference between the growth rate of manufacturing output and biocarbon output, i.e., by  $\gamma_Y - \gamma_F$ . Thus, food prices grow slower than fuel prices if  $\gamma_F > \ln \beta$ . When the fossil price grows faster than the biocarbon price, eventually the former must reach the latter and phase 2 ends. During phase 3 the fuel price continues to rise, now together with the biocarbon (food) price. The price increase of biocarbon is driven by a falling use of fossil carbon that leads to a growing share of biocarbon used as fuel. However, this increase is bounded by the fact that also when no fossil fuel is used, the share of biocarbon used as fuel is interior implying a finite biocarbon price. Thus, phase 3 is also transitory.

Finally, under exactly the same condition as made in phase 2 transitory, the fuel (and food) price grows more slowly in phase 4 than the price that would be required for it to be worthwhile to store fossil fuel for use in some period in phase 4. Under these conditions, phase 4 is absorbing and fossil fuel will never be used again. All fossil fuel reserves are exhausted during phases 2 and 3, since it is obviously not optimal to save reserves that will never be used ( $\lambda^R > 0$ ). Finally, we note that we can easily add the pre-industrial phase 1, in which fossil fuel (or the technology to use fossil fuel) has not yet been discovered. In key respects, it is identical to phase 4, where the fossil fuel is exhausted. In that sense, *the fossil era*, consisting of phases 2 and 3 (or only a phase 3), is a parenthesis, *just an episode in history*.

## 4. Quantitative implications

Let us now analyze the dynamics of food prices along the transition to the biocarbon phase. We normalize the aggregate labor supply to unity and disregard population growth.<sup>10</sup> Also normalize the supply of land,  $G$ , to unity. Clearly, the price dynamics in general depends on how the capital stock and technology develop during the transition. Specifically, given the allocation variables ( $s_t, \kappa_t, \Lambda_t$  and  $\Phi_t$ ), the ratio of prices in period  $t'$  and  $t$  is proportional to

$$\frac{A_{1,t'}}{A_{1,t}} \left( \frac{A_{2,t'}}{A_{2,t}} \right)^{1-\nu_1} \left( \frac{K_{t'}}{K_t} \right)^{\alpha_1 - \alpha_2(1-\nu_1)}. \quad (11)$$

Thus, if  $\gamma_{A_1} > (<) (1 - \nu_1) \gamma_{A_2}$ , technological developments tend to push food prices up (down). Similarly, if  $\frac{\alpha_1}{1-\nu_1} > (<) \alpha_2$ , i.e., capital's share of income net of energy payments in manufacturing good production is higher (lower) than capital's income share in biocarbon production, capital accumulation tends to increase (reduce) the food price by making biocarbon relatively more scarce. Let us now focus on the effect on food prices of the transition from phase 2 to 4, disregarding the direct effect of technology and capital accumulation. Thus, hold technology and capital constant and focus on the effect on

<sup>9</sup> We define the growth rate of a variable  $x$  as  $\gamma_x \equiv \ln \left( \frac{x_t}{x_{t-1}} \right)$ . To denote balanced growth rates, we omit the time subscript.

<sup>10</sup> Of course, population growth may itself be important for the development of food prices. But this is not the focus of this paper.

the food price by the change in demand for fuel due to the increase in the share of biocarbon used as energy input from zero to  $\Phi_B$ .<sup>11</sup>

The food price in phase 2 depends on fossil fuel use. Consider the situation at the end of phase 2, when food and energy prices have become equal, implying

$$\frac{\theta(1-s_t)Y_t}{F_t} = \frac{\nu_1 Y_t}{O_t} \Rightarrow O_t = \frac{\nu_1 F_t}{\theta(1-s_t)}. \tag{12}$$

Using (12) and the production functions allow us to compute the ratio of food prices in phase 4 and at the end of phase 2. It is straightforward to show that (apart from the proportionality factor (11)) this ratio is given by

$$\frac{1}{1-\Phi_B} \left( \frac{1-\kappa_B}{1-\kappa_t} \right)^{\alpha_1} \left( \frac{\kappa_B}{\kappa_t} \right)^{-\alpha_2(1-\nu_1)} \left( \frac{1-\Lambda_B}{1-\Lambda_t} \right)^{\varepsilon_1} \left( \frac{\Lambda_B}{\Lambda_t} \right)^{-\varepsilon_2(1-\nu_1)} \frac{1-s_B}{1-s_t} \left( \frac{\theta(1-s_t)\Phi_B}{\nu_1} \right)^{\nu_1} \tag{13}$$

where  $s_t, \kappa_t$  and  $\Lambda_t$  denote the savings rate, capital and labor allocations at the end of phase 2.<sup>12</sup>

Several things are worth noting in this expression. For reasonable parameters, the two last factors turn out to be close to unity. Comparing the steady state savings rates in Proposition 3, we see that the savings rate increases by an amount smaller than  $\beta \frac{\alpha_2 \nu_1}{1+\beta \theta \alpha_2}$  over the transition. Since  $\nu_1$  is on the order of a few percent, this is small. For the same reason the last factor is also close to unity. The first term  $\frac{1}{1-\Phi_B}$  can then be interpreted as the direct effect on the food price of the increased fuel demand occurring during phase 3. If capital and labor are not allowed to relocate towards biocarbon production, all the four factors involving the allocation variables  $\kappa$  and  $\Lambda$  would be unity and the price increase would be given (approximately) by  $\frac{1}{1-\Phi_B}$ . In our baseline benchmark calibration  $\Phi_B$  will be around 0.71, i.e., in phase 4, 71% of biocarbon output is used as fuel. Thus, in absence of endogenous capital and labor relocation, the increase in the demand for biocarbon to be used as fuel would imply that the price of food would increase by a factor  $\frac{1}{0.29} \approx 3.45$ , i.e., an increase by close to 250%!

The remaining four terms in (13) mitigate the price increase. Since capital is moving towards biocarbon production,  $\kappa_B > \kappa_t$ . This reduces the price increase of food by making  $\left( \frac{1-\kappa_B}{1-\kappa_t} \right)^{\alpha_1}$  and  $\left( \frac{\kappa_B}{\kappa_t} \right)^{-\alpha_2(1-\nu_1)}$  smaller than unity. Similarly, moving labor to biocarbon production implies that  $\Lambda_B > \Lambda_t$ , makes the factors  $\left( \frac{1-\Lambda_B}{1-\Lambda_t} \right)^{\varepsilon_1}$  and  $\left( \frac{\Lambda_B}{\Lambda_t} \right)^{-\varepsilon_2(1-\nu_1)}$  smaller than unity to an extent determined by the labor shares.

Finally, note that the expression for the price increase in (13) implicitly determines the duration of phase 3. During phase 3, the price of biocarbon grows at the rate of interest. Given an interest rate, the price increase over phase 3 therefore determines the duration of phase 3. However, while the formula in (13) is independent of technological growth rates, clearly this is not the case for interest rates. Specifically, the interest rate tends to be higher the higher the growth rate of technology. Thus, while exogenous growth does not enter (13), it affects the duration of phase 3.

Let us now calibrate  $\alpha_1 = \alpha_2 = 0.3, \beta = 0.99$ <sup>10</sup> and study how the other parameters affect the price ratio. As a benchmark, the income share of energy  $\nu_1$  is set to 0.05 and the income share of land  $\nu_2$  to 0.3. The parameter  $\theta$  is set to match the world share of GDP in agriculture, which was 2.8% in 2010.<sup>13</sup> Noting that the income share is  $\theta/(1+\theta)$ , this yields  $\theta = 0.029$ .

Without a closed form expression for  $s_t$  in the last period of phase 2, an exact expression for the price increase cannot be derived. However,  $s_B > s_t > s_F$  since the positive root and unstable root in the dynamic equation for savings implies monotone convergence. This provides a narrow interval for the price increase. Using  $s_t = s_F$  gives a price increase of 40.1%, while if instead  $s_t = s_B$  the price increase is 40.8%. We proceed the analysis under the approximation  $s_t \approx s_F$ , which also pins down  $\kappa_t$  and  $\Lambda_t$  from their formulas in (6) to  $\kappa_F$  and  $\Lambda_F$ .

As already noted, the direct effect, given by  $\frac{1}{1-\Phi_B}$ , implies a price increase of 243% since  $\Phi_B = 0.7085$ . The mitigation of this effect is largely due to the increase in  $\kappa$  from  $\kappa_F = 2.05\%$  to  $\kappa_B = 6.59\%$  and  $\Lambda$  from  $\Lambda_F = 1.27\%$  to  $\Lambda_B = 4.16\%$ . The effect of these two factors alone through the factor  $\left( \frac{\kappa_B}{\kappa_F} \right)^{-\alpha_2(1-\nu_1)} \left( \frac{\Lambda_B}{\Lambda_F} \right)^{-\varepsilon_2(1-\nu_1)}$  is to reduce the price increase to 56.8%.

Let us now do some sensitivity analysis. In particular, with respect to how the food price increase during phase 3 responds to variation in the energy share in sector 1 ( $\nu_1$ ) and the income share of food in consumption ( $\theta$ ). Fig. 4 shows that the price increase vs. these two parameters holding the other parameters constant.<sup>14</sup> The price increase is increasing in the income share of energy in sector 1. This is not surprising since a higher income share of energy leads to a larger increase in the demand for biocarbon as fossil fuel is exhausted. However, the sensitivity is (perhaps surprisingly) moderate – a doubling of the income share to 10% leads to a price increase of 57% rather than 40%.

The other curve in Fig. 4 shows that the price increase falls in the income share of food. With a high income share of food, a large share of capital and labor is allocated to the biocarbon sector already in phase 2 and the extra increase in the demand

<sup>11</sup> Alternatively, assume  $\gamma_{A_c} = (1-\nu_1)\gamma_{A_f}$  and  $\alpha_1 = (1-\nu_1)\alpha_2$ , in which case technology and capital have no effect on the food price.

<sup>12</sup> Inspection of the formulas for  $\kappa_t, \kappa_B, \Lambda_t$ , and  $\Lambda_B$  in (6) and Proposition 3, shows that  $\kappa_t < \kappa_F$  and  $\Lambda_t < \Lambda_F$ . This follows from the fact that  $s_B > s_F$  and that the positive root in the law-of-motion for the savings rate implies a monotone convergence to the higher savings rate. The forward looking behavior of savings implies that capital and labor allocations start moving slightly already before phase 3 is entered. However, in our calibration, we show that  $s_t \approx s_F, \kappa_t \approx \kappa_F$  and  $\Lambda_t \approx \Lambda_F$  to the third decimal point.

<sup>13</sup> Source: Worldbank, data downloaded from <http://data.worldbank.org/> October 8, 2012. The value includes forestry.

<sup>14</sup> We maintain constant returns to scale by changing  $\varepsilon_1$  as we change  $\nu_1$ .



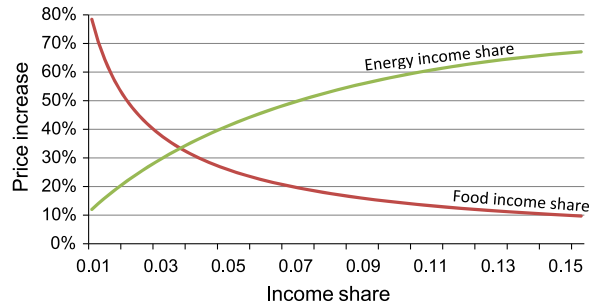


Fig. 4. Price increase of food during phase 3 as functions of food and energy income shares, respectively.

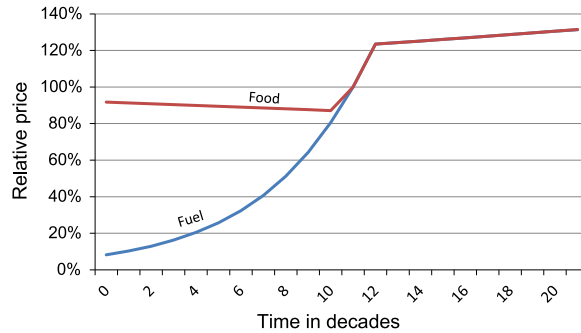


Fig. 5. Food and fuel prices relative to their values at the start of phase 3.

for biocarbon therefore has small effects. At low income shares, the price increase can be quite substantial, implying a long transition period. If, for example, we calibrate  $\theta$  to the US income share of agriculture of 1.2%, the price increase is a hefty 70.2%.

It is also straightforward that the price increase is higher, the higher the income share of land is. A higher income share of the fixed factor implies faster declining marginal products when output is increased by moving labor and capital to the sector. Although standard estimates of this income share are fairly low, higher values may be reasonable if one takes into account that higher demand for biocarbon leads to less productive land being taken into use.<sup>15</sup>

Finally, note that the model is able to generate quite sizeable increases in the price of food if the income share of food is low and the energy and land shares are high. Setting  $\theta = 0.012$ ,  $\nu_1 = 0.08$ , and  $\nu_2 = 0.5$ , which arguably is somewhat extreme, gives a price increase of 187%. In this case, over 90% of biocarbon production would be used as fuel in phase 4.

#### 4.1. Price dynamics

Our model easily lends itself to calibrated simulations. The details of this are discussed in the appendix available on-line. Here, we simply note that under the weak assumptions specified above, phase 4 is absorbing. Once phase 4 is reached, the economy evolves along the balanced growth path given by Proposition 3. Thus, backward induction from the beginning of phase 4 is straightforward. The balanced growth path in phase 2 is approached asymptotically backward in time.

Fig. 5 shows the evolution of food and fuel prices. Both prices are expressed relative to their values at the start of phase 3. The technological growth rates are set to  $\gamma_{A_1} = \gamma_{A_2} = 1.1$  per decade,  $A_{1,0} = A_{2,0} = 1$  and  $K_0$  so that the marginal product of capital is approximately constant from the beginning of the simulation period. Interest rates during the transition are then between 2.2% and 2.5% per year. The economy is in phase 2 until period 10. During this phase, energy prices grow at the rate of interest (on average by 2.3% per year) while food prices fall very slowly (about 0.05% per year). Phase 3 takes two decades and during this, prices of energy and food rise in parallel by 42% from the last period of phase 2 to the first period of phase 4.<sup>16</sup> During phase 4, biocarbon prices increase slowly at close to 0.1% per year.

<sup>15</sup> Land share in agriculture is 0.18 in Herrendorf and Valentinyi (2008).

<sup>16</sup> The transition to the fossil fuel-free economy is likely to be longer in reality, of course. The discrepancy is due to the fact that we disregard fossil fuel sources with non-negligible extraction costs. We briefly discuss the consequences of this omission in the conclusion.

## 5. Biocarbon and the climate

Proponents of biofuel argue that it provides a powerful possibility of slowing down global warming. Indeed, biofuel is by far the most important green energy source in the OECD countries, accounting for 55% of all green energy used in the OECD countries (see [Sinn, 2012](#), chapter 2). Antagonists, however, point out that the use of biofuel has increased the food price and should therefore be banned. Our model can easily be extended so that it can be used to analyze this trade-off. In order to do that, we must add specifications of two mechanisms. First, a specification of how fossil fuel emissions affect the path of the atmospheric CO<sub>2</sub> concentration (carbon circulation) is needed. Second, a mapping from the path of CO<sub>2</sub> concentration to a measure of climate change must be specified. The accumulation of CO<sub>2</sub> in the atmosphere is assumed to affect the productivity trends  $A_1$  and  $A_2$ . As demonstrated above, private choices will then be identical to the solution to the central planner solution where the externality is ignored. Therefore, the economic model described above need not be changed and we can find the decentralized solution to the model by solving the central planner solution ignoring any climate externalities.<sup>17</sup>

### 5.1. Climate components

The specification of the carbon cycle is as in [Golosov et al. \(2014\)](#). There is a linear relation between fossil fuel use and the stock of atmospheric carbon,<sup>18</sup>

$$S_t = \sum_{s=0}^{t+T} (1 - d_s)(O_{t-s} + X_{t-s}) \tag{14}$$

where  $d_s \in [0, 1]$  for all  $s$ . Here,  $X_{t-s}$  is a sequence of exogenous emissions of greenhouse gas not coming from the use of  $O_t$ , and  $1 - d_s$  represents the share of a unit of carbon remaining in the atmosphere  $s$  periods after it was emitted. As discussed in detail in [Archer \(2005\)](#) and [Golosov et al. \(2014\)](#), a reasonable approximation to the carbon cycle is a process under which a share  $\varphi_L$  of carbon emitted into the atmosphere stays there forever, a share  $1 - \varphi_0$  of the remainder quickly exits the atmosphere into the biosphere and the surface oceans, and the remaining part decays at a geometric rate  $\varphi$ . Such a process implies that  $1 - d_s$  can be represented as

$$1 - d_s = \varphi_L + (1 - \varphi_L)\varphi_0(1 - \varphi)^s, \tag{15}$$

The so-called Arrhenius relation

$$Z_{t+1} = Z(S_t) = \lambda \ln \left( \frac{S_t}{\bar{S}} \right) / \ln 2, \tag{16}$$

where  $Z_t$  is global mean temperature above the pre-industrial steady state and  $\bar{S}$  is the pre-industrial atmospheric stock of CO<sub>2</sub> determines global warming. This is a simplified version of the climate module used by [Nordhaus \(2008\)](#). Compared to Nordhaus, we abstract from dynamic effects due to a drag on temperature induced by the fact that it takes longer to heat the oceans than to heat the atmosphere. However, except for quite extreme scenarios, these dynamics are not likely to be very important. A standard value for the climate sensitivity parameter  $\lambda$  here is 3.0 °C. Thus, we assume that a doubling of the stock of atmospheric carbon leads to a 3 °C increase in the global mean temperature.

### 5.2. Additional calibration

Again following [Golosov et al. \(2014\)](#),  $\varphi = 0.0228$ ,  $\varphi_L = 0.2$ , and  $\varphi_0 = 0.393$  implying that 50% of an emitted unit of carbon has been absorbed by oceans and plants after three decades, 20% remains for ever in the atmosphere and the remainder is slowly absorbed by deep oceans with a half-life of 300 years. The pre-industrial stock of atmospheric CO<sub>2</sub> is 581 GtC (gigaton of carbon). Furthermore, we set the current value of CO<sub>2</sub> in the atmosphere to 802 GtC of which 103 is not depreciating (see [Golosov et al., 2014](#)). In order to capture other sources of greenhouse gas emissions, we set  $X_t = 6$  GtC per year the first decade in line with actual global use ([IEA, 2010](#) reports a global use of coal and gas in 2008 of 5.9 GtC). We assume this increases to 7.5 GtC per year the second decade and then to 10 GtC per year for three decades. After that, exogenous emissions are assumed to fall to 7.5 GtC per year for three decades and then to 5. The exogenous carbon emissions increase the temperature in all scenarios and also slightly reduce the difference between the temperature impacts in the studied scenarios. This is due to the logarithmic relation between CO<sub>2</sub> concentration and temperature in (16), implying that the marginal effect of carbon emissions on temperature is lower at higher CO<sub>2</sub> concentrations.

<sup>17</sup> By specifying the climate externality we could easily find the optimal tax on fossil fuel using the optimal tax formula in [Golosov et al. \(2014\)](#). A more ambitious task would be to solve a constrained optimization problem where we specify a climate externality but restrict the policy options of the planner so that the first best cannot be achieved. For example, we could ask the question of what is the optimal policy with respect to biocarbon use if the fossil carbon tax is set suboptimally? We leave this for future work.

<sup>18</sup> We measure the atmospheric CO<sub>2</sub> concentration in terms of the global stock of atmospheric carbon  $S_t$ , measured in GtC (gigatons of carbon). It may be noted that 2.12 atmospheric GtC corresponds to 1 ppm atmospheric CO<sub>2</sub>.

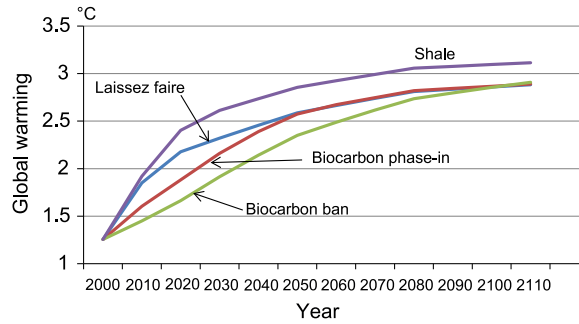


Fig. 6. Global warming in degrees Celsius over preindustrial in different scenarios.

The simulation assumes that the economy entered phase 3 in the previous decade (2000–10) and that the experiments start in the current (2010–2020). Finally,  $R_0$ , the current the current stock of fossil fuel reserves below ground, is set to 300 GtC. As discussed in Golosov et al. (2014), this is well in between current estimates of, on the one hand, estimates of existing oil that is economically profitable to extract at current economic and technical conditions, and on the other, estimates of total reserves.<sup>19</sup> All other parameters are unchanged.

### 5.3. Experiments

Our model is used for three experiments. Two involve policies towards the use of biofuels and the third quantifies the consequences of finding more oil. The latter is motivated by the recent improvement in hydraulic fracturing (“fracking”) that has increased the effective supply of oil.

The first experiment is a ban on the use of biocarbon as fuel. The second involves a milder restriction, namely  $\Phi_t \leq \bar{\Phi}$  for all  $t$  when fossil carbon is still used. If  $\bar{\Phi} > 0$ , biocarbon will eventually replace fossil fuel and we allow the restriction to be lifted when all fossil fuel is used up. The second experiment is labelled a *biocarbon phase-in*. In both cases, the restriction is introduced unexpectedly just as the economy were to enter phase 3. Arbitrarily, we set  $\bar{\Phi} = 0.3$ .

Solving the constrained problem is straightforward. Under the constraint  $\Phi_t = 0 \forall t$ , the economy is simply in phase 2 forever, with  $s_t = s_F$ ,  $\lambda_t = \lambda_F$  and  $\kappa_t = \kappa_F$  as defined in Proposition 3. Fossil carbon use satisfies the Hotelling equation, which simplifies to  $\frac{\partial \lambda_t}{\partial t} = \beta$ . Compared to the unconstrained case, the price of fossil carbon must increase when the constraint is unexpectedly introduced. The increase is necessary to reduce resource extraction at all points in time so as to exclude exhaustion in finite time. Given that the extraction of fossil carbon slows down, it is clear that the ban on biocarbon use as fuel will slow down climate change. Not a subsidy, but a ban on biofuel will be better for the climate. This is a variant of the Green Paradox (Sinn, 2008, 2012), according to which measures that restrict the future demand for fossil fuel will accelerate global warming whereas measures that stimulate the future demand for fossil fuel will slow it down. In our model, phase 4, with its biocarbon-only regime, acts as an endogenously determined backstop technology. Banning this technology means increasing the future demand for fossil fuel, which provides an incentive to postpone fossil carbon sales.

Under the second restriction, the economy goes through distinct phases as in the unconstrained case. If there is a sufficient amount of fossil fuel reserves still in ground when the restriction is introduced, the fossil fuel price is initially below the biocarbon price but grows over time at the rate of interest. Eventually, the price of fossil fuel reaches the biocarbon price and then phase 3 is reached and  $\Phi_t$  becomes strictly positive, i.e., some biocarbon is used as fuel. The price of biocarbon and fossil carbon grow together at the rate of interest.  $\Phi_t$  then increases over time and eventually  $\Phi_t \leq \bar{\Phi}$  becomes binding. When this happens, Eqs. (6), (8) and (10) hold with  $\Phi_t = \bar{\Phi}$ . This, together with Eq. (10), determines the dynamic path of the economy. When the constraint  $\Phi_t \leq \bar{\Phi}$  binds, fuel prices are higher than food prices and the former continue to grow at the rate of interest. Fossil fuel will be fully exhausted in finite time but the date of fossil carbon exhaustion is postponed relative to the unconstrained case, which again is an example of the Green Paradox.

The third experiment is to unexpectedly increase the stock of oil in ground by 47%. This value is chosen in order to be of comparable size as the addition to the oil stock due to recent estimates of technically recoverable shale gas and shale oil resources.<sup>20</sup> In reality, the shale revolution was of course not perfectly unanticipated and an important fact about shale gas and shale oil is that they are relatively expensive to extract. Both these features would tend to dampen the real world effects of the shale revolution.

Fig. 6 shows the global mean temperature in four scenarios: laissez faire, biocarbon ban, biocarbon phase-in and the increase in (shale) reserves. The two constrained policies lead to a substantial reduction in the speed of global warming. Our

<sup>19</sup> Recall that we do not model endogenous coal use here. Coal exists in much larger supply and has a price that, at least currently, is largely determined by extraction costs rather than Hotelling rents. Including endogenous coal use in the analysis is potentially important but left for future work.

<sup>20</sup> See EIA (2013). We set the emissions of shale gas and oil to be equal to that of oil per unit of energy. In reality, shale gas has a lower carbon content than conventional oil. On the other hand, shale gas leads to emissions of other greenhouse gases. Since the purpose of exercise is not to give an exact description of the consequences of the shale revolution we abstain from dealing with these consequences.

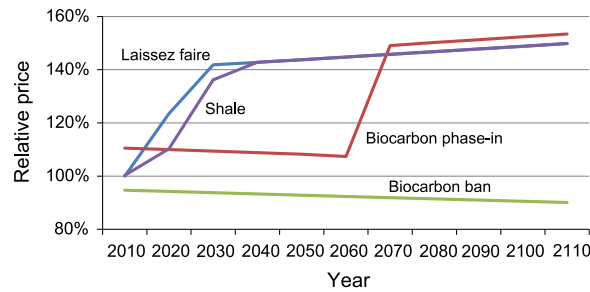


Fig. 7. Food price relative to its laissez faire value at start of phase 3 in different scenarios.

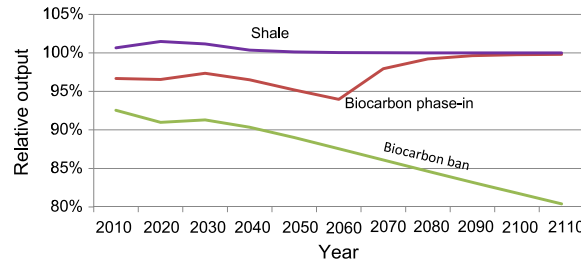


Fig. 8. Manufacturing output relative to laissez faire in different scenarios.

model, as well as most other standard climate models, somewhat overpredicts the current global mean temperature. A likely reason for this is the omission of the effect of short-lived aerosols that reflect incoming solar radiation (see Schwartz et al., 2010). During the initial decades, the biocarbon phase-in leads less global warming in the order of one third degree Celsius relative to laissez faire, while the corresponding figure for the complete ban is about half a degree. The increase in oil reserves leads to accelerated global warming. The temperature rises faster and is close to 0.3° higher than without the addition to oil reserves after a few decades. In the longer run, the increase is slightly smaller, about 0.23°.

Fig. 7 shows the food price relative to the price prevailing at the switch from phase 2 to 3 under laissez faire. Under the complete ban on using biocarbon as fuel, the food price fall initially and continues to fall over the whole simulation period. Under the biocarbon phase-in, the food price in the initial period is actually slightly higher than in laissez faire. The reason for this is that the expected future restrictions on biocarbon for fuel leads to an increase in the current fuel price. This in turn leads to a small increase in the share of biocarbon for fuel  $\Phi_{2010}$  up to the limit  $\bar{\Phi}$ , implying higher food prices. After this fairly modest increase, food prices remain almost constant until the phase of mixed use of biocarbon and fossil fuel is over in 2070.

With more oil, the food price increase is delayed by approximately a decade. Interestingly, although there is almost 50% more oil in the shale scenario, oil use increases only by 14% in the first decade. This contrasts sharply with the standard Dasgupta–Heal model where oil use is proportional to the size of remaining oil reserves. The lower response is due to the fact that biocarbon is an endogenous backstop for oil in our model. More oil and therefore lower current prices postpones the time at which biocarbon becomes competitive as energy input. The stock of oil must therefore be spread over a longer period.

Fig. 8 shows output  $Y_t$  relative to laissez faire. The biocarbon ban leads to quite substantial losses, in particular towards the end of the simulation, while the phase in leads to more modest losses. Recall, however, that we have not specified the climate externality here. Thus, any gains in output due to slower global warming are disregarded. To analyze whether these losses are worth taking in order to achieve a slower global warming depends crucially on assumptions on the costs of climate change. This is left for future work.

## 6. Concluding remarks

A simple two-sector growth model with two potential sources of energy has been constructed to analyze the dynamic relation between food and energy prices in general equilibrium. The model was used to study consequences of policies related to the use of biocarbon to substitute for fossil fuel and of unexpected increases in the supply of fossil fuel. To keep the model tractable and transparent we have abstracted from costs associated with the extraction of fossil fuel. This of course is not realistic, particularly not for coal. Coal exists in substantially larger quantities than cheap oil and its extraction costs are currently high relative to the coal price. Furthermore, the price of coal per unit of energy is much lower than for oil and biocarbon. The inclusion of stock-dependent extraction costs is a potentially interesting avenue for future research. We conjecture that adding extraction costs will increase the length of the transition phase 3 in which both fossil and biocarbon

are used. The reason for this is that the Hotelling–Solow–Stiglitz rule implies that the price net of the unit extraction cost should rise at the rate of interest rate, which allows a rate of gross price increase below the rate of interest. With a lower rate of price increase, the transition phase between the fossil-carbon-only phase and the biocarbon-only-phase is longer.

Let us finish by shortly discussing distributional issues, obviously also absent from our representative agent framework. Due to the assumption of homothetic preferences, the model is compatible with an uneven allocation of property rights in land, capital, labor and the stock of the fossil fuel resource, provided everyone holds these endowments in equal proportions. However, although homotheticity is a convenient and standard assumption in growth theory, it may be an imperfect description of reality. Income shares for food tend to be negatively associated with income. If the price of food in phase 3 keeps rising, this will then have more problematic implications for the well-being of individuals poor countries. This would strengthen the case against the use of biocarbon for energy purposes.

## Acknowledgments

We are grateful for comments from Per Krusell and Rick van der Ploeg as well as from seminar participants in Barcelona, Cambridge, Edinburgh, London, Madrid, Izmir, Oxford, Sigtuna, Stockholm and Umeå. We thank Wolfgang Meister and Julio Saavedra for excellent editorial assistance. The referees and the editor of this journal have provided very important input. John Hassler acknowledges financial support from Mistra-SWECIA.

## Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at <http://dx.doi.org/10.1016/j.jmoneco.2016.08.001>.

## References

- Archer, D., 2005. The fate of fossil fuel CO<sub>2</sub> in geologic time. *J. Geophys. Res.* 110 (C9).
- Avolos, F., 2014. Do oil prices drive food prices? The tale of a structural break. *J. Int. Money Finance* 42, 253–271.
- von Braun, J., 2008. High Food Prices: The What, Who and How Proposed Policy Actions. International Food Policy Research Institute. IFPRI, Washington DC.
- Dasgupta, P., Heal, G., 1974. The optimal depletion of exhaustible resources. *Rev. Econ. Stud.* 41, 3–28.
- Desmet, K., Rossi-Hansberg, E., 2015. On the spatial economic impact of global warming. *J. Urban Econ.* 88, 16–37.
- EIA, 2013. Technically Recoverable Shale Oil and Shale Gas Resources: An Assessment of 137 Shale Formations in 41 Countries Outside the United States. U.S. Energy Information Administration.
- Golosov, A., Hassler, J., Krusell, P., Tsyvinski, A., 2014. Optimal taxes on fossil fuel in general equilibrium. *Econometrica* 82 (1), 41–88.
- Hassler, J., Krusell, P., Olovsson, C., 2010. Oil monopoly and the climate. *Am. Econ. Rev.: Pap. Proc.* 100 (2), 460–64.
- Herrendorf, B., Valentinyi, A., 2008. Measuring factor income shares at the sector level. *Rev. Econ. Dyn.* 11, 820–835.
- IEA, 2010. Key World Energy Statistics. International Energy Agency, Paris.
- Nordhaus, W., 2008. *A Question of Balance: Weighing the Options on Global Warming Policies*. Yale University Press, New Haven, CT.
- Penn State, College of Agricultural Sciences, 2012. (<http://extension.psu.edu/energy/field-crops/shelled-corn/heat-energy-content-of-shelled-corn>), downloaded November 20, 2012.
- Schwartz, S., Charlson, R., Kahn, R., Ogren, J., Rodhe, H., 2010. Why hasn't earth warmed as much as expected? *J. Clim.* 23, 2453–2464.
- Schnepf, R., 2004. Energy Use in Agriculture: Background and Issues. CRS Report for Congress. Congressional Research Service.
- Sinn, H.-W., 2008. Public policies against global warming: a supply side approach. *Int. Tax Publ. Finance* 15, 360–394.
- Sinn, H.-W., 2012. *The Green Paradox. A Supply Side Approach to Global Warming*. MIT Press, Cambridge, Mass.
- Stout, B.A., 1990. Handbook of Energy for World Agriculture. Elsevier Science Publishers, London (Chapter II).